



# A BAYESIAN INFERENCE HISTORY

Universidad Complutense  
METODOS BAYESIANOS'17.MADRID,  
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# Summary

- The second book of probabilities in the world
- The first statisticians
- Application to social sciences
- Application to Inheritance Studies
- Application to the experimental sciences
- The next step
- Bibliographic reviews

# The first probabilistics

- Cardano (1501-1576)
- Galileo (1564-1642)
- Fermat (1601-1665) and Pascal (1623-1662) correspondence in the period July and October 1654
- Christiaan Huygens (1629-1695) *De Ratiociniis in Ludo Aleae* in 1656
- Juan Caramuel (1606-1682) *La Kybeia de Ludis* in 1670
- Jakob Bernoulli (1654-1705) *Ars Conjectandi* in 1708
- De Moivre (1667-1754) *The Doctrine of Chance* in 1715
- Laplace (1749-1827) *Théorie Analytique des Probabilités* in 1812
- Gauss (1777-1855) and Poisson (1781-1840)

## Blas Pascal (1623-1662)

- He was born in Clermont-Ferrand
- His mother died when he was 3 years old
- He was a child prodigy
- At age of 11 he composed a Treatise on sounds
- At the age of 16 he put the bases of the projective geometry
- On 1646 he converts to Jansenism
- On 1658 he retired to the abbey of Port-Royal where he wrote his provincial letters

# Pascal (1623-1662)



A History of Bayesian  
Statistical Inference

# Fermat (1601-1665)



A History of Bayesian  
Statistical Inference

- He was born in Beaumont-de-Lomagne, in the Gascony
- He studied Law at the University of Toulouse
- He was a parliamentarian and a great scholar, speaking Greek, Latin, English and Spanish
- He participated in meetings between Catholics and Huguenots
- He had mathematics as an entertainment



## Correspondence between Fermat (1601-1665) and Pascal (1623-1662) during July and October 1654

- It is considered the birth of the Probability Calculus
- Contain aspects of equiprobabilities
- Problem of points
- Pascal reduces it to its triangle and Fermat to random paths
- **Infini-Rien (1670) o Le pari**

## Juan Caramuel Lobkowitz (1605-1682)

- He was born in Madrid in 1605
- He studies Philosophy and Humanities in Alcala de Henares
- He joined the Cistercian
- He hold a PhD in Theology from the University of Louvain
- He was abbot of Melrose in Scotland and of the Benedictines in Vienna
- He was called the Spanish Leibniz
- He wrote 272 titles from which 70 were printed
- He was the bishop of Vigevano



- Todhunter (1865) in his VI chapter notes:

“A Jesuit named John Caramuel published in 1670, under the title of *Mathesis Biceps*, two volumes of a course of Mathematics; it appears in the list of the author’s works that the entire course was to have comprised four volumes.

There is a section called *Combinatoria* which occupies pages 921--1036 and part is devoted to probabilities and gives an account of combinations in the modern sense of the word [after Todhunter writes ] the tract of Caramuel named the *Kybeia de Ludis* (1670) it contains a reprint of the tract of Huygens *De Ratiociniis in Ludo Aleae (Del Razonamiento en los Juegos de Azar)* (1656) [it is the first text on probabilities]”.

[The opinion of the Todhunter about Caramuel is enough positive, although no Nicolás Bernoulli, that I think must defend the *Ars Conjectandi* (1713)]

IOANNIS CARAMVELIS  
**MATHESIS**  
**BICEPS.**

VETVS, ET NOVA.

