Historia de los Poliedros

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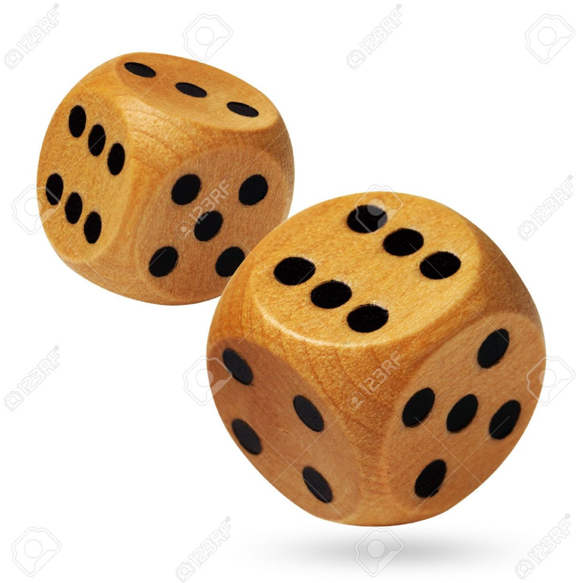
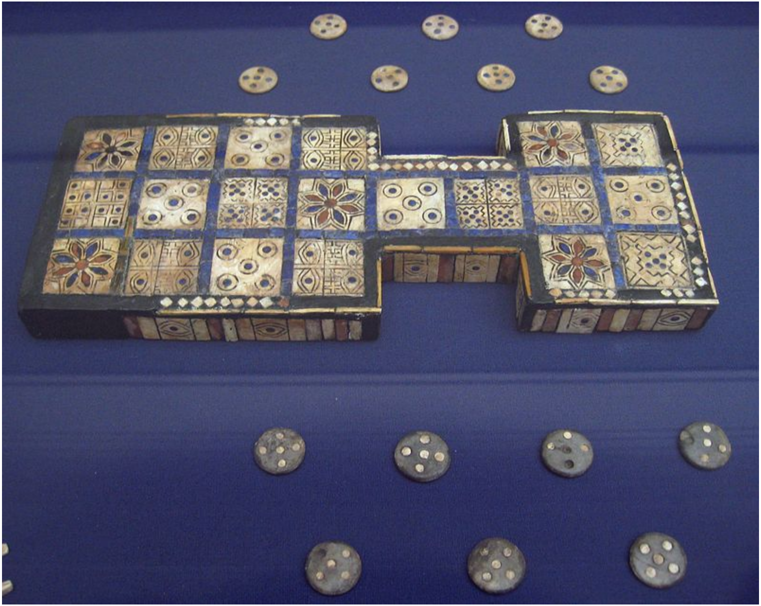
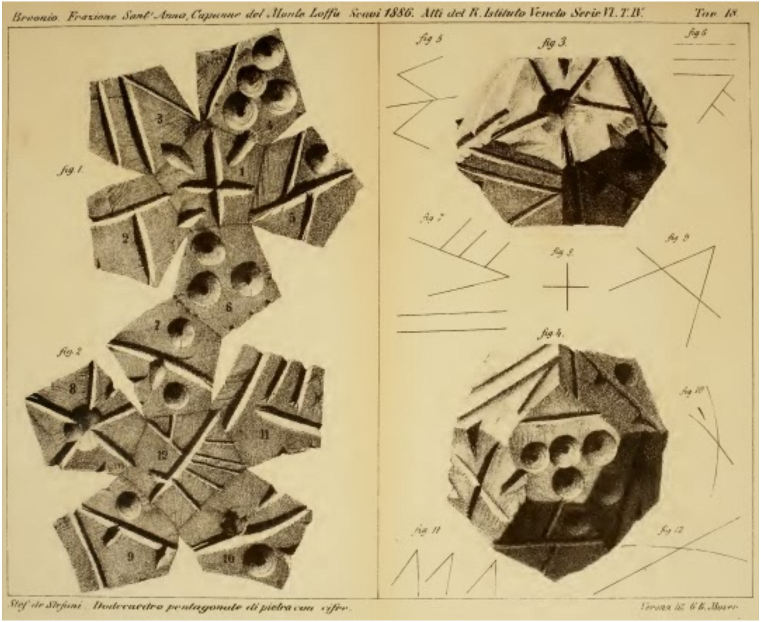
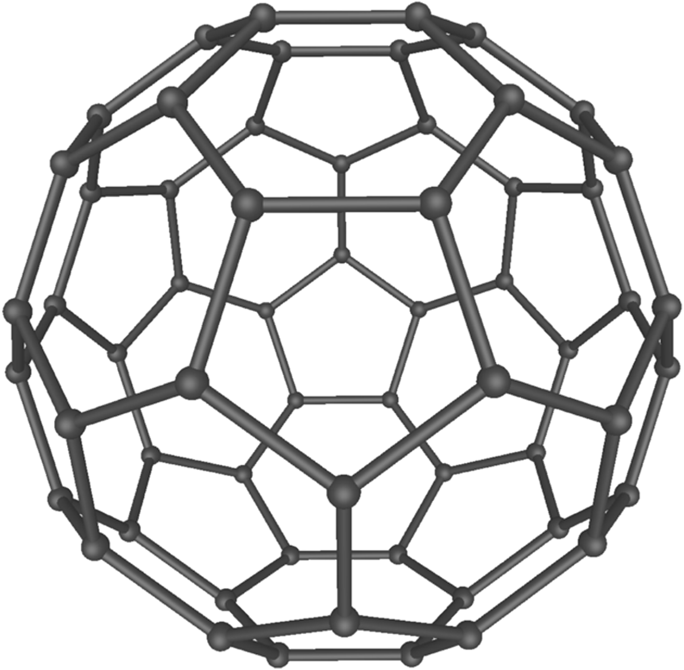
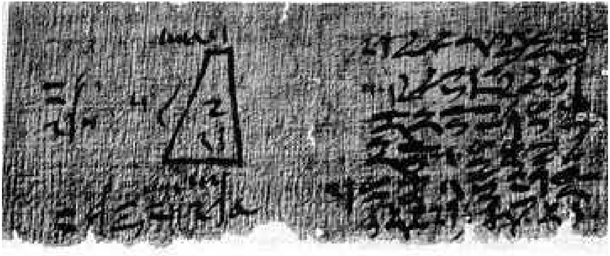
Formación del Profesorado, CTIF Madrid–Sur, 2016–17

