# ON THE BIRATIONALITY OF PLURICANONICAL MAPS OF THREEFOLDS AND FOURFOLDS OF LARGE VOLUME

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ABSTRACT. A celebrated result of Hacon-McKernan and Takayama asserts that, for every $n \geq 1$, there exists an integer $m_n$ such that, for any smooth complex projective variety $X$ of general type, the $m$-th pluricanonical map $\varphi_m : X \dashrightarrow \mathbb{P}H^0(mK_X)$ is birational onto its image for $m \geq m_n$. It is known that one can take $m_1 = 3, m_2 = 5$ and $m_3 = 73$.

On the other hand it is reasonable to assume that if the volume of $X$ is large enough one should get better results.

In the talk we will give some explicit bounds for $n = 3, 4$: if $n = 3$ and $\mathrm{vol}(X) > 2(1917)^3$ then $\varphi_m$ is birational for $m \geq 5$; if $n = 4$ and $\mathrm{vol}(X) > (2816)^4$ then $\varphi_m$ is birational for $m \geq 817$. This improves a result obtained by Todorov for $n = 3$.

Moreover these bounds are, in some sense, optimal. For example, if a threefold $X$ is birationally fibered by surfaces of general type with nonbirational fourth-canonical map, then certainly $\varphi_4$ is not birational for $X$. We show that if $\mathrm{vol}(X) > 2(6141)^3$ and $\varphi_4$ is not birational, then $X$ is birationally fibered as above. Similar results hold for $\varphi_3$ and $\varphi_2$.

This is work in collaboration with Lorenzo Di Biagio, most of which being part of his Ph.D. thesis.