I.	ARITHMETICA.	XXI.	LOGARITHMICA FLVENS.
II.	ALGEBRA.	XXII.	LOGARITHMICA REFLVENS.
III.	GEOMETRIA GENERALIS.	XXIII.	COMBINATORIA.
IV.	COSMOGRAPHIA.	XXIV.	KYBELA: DE LVDIS.
V.	GEODÆSIA.	XXV.	ARITHMOMANTICA.
VI.	GEOGRAPHIA.	XXVI.	TRIGONOMETR. GENERALIS.
VII.	CENTROSCOPIA.	XXVII.	TRIGONOMETR. RECVRRENS.
VIII.	OROMETRIA.	XXVIII.	TRIGONOM. ASTRONOMICA.
IX.	HYDROGRAPHIA.	XXIX.	ÆTHEREVS RECTANGVLVS.
X.	HISTIODROMICA.	XXX.	ΔΙΑΒΗΤΗC. CIRCINVS.
XI.	HYPOTHALATICA.	XXXI.	ARCHITECTVRA MILITARIS.
XII.	NECTICA.	XXXII.	MVSICA.
XIII.	NAVITICA SVBLVNARIS.	XXXIII.	METALLARIA.
XIV.	NAVITICA ÆTHEREA.	XXXIV.	PEDARSICA.
XV.	POTAMOGRAPHA.	XXXV.	STATICA.
XVI.	HYDRAVLICA.	XXXVI.	HYDROSTATICA.
XVII.	AEROGRAPHIA.	XXXVII.	METEOROLOGIA.
XVIII.	ANEMOMETRIA.	XXXVIII.	SPHOERICÆ
XIX.	PYETICA.	XXXIX.	OSCILLATORIÆ
XX.	SCIOGRAPHIA.	XL.	RECTILINEÆ

} Planetarum  
Hypothes.

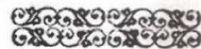
IN OMNIBVS, ET SINGVLIS

*Veterum, & Receniorum Placita examinantur, interdum corriguntur, semper dilucidantur:  
 & pleraque omnia Mathematica reducuntur speculative & practice ad facillimos,  
 & expeditissimos Canones.*

ACCEDENT ALII TOMI, VIDELICET:

ARCHITECTVRA RECTA, symmetrias à Veteribus traditas corrigens & exornans.  
 ARCHITECTVRA OBLIQA, de quâ nemo scripsit hucusque. Est Ars sume necessaria, ut errores à Iunioribus passim admissi cognoscatur.  
 ARCHITECTVRA MILITARIS, Canones Artificum ingenio & captui attemperans, re-

ducensque ad exquisitissimam facilitatem.  
 MVSICA, Vocalis, & Organica, reiectis Giudonis Aretini Mutationibus per viam liberam & expeditam Philomusos conducens.  
 ASTRONOMIA PHYSICA, multos Tractatus & Dissertationes de motibus Astrorum continens.



CAMPANIAE,

In Officinâ Episcopali Anno M.DC.LXX. SVPERIORVM PERMISSV.  
 Prostant Lugduni apud Laurentium Anisson.

A History of Bayesian  
 Statistical Inference

## Jacob y Daniel Bernoulli: *Direct Probabilities*

- Jacob (1164-1705) was a student of philosophy and theology in Basel where he simultaneously studied Mathematics and Astronomy
- In 1687 he became Professor of Mathematics at the University of Basel
- He was an excellent teacher who taught his brother John and his nephew Nicolas
- In 1684 Leibniz published on differential calculus



# Jakob y Daniel Bernoulli:

## *Direct Probabilities*

- The Bernoulli clarify differential calculus

*Ars Conjectandi* (1713) has 5 parts:

- The comments on the Huygens treaty
- The doctrine of combinations and permutations
- The application to gambling and dices
- The application to civil, moral and economics
- The study of infinite series



# The first Statisticians

- Graunt (1620-1674) *Natural and Political Observations, made upon the Bills of Mortality (1662)*
- Thomas Bayes (1701?-1761) *An Essay Towards Solving a Problem in the Doctrine of Chances (1763)*
- Laplace (1749-1827) *Théorie Analytique des Probabilités (1774)*
- Quetelet (1796-1874) *Lettres à S.A.R. Le Duc Règnant de Saxe-Coburg et Gotha, sur la Théorie des Probabilités Appliquée aux Sciences Morales et Politiques (1846)*

## John Graunt (1629-1674)

- He was born in London where he was educated in English Learning
- He was an merchant apprentice and merchant in cloths in the City of London
- He hold administrative positions and was Major of the Train'd Bands





- In 1662 he published *Natural and Political Observations, Made upon the Bills of Mortality*
- He was a *fellow* from the *Royal Society* recommended by the king Charles III
- He was dearly beloved among the citizens of London
- He was Protestant, became Socinian (denied the divinity of Christ) and later Catholic
- He was accused of provoking the great London fire of 1666
- He was related to William Petty, doctor and economist



**A general Bill for this present year,**  
ending the 19 of December 1665. according to  
the Report made to the KING'S most Excellent Majesty.