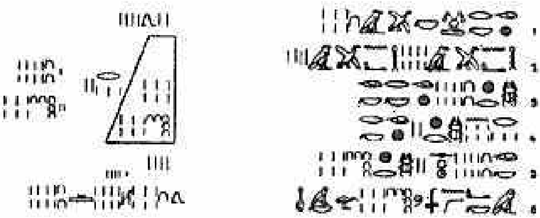




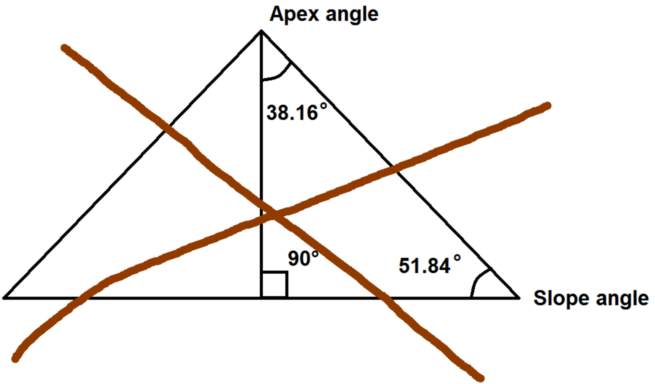

Breonio. Frazione Sant' Anna, Capanne del Monte Loffa Scavi 1886. Atti del R. Istituto Veneto Serie VI. T. IV.
Tav. 18.
fig. 1.
fig. 2
fig. 3.
fig. 4.
fig 6
fig. 7
fig. 10
fig. 11
fig. 12
Stef. de Stefani. Dodecaedro pentagonale di pietra con cifre.
Verona lit. G.B. Moser







$$V = \frac{h}{3}(a^2 + ab + b^2)$$



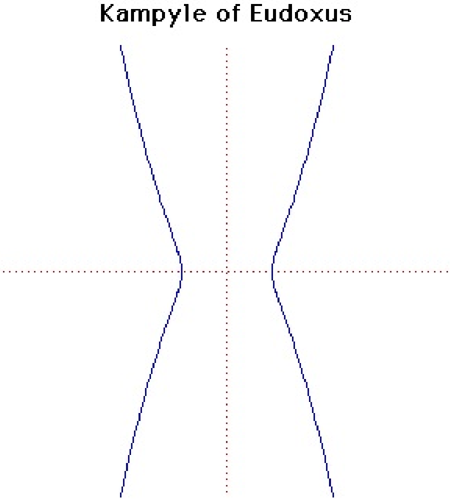
Triángulo de la derecha ¿isósceles? **¡Ojo con las falsedades de Internet!**

Griegos y poliedros

Demócrito (de Abdera), finales s. V a.C.
Eudoxo (de Cnido), s. IV a.C.
Teeteto (de Atenas), s. IV a.C.
Platón (de Atenas), s. V y IV a.C.
Euclides (de Alejandría), s. IV y III a.C.
Arquímedes (de Siracusa), s. III a.C.

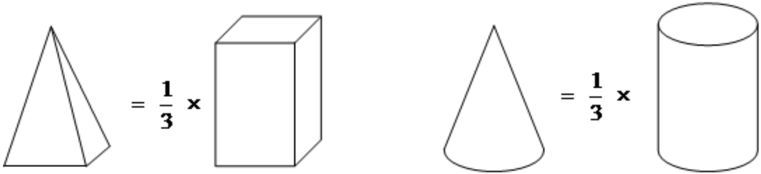
Pappus (de Alejandría), s. III d.C.

Demócrito (fin s.V a.C.) y Eudoxo (409–356 a.C. aprox)



$$\mathbf{a^2x^4 = b^4(x^2 + y^2)}$$ Campila de Eudoxo

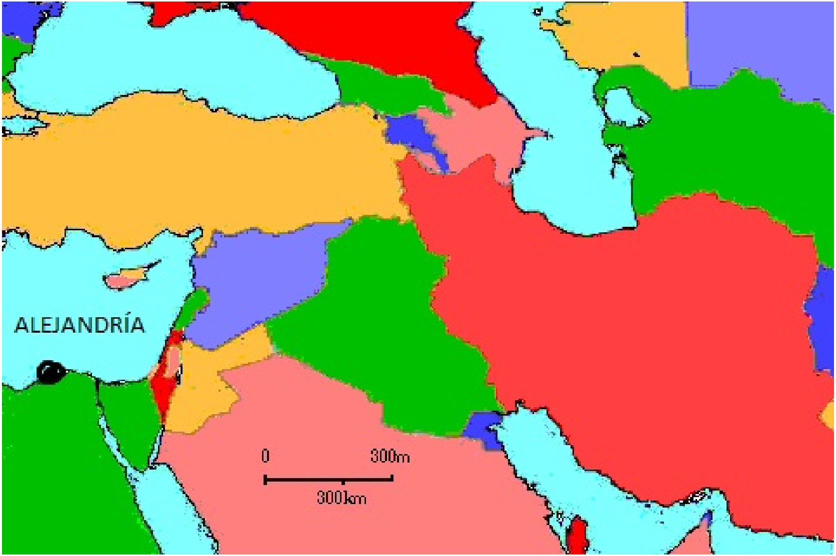
Demócrito (fin s.V a.C.) y Eudoxo (409–356 a.C. aprox)

