Workshop on Operators and Banach lattices

25-26 October 2012

A non-linear Banach–Stone theorem for lattices of uniformly continuous functions

Félix Cabello, Universidad de Extremadura

Abstract: We give an explicit representation of the order isomorphisms between lattices of uniformly continuous functions on complete metric spaces:

THEOREM: Every lattice isomorphism $T : U(Y) \to U(X)$ is given by the formula $Tf(x) = t(x, f(\tau(x)))$, where $\tau : X \to Y$ is a uniform homeomorphism and $t : X \times \mathbb{R} \to \mathbb{R}$ is defined by $t(x, c) = (Tc)(x)$.

This provides a correct proof for a statement made by Shirota sixty years ago. We emphasize that lattice isomorphisms are not assumed to be linear, they are just bijections that preserve the order in both directions. Of course the preceding Theorem implies that each linear lattice isomorphism $T : U(Y) \to U(X)$ is a weighted composition operator of the form $(Tf)(x) = w(x)f(\tau(x))$, where $\tau : X \to Y$ is a uniform homeomorphism and $w = T(1)$. To the best of our knowledge, even this specialization is new.

Boyd Indices on Banach Function Spaces

María J. Carro, Universidad de Barcelona

Abstract: The Lorentz-Shimogaki theorem characterizes the boundedness of the Hardy-Littlewood operator on a rearrangement invariant space X in terms of the upper Boyd index. Similarly, Boyd's theorem characterizes the boundedness of the Hilbert transform in terms of both the upper and lower Boyd indices. In this talk, we shall revisit these two theorems for spaces X non-necessarily r.i. such as weighted Lebesgue spaces or weighted Lorentz spaces.

On weak-$*$ convergent sequences in duals of Banach lattices and symmetric spaces

Ben de Pagter, Delf University of Technology

Abstract: In this talk we shall discuss, in particular, some properties of weak-$*$ convergent sequences in the dual spaces of Banach lattices and non-commutative symmetric spaces associated with a semi-finite von Neumann algebra. The results are based on joint work with Peter Dodds and Fedor Sukochev.

Following Kalton and Pelczyńsky (1997), we say that a Banach space X has *property* (K) if for every sequence $(x_n^*)_{n=1}^{\infty}$ in the Banach dual X^* satisfying $x_n^* \to 0$ with respect to $\sigma(X^*, X)$, there is a sequence $(y_k^*)_{k=1}^{\infty}$ of consecutive convex combinations of (x_n^*) such that $\langle x_k, y_k^* \rangle \to 0$ for every sequence (x_k) in X satisfying $x_k \to 0$ with respect to $\sigma(X, X^*)$. Several versions of this property will be discussed in Banach lattices and symmetric spaces. In particular, it will be indicated that every predual of a σ-finite von Neumann algebra has property (K), solving a problem posed by Figiel, Johnson and Pelczyńsky (2011).

Interpolation of spaces of integrable functions with respect to a vector measure

Antonio Fernández-Carrión, Universidad de Sevilla

Abstract: Let ν be a countably additive vector measure with values in a Banach space. Associated with ν are the Banach lattices $L^p(\nu)$ (and $L^p_w(\nu)$), with $1 \leq p < \infty$, of equivalence classes of scalar measurable functions p-integrable (or weak integrable) with respect to ν, equipped with the topology of convergence in p-mean. In this talk we analyze the results obtained by applying different interpolation methods (real and complex) to different couples of Banach spaces formed by spaces $L^p_w(\nu)$ or $L^p(\nu)$.

Perturbation classes for semi-Fredholm operators in some Banach lattices

Manuel González, Universidad de Cantabria

Abstract: The perturbation classes problem asks if the set of strictly singular operators $SS(X,Y)$ coincides with the perturbation class of the upper semi-Fredholm operators $SF_+(X,Y)$, or the set of strictly cosingular operators $SC(X,Y)$ coincides with the perturbation class of the lower semi-Fredholm operators $SF_-(X,Y)$. The answer is negative in general, but it is interesting to find pairs X,Y of Banach spaces for which the answers are positive, because in those cases we have intrinsic characterizations of the perturbations classes. We give some conditions for a Banach lattice X and a Banach space Y that imply a positive answer for $SF_+(X,Y)$, and derive a dual result for SF_-. Joint work with Javier Pello, Universidad Rey Juan Carlos, Móstoles.

Daugavet-like properties and numerical indices in some function spaces

Miguel Martín, Universidad de Granada

Abstract: For function spaces, we show that the only lush r.i. separable function space on $[0,1]$ is $L_1[0,1]$ and that the same space is the only r.i. separable function space on $[0,1]$ with the Daugavet property over the reals. For sequence spaces, we show that lushness, the alternative Daugavet property and numerical index 1 are equivalent for spaces with 1-unconditional bases. Also, the only examples of r.i. sequence spaces with these properties are c_0, ℓ_1 and ℓ_∞. On the other hand, we show that c_0 is the unique complex Banach space with 1-unconditional basis whose polynomial numerical index of order 2 is equal to 1.