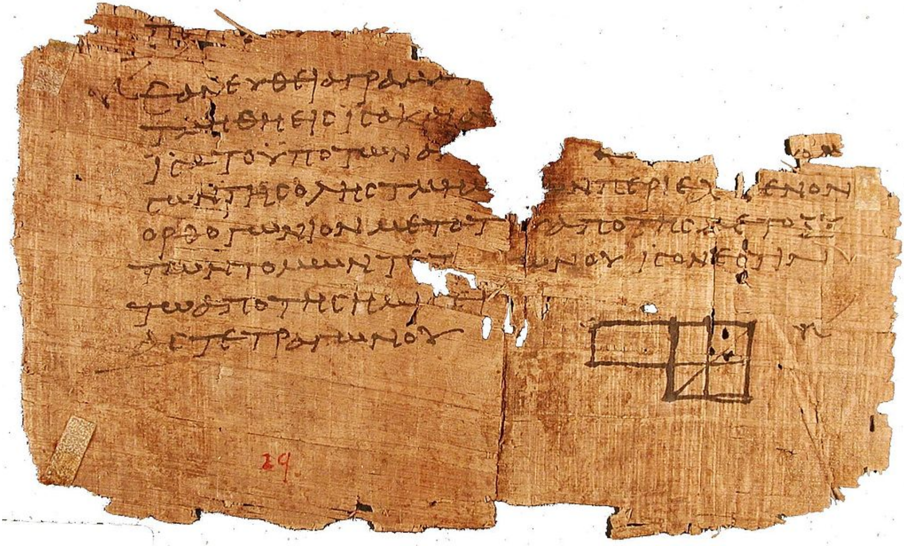


$$\mathbf{V} = \frac{\mathbf{Bh}}{\mathbf{3}}$$

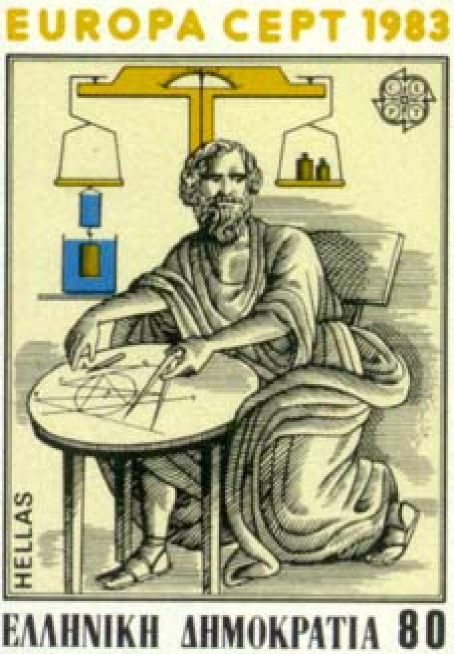
Teeteto (415–369 a.C. aprox) y Platón (427–347 a.C.)

Euclides (323–285 a.C. aprox. activo)





Arquímedes (287–212 a.C. aprox)


EUROPA CEPT 1983
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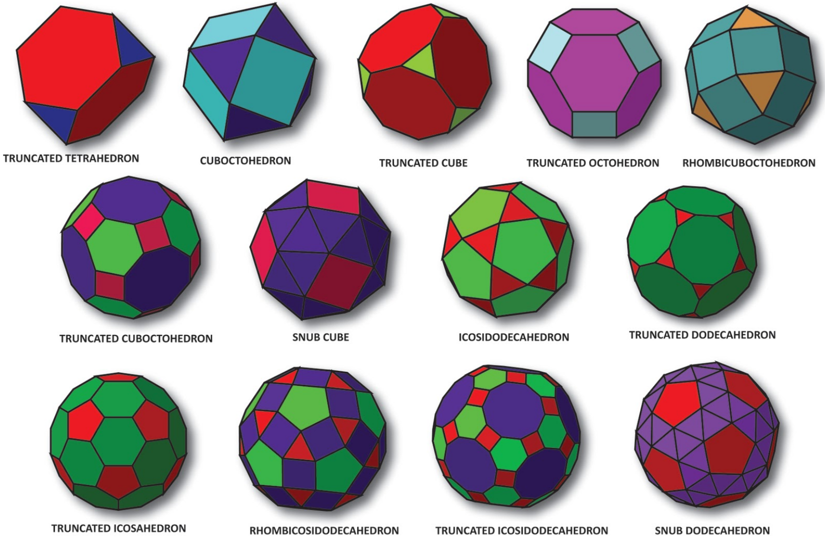
TRUNCATED TETRAHEDRON
CUBOCTOHEDRON
TRUNCATED CUBE
TRUNCATED OCTOHEDRON
RHOMBICUBOCTOHEDRON
TRUNCATED CUBOCTOHEDRON
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ICOSIDODECAHEDRON
TRUNCATED DODECAHEDRON
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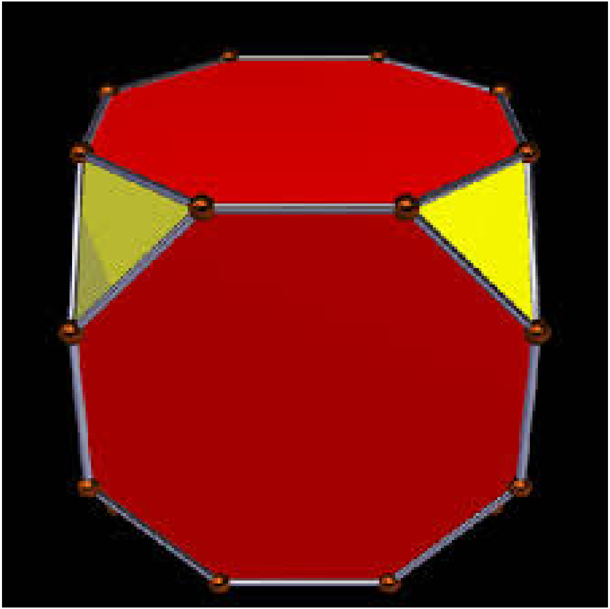
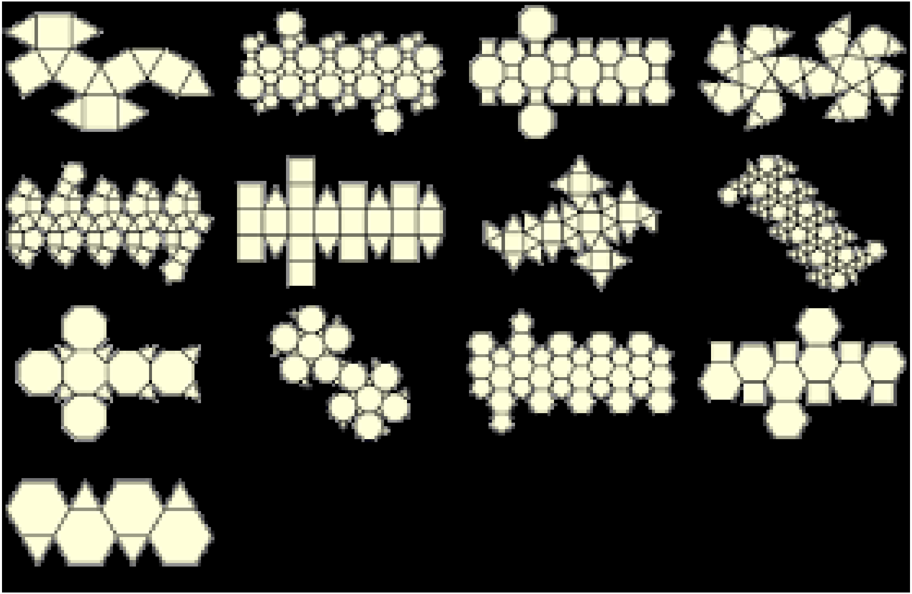
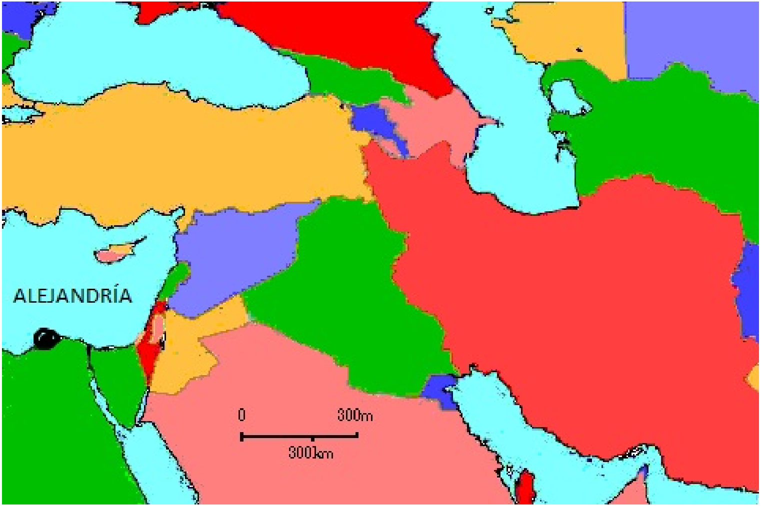
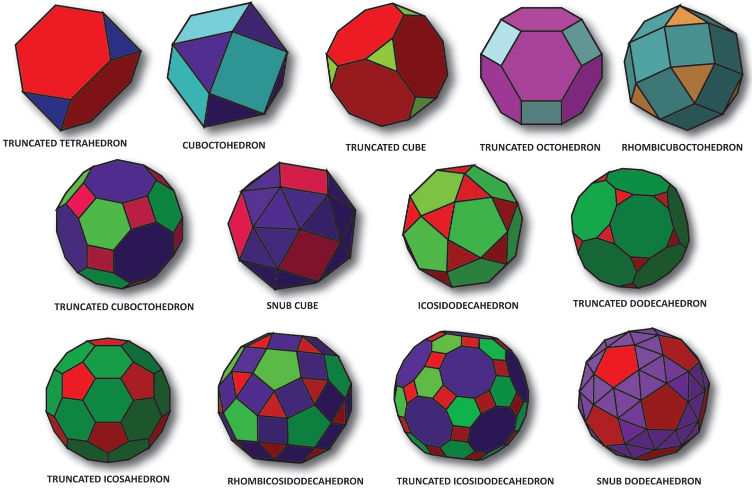




Pappus (290–350 d.C. aprox)

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SNUB DODECAHEDRON

Durero (1471–1528)

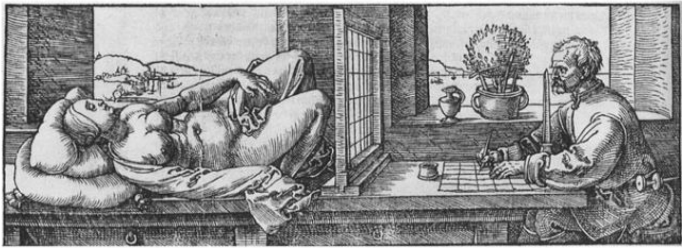


Fig. 34. Albrecht Dürer, "A Draftsman Making a Perspective Drawing of a Woman." Woodcut. From *Underweysung der Messung.* All rights reserved, The Metropolitan Museum of Art. Gift of Felix M. Warburg, 1918. (18.58.3 [recto]).

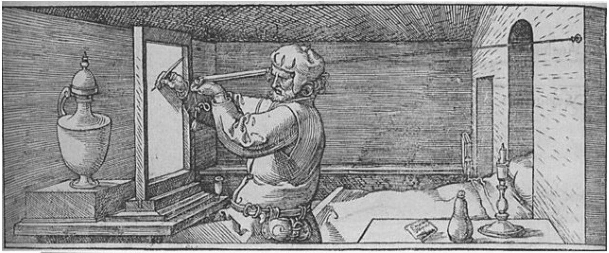


Fig. 32. Albrecht Dürer, "A Man Drawing an Urn." Woodcut. From *Underweysung der Messung.* All rights reserved, The Metropolitan Museum of Art, Harris Brisbane Dick Fund, 1941. (41.48.3).