Dual Orlicz-Lorentz spaces

Yves Raynaud, CNRS/Paris VI

Abstract: We present an Orlicz version of the well known Hardy-Littlewood inequality and its applications to the structural study of certain Orlicz-Lorentz spaces (which, when convex, are in duality with the usual ones).

On weak compactness in Lebesgue-Bochner spaces

José Rodríguez, Universidad de Murcia

Abstract: Let X be a Banach space and μ a probability measure. Schlüchtermann and Wheeler asked whether the Lebesgue-Bochner space $L^1(\mu, X)$ is strongly weakly compactly generated if X is. We shall focus on this open problem and related questions, presenting a partial answer (joint work with S. Lajara).

Some finitely strictly singular operators in Analysis

Luis Rodríguez-Piazza, Universidad de Sevilla

Abstract: Recall that a bounded operator between two Banach spaces $T\colon X \to Y$ is called *strictly singular* if its restriction to every infinite-dimensional subspace F of X is not an isomorphism between F and $T(F)$. Equivalently, if for every such F,

$$\inf_{x\in F, \|x\|=1} \|Tx\| = 0\,.$$

The operator T is called *finitely strictly singular* or *super strictly singular* if, for every $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that, for every subspace F of X with dimension greater than N, we have

$$\inf_{x\in F, \|x\|=1} \|Tx\| \leq \varepsilon\,.$$

Of course, finite strict singularity is a stronger property than strict singularity.

In this talk we review some facts about finitely strictly singular operators, and present new examples (obtained in collaboration with Pascal Lefèvre, from Université d'Artois, France) in the setting of classical spaces and operators. For instance, we will see that the Fourier transform $\mathcal{F}\colon L^p(\mathbb{T}) \to \ell^{p'}(\mathbb{Z})$ is finitely strictly singular for $1 < p < 2$, but it is only strictly singular for $p = 1$.

We will also see that the injection of the Hardy space $H^p(\mathbb{D})$ $(1 \leq p < +\infty)$ into the Bergman space $B^{2p}(\mathbb{D})$ is strictly singular and not compact. For obtaining this last result, we will present some (up to our knowledge new) estimates of the norm in $B^{2p}(\mathbb{D})$ in terms of the norm in Hardy spaces. Along the talk it will appear several times the notion of a uniformly finitely strictly singular family of operators.

Vector measures and classical disjointification methods

Enrique A. Sánchez-Pérez, Universidad Politécnica de Valencia

Abstract: In this talk we show how the classical disjointification methods (Bessaga-Pelczynski, Kadec-Pelczynski) can be applied in the setting of the spaces of p-integrable functions with respect to vector measures. These spaces provide in fact a representation of p-convex order continuous Banach lattices with weak unit; consequently, our results can be directly extended to a broad class of Banach lattices. The additional tool of the vector valued integral for each function has already shown to be fruitful for the analysis of these spaces. Following this well-known technique, we show that combining Kadec-Pelczynski Dichotomy, vector measure orthogonality and with disjointness in the range of the integration map, we can determine the structure of the subspaces of our family of Banach function spaces. These results can already be found in some recent papers and preprints in collaboration with J.M. Calabuig, E. Jiménez, S. Okada, J. Rodríguez and P. Tradacete.

Domination problem on C^*-algebras and noncommutative function spaces

Eugeniu Spinu, University of Alberta

Abstract: Let X and Y be ordered Banach spaces. Suppose $0 \leq A \leq B : X \to Y$ are linear bounded operators and B is (weakly) compact. Is A a (weakly) compact operator? We will consider this question in the case when at least one of the spaces is either a C^*-algebra or a noncommutative function space, establishing results similar to the ones, obtained by Dodds-Fremlin, Wickstead and Aliprantis-Burkinshaw for Banach lattices. This is a joint work with T. Oikhberg.

Irreducible semigroups of positive operators on Banach lattices

Vladimir Troitsky, University of Alberta

Abstract: An operator T on a Banach lattice is said to be peripherally Riesz if the peripheral spectrum of T is separated from the rest of the spectrum and the corresponding spectral subspace is finite dimensional. We study the structure of irreducible semigroups of positive operators containing a peripherally Riesz operator. In particular, we extend some facts of Perron-Frobenius Theory and results of H.Radjavi for matrices and compact operators on L_p-spaces.

Selected topics in positive and regular operators acting between Banach lattices

Witold Wnuk, Adam Mickiewicz University

Abstract: Let us recall that a linear operator T mapping a Banach lattice E into a Banach lattice F is said to be regular if $T = T_1 - T_2$ where $T_i : E \to F$ are positive, i.e., $T_i(E_+) \subset F_+$. We will discuss various results related to an old question when the space $L^r(E, F)$ of regular operators is a Riesz space with respect to the natural partial ordering $T \leqslant S$ iff $S - T$ is positive, and when continuous linear operators $T : E \to F$ are regular. Additionally we will investigate how order continuity of a norm and Schur type properties of a Banach lattice E are related to properties of operators acting on E or taking values in E.