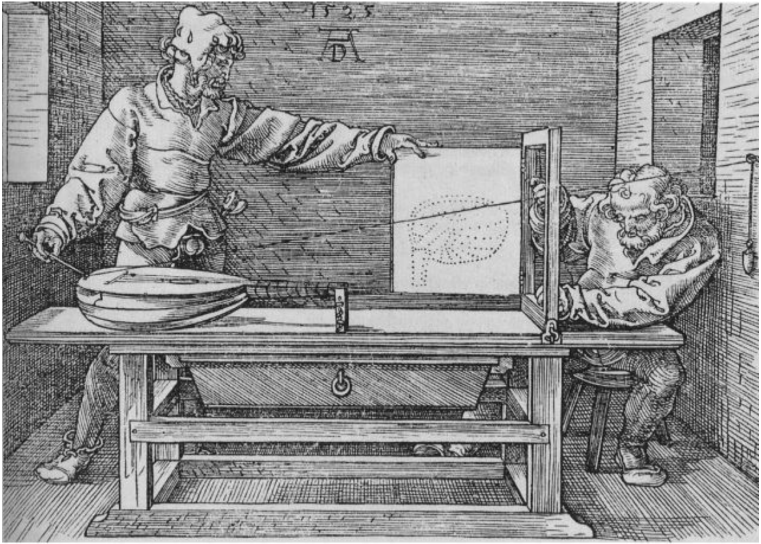
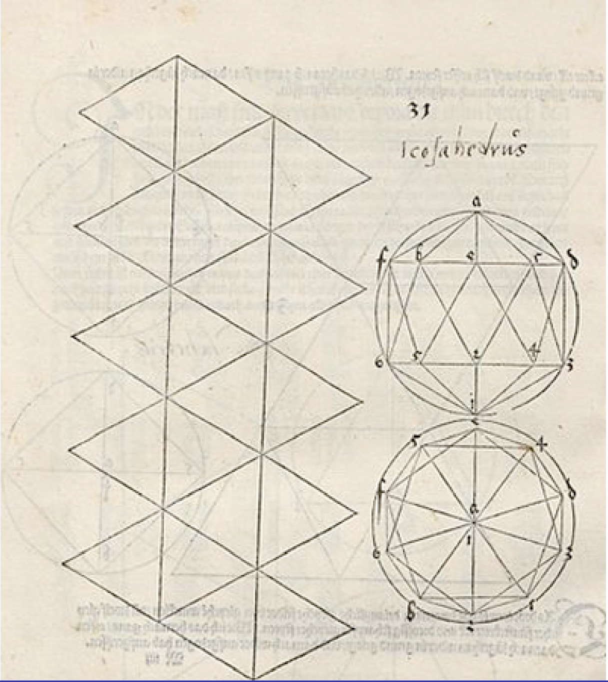
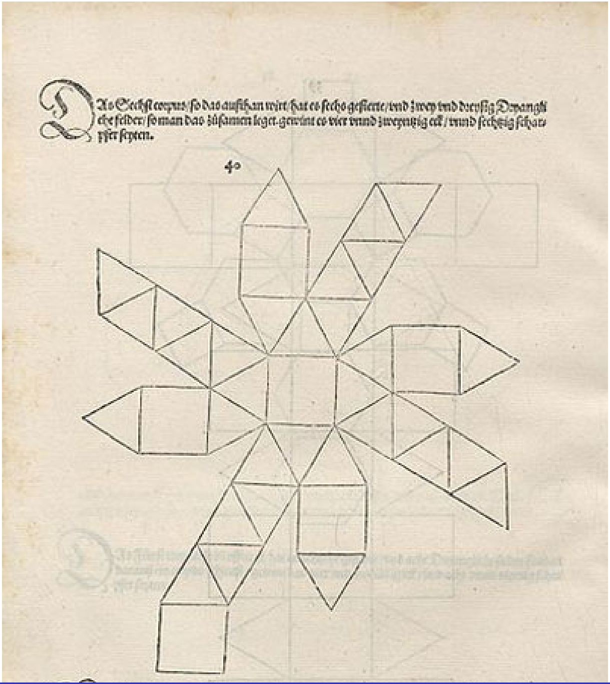


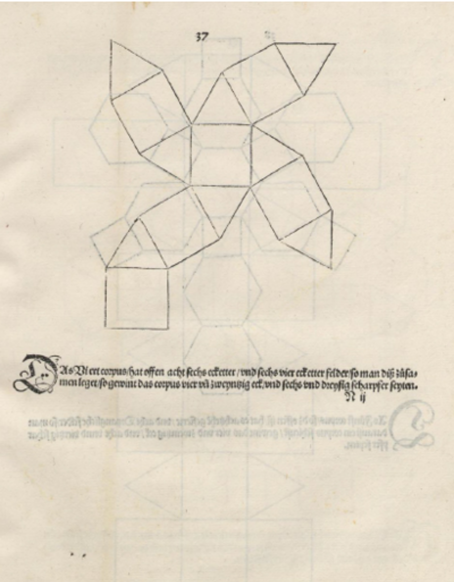
Fig. 31. Albrecht Dürer, "Draughtsman Drawing a Lute." Woodcut. Collection Centre Canadien d'Architecture/Canadian Centre for Architecture, Montréal.


31
Icosahedrus
a
f
b
e
c
d
6
5
2
4
3
i
5
4
f
d
a
6
3
b
1



Das Sechst corpus/ so das aufthan wirt/ hat es sechs gefierte/ vnd zwey vnd dreyssig Dryangkliche felder/ so man das zůsamen leget. gewint es vier vnnd zweyntzig eck/ vnnd sechtzig scharpffer seyten.

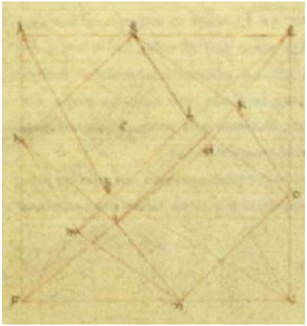
40

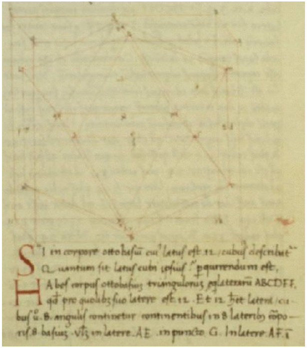


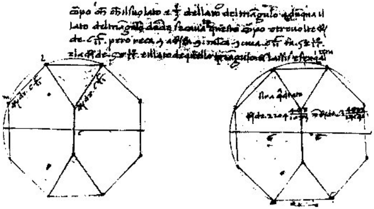
¿Existe siempre un desarrollo? ¡Problema abierto! (Problema de Durero o Conjet. de Shephard)

Artistas del Renacimiento (Italia) Paollo Uccello, Piero della Francesca, Luca Pacioli, L.da Vinci etc.

Uccello

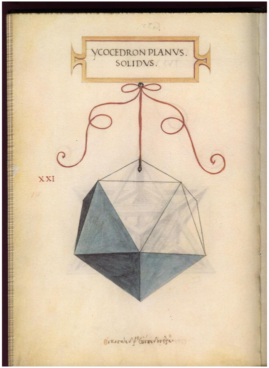


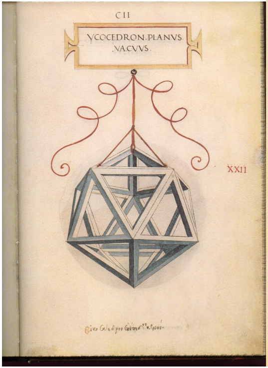


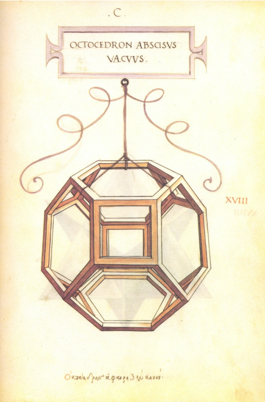


y fórmula volumen tetraedro arbitrario

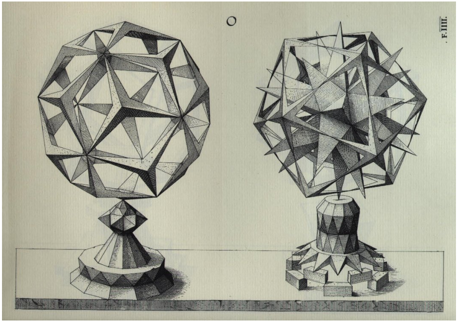
Leonardo da Vinci

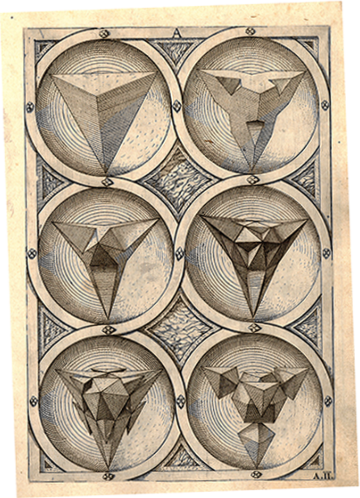






Maestros alemanes: Stöer, Jamintzer etc., s. XVI y XVII

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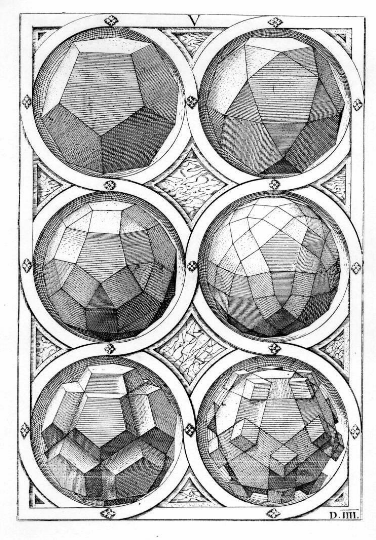
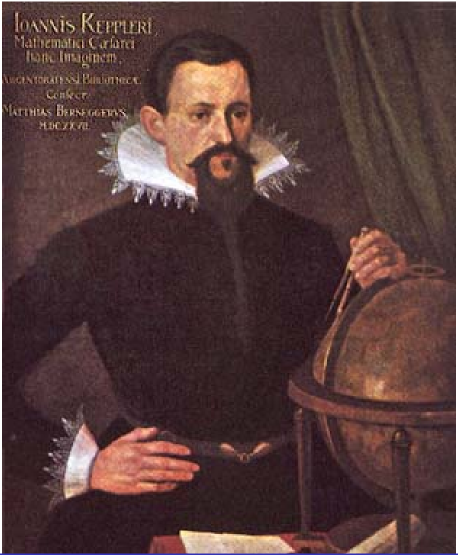
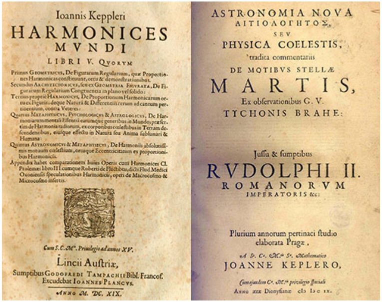

V
D.IIII.

J. Kepler (1571–1630) Misterium Cosmographicum (1596) y Harmonices Mundi II (1619)





Ioannis Keppleri
HARMONICES
MVNDI
LIBRI V. QVORVM

Primus GEOMETRICVS, De Figurarum Regularium, quæ Proportiones Harmonicas constituunt, ortu & demonstrationibus.
Secundus ARCHITECTONICVS, seu ex GEOMETRIA FIGVRATA, De Figurarum Regularium Congruentia in plano vel solido:
Tertius propriè HARMONICVS, De Proportionum Harmonicarum ortu ex Figuris; deque Naturâ & Differentiis rerum ad cantum pertinentium, contra Veteres:
Quartus METAPHYSICVS, PSYCHOLOGICVS & ASTROLOGICVS, De Harmoniarum mentali Essentiâ earumque generibus in Mundo; præsertim de Harmonia radiorum, ex corporibus cœlestibus in Terram descendentibus, eiusque effectu in Natura seu Anima sublunari & Humana:
Quintus ASTRONOMICVS & METAPHYSICVS, De Harmoniis absolutissimis motuum cœlestium, ortuque Eccentricitatum ex proportionibus Harmonicis.
Appendix habet comparationem huius Operis cum Harmonices Cl. Ptolemæi libro III cumque Roberti de Fluctibus, dicti Flud Medici Oxoniensis speculationibus Harmonicis, operi de Macrocosmo & Microcosmo insertis.

Cum S. C. M^tis. Privilegio ad annos XV.

Lincii Austriæ,
Sumptibus GODOFREDI TAMPACHII Bibl. Francof.
Excudebat IOANNES PLANCVS.

ANNO M. DC. XIX.

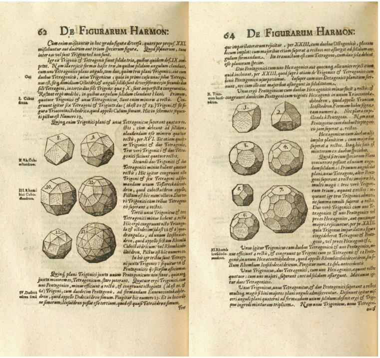
ASTRONOMIA NOVA
ΑΙΤΙΟΛΟΓΗΤΟΣ,
SEV
PHYSICA COELESTIS,
tradita commentariis
DE MOTIBVS STELLÆ
MARTIS,
Ex observationibus G. V.
TYCHONIS BRAHE:

Jussu & sumptibus
RVDOLPHI II.
ROMANORVM
IMPERATORIS &c:

Plurium annorum pertinaci studio
elaborata Pragæ,

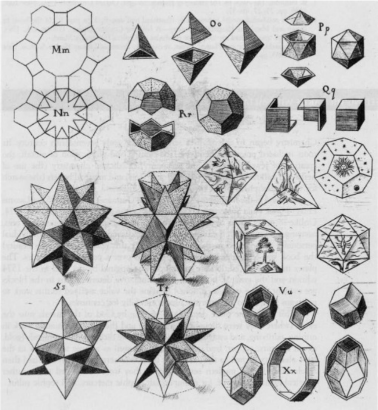
A S^æ. C^æ. M^tis S^æ. Mathematico
JOANNE KEPLERO,

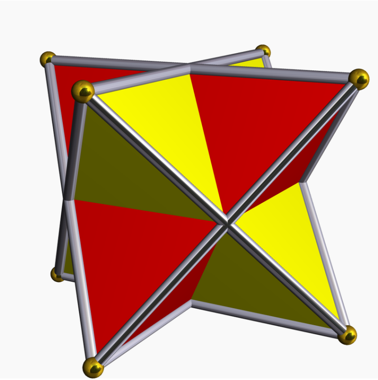
Cum ejusdem C^æ. M^tis privilegio speciali
ANNO æræ Dionysianæ CIↃ IↃC IX.



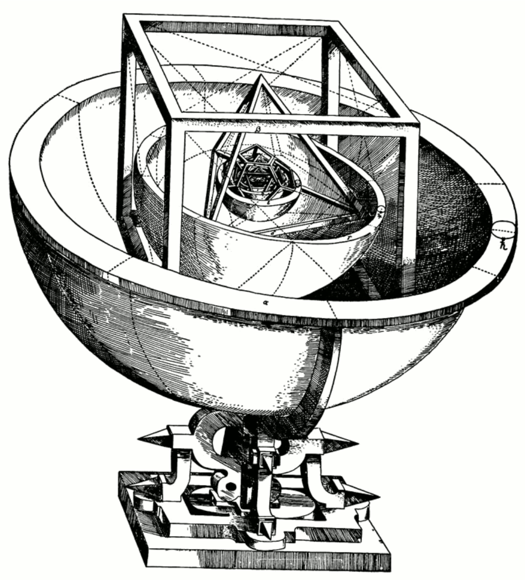
62 DE FIGURARUM HARMON:

64 DE FIGURARUM HARMON:





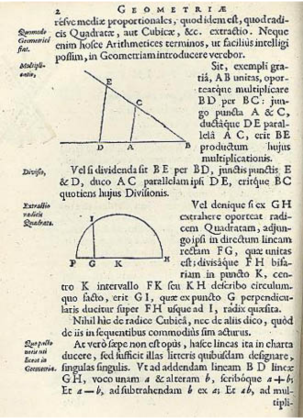
Poliedros estrellados, ¿Dualidad?



R. Descartes (1596–1650) Progymnasmata de solidorum elementis

DISCOURS
DE LA METHODE
Pour bien conduire sa raison, & chercher
la verité dans les sciences.
PLUS
LA DIOPTRIQUE.
LES METEORES.
ET
LA GEOMETRIE.
Qui sont des essais de cete METHODE.

A LEYDE
De l'Imprimerie de IAN MAIRE.
CIƆ IƆ C XXXVII.
Avec Privilege.



2 GEOMETRIÆ

réſve mediæ proportionales, quod idem eſt, quod radicis Quadratæ, aut Cubicæ, &c. extractio. Neque enim hoſce Arithmetices terminos, ut faciliùs intelligi poſſim, in Geometriam introducere verebor.

Quomodo Geometricè fiat.

Multiplicatio,

Sit, exempli gratiá, AB unitas, oporteatque multiplicare BD per BC: jungo puncta A & C, ductâque DE parallelâ AC, erit BE productum hujus multiplicationis.

Divisio,

Vel ſi dividenda ſit BE per BD, junctis punctis E & D, duco AC parallelam ipſi DE, eritque BC quotiens hujus Diviſionis.

Extractio radicis Quadratæ.

Vel denique ſi ex GH extrahere oporteat radicem Quadratam, adjungo ipſi in directum lineam rectam FG, quæ unitas eſt; diviſâque FH bifariam in puncto K, centro K intervallo FK ſeu KH deſcribo circulum. quo facto, erit GI, quæ ex puncto G perpendicularis ducitur ſuper FH uſque ad I, radix quæſita.

Nihil hìc de radice Cubicâ, nec de aliis dico, quòd de iis in ſequentibus commodiùs ſim acturus.

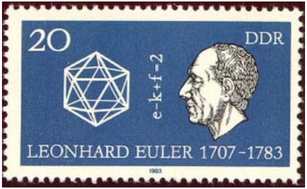
Quo pacto notis uti liceat in Geometria.

At verò ſæpe non eſt opùs, haſce lineas ita in charta ducere, ſed ſufficit illas litteris quibuſdam deſignare, ſingulas ſingulis. Vt ad addendam lineam BD lineæ GH, voco unam *a* & alteram *b*, ſcribóque *a* + *b*; Et *a* — *b*, ad ſubtrahendam *b* ex *a*; Et *ab*, ad multipli-

Teorema : la **suma** de los **defectos** en los vértices es $\mathbf{4}\pi$, para todo poliedro 3–dim

Defecto (o curvatura) **en vértice** V : lo que falta a la suma de ángulos faciales en V para llegar a $\mathbf{2}\pi = \mathbf{360}^\circ$

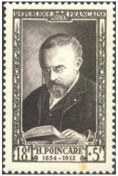
Leonardo Euler (1707–1783)



Fórmula de Euler

$$\mathbf{c} - \mathbf{a} + \mathbf{v} = \mathbf{2} \text{ (1750)}$$

$$\xi(\mathbf{S}) = \mathbf{2g} - \mathbf{2}, \quad \xi(\mathbf{X}) = \mathbf{b_0} - \mathbf{b_1} + \mathbf{b_2} - \mathbf{b_3} + \cdots \text{ (1895)}$$

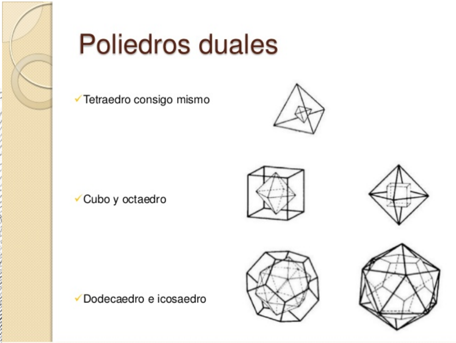


Característica Euler–Poincaré

Agustín Luis Cauchy (1789–1857)

Teorema de rigidez (1913) : si P, P' poliedros 3–dim con misma estructura combinatoria y con caras correspondientes congruentes $\Rightarrow P, P'$ congruentes.

Eugenio Carlos Catalan (1814–1894)



duales de sólidos platónicos

David Hilbert (1879–1934) y Max Dehn (1878–1952)

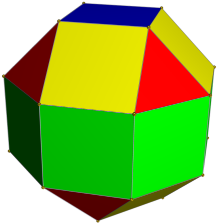

2–dim: Dado S polígono plano, cortamos S en una cantidad finita de polígonos y con los trozos armamos $T \Rightarrow area(S) = area(T)$ ¿Recíproco? SI

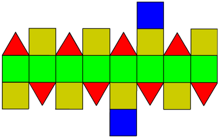
Problema 3 de Hilbert :¿Se puede descomponer un tetraedro de vol 1 en una cantidad finita de poliedros y con ellos armar un cubo de vol 1? NO.

En 1901 Dehn introduce un invariante y calcula $D(cubo) = 0$, $D(tetra) \neq 0$

Ecuaciones de Dehn–Sommerville

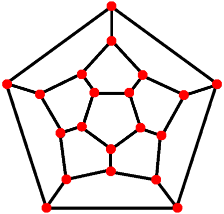
D.M.Y. Sommerville (1879–1934)





Girobicúpula cuadrada elongada o J 37: es localmente regular por vértices, pero no transitivo en vértices.

Ernesto Steinitz (1871–1928) y Luis Schläfli (1814–1895)

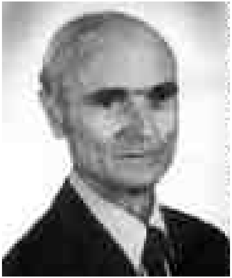




- Steinitz: 1916 Caracterización **combinatoria** de poliedros convexos 3–dim
  Teorema : Todo poliedro convexo forma un grafo plano 3–conexo y recíprocamente.
- Schläfli: en dim–4 hay 6 regulares; en dim superiores hay 3 regulares

Alejandro D. Aleksandrov (1912–1999)

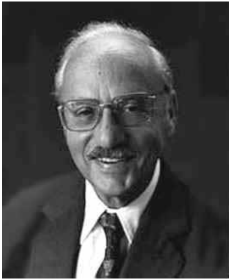
- 1941 **Teorema de unicidad** : Si X espacio métrico geodésico homeomorfo a esfera y localmente euclídeo salvo en conjunto finito de puntos con defecto angular positivo y suma de defectos igual a $\mathbf{4}\pi$ (rec. Descartes) $\Rightarrow$ X es desarrollo (rec. Durero) de un único poliedro convexo
- 1950 libro (en ruso) traducción alemana 1958, traducción inglesa 2005 Convex polyhedra

H.S.M. Coxeter (1907–2003) y Alicia Boole Stott (1860–1940)

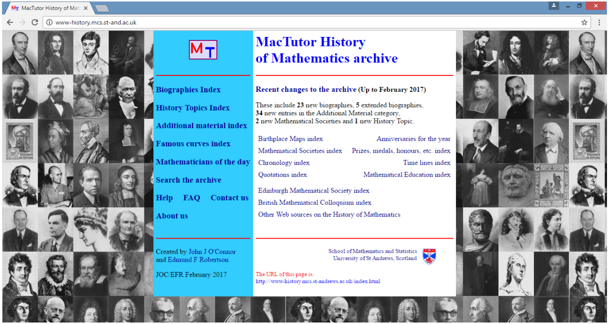



- Coxeter: Años 70 varios libros : variantes concepto de regularidad, generalización a dimensión arbitraria, teoría poliedral de grupos (Klein), Convex Polytopes (1967): compendio, enfoque combinatorio
- Boole: en 4–dim, hay 6 politopos regulares

Jorge Dantzig (1914–2005) y más



1947 Método del simplex, optimización, caminos sobre poliedros
Otros: : Richard Buckminster "Bucky" Fuller (1895–1983) (aplicaciones), N.W. Johnson, Branko Grünbaum (Convex polytopes, 1967) y Günter M. Ziegler (Lectures on Polytopes, 1995)



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