By the Company of Parish Clerks of London, &c.

Parish	Number	Parish	Number	Parish	Number	Parish	Number
S <sup>t</sup> A'ban's Woodstreet	100	S <sup>t</sup> Clement's Eastcheap	18	S <sup>t</sup> Margaret Mole	15	S <sup>t</sup> Michael Cornhill	104
S <sup>t</sup> A'lhallowes Barking	114	S <sup>t</sup> Dunstons Backchurch	28	S <sup>t</sup> Margaret Newfish	114	S <sup>t</sup> Michael Crookhill	179
S <sup>t</sup> A'lhallowes Breadst	15	S <sup>t</sup> Dunstons East	165	S <sup>t</sup> Margaret Pastons	49	S <sup>t</sup> Michael Ogerhills	103
S <sup>t</sup> A'lhallowes Great	455	S <sup>t</sup> Edmunds Lombard	170	S <sup>t</sup> Mary Abchurch	99	S <sup>t</sup> Michael Queene	11
S <sup>t</sup> A'lhallowes Honils	10	S <sup>t</sup> Ethelborough	195	S <sup>t</sup> Mary Aldermanbury	101	S <sup>t</sup> Michael Leys	132
S <sup>t</sup> A'lhallowes Little	119	S <sup>t</sup> Fairs	104	S <sup>t</sup> Mary Aldermary	105	S <sup>t</sup> Michael Woodstreet	113
S <sup>t</sup> A'lhallowes Lombard	90	S <sup>t</sup> Fettes	144	S <sup>t</sup> Mary le Bow	105	S <sup>t</sup> Michael Broadstreet	59
S <sup>t</sup> A'lhallowes Staining	185	S <sup>t</sup> Gabriel Fen-church	65	S <sup>t</sup> Mary Stock	15	S <sup>t</sup> Michael Postney	68
S <sup>t</sup> A'lhallowes the Wall	500	S <sup>t</sup> George Southwark	121	S <sup>t</sup> Mary Colechurch	17	S <sup>t</sup> Nicholas Acons	46
S <sup>t</sup> Alphege	171	S <sup>t</sup> Gregories by Pauls	125	S <sup>t</sup> Mary Hill	94	S <sup>t</sup> Nicholas Colechey	135
S <sup>t</sup> Andrew Hubbard	71	S <sup>t</sup> Hzens	126	S <sup>t</sup> Mary Monnow	10	S <sup>t</sup> Nicholas Olivers	90
S <sup>t</sup> Andrew Underhill	174	S <sup>t</sup> James Dukes place	163	S <sup>t</sup> Mary Sumner	143	S <sup>t</sup> Olives Martines	137
S <sup>t</sup> Andrew Wardrobe	176	S <sup>t</sup> James Garlickhithe	150	S <sup>t</sup> Mary Seppings	47	S <sup>t</sup> Olives Jewry	14
S <sup>t</sup> Anne Alderigate	55	S <sup>t</sup> John Baptists	118	S <sup>t</sup> Mary Workburch	65	S <sup>t</sup> Olives Fisherers	150
S <sup>t</sup> Anne Blacke Friars	453	S <sup>t</sup> John Evangelist	119	S <sup>t</sup> Mary Woolburch	75	S <sup>t</sup> Pancras Repentance	15
S <sup>t</sup> Anthons Parishes	58	S <sup>t</sup> Iohn Zacharie	155	S <sup>t</sup> Martin Ironmonger	11	S <sup>t</sup> Peters Cheap	61
S <sup>t</sup> Austons Parishes	41	S <sup>t</sup> Katherine Coleman	199	S <sup>t</sup> Martin Ludgate	195	S <sup>t</sup> Peters Cornhill	116
S <sup>t</sup> Barthol. Exchange	73	S <sup>t</sup> Katherine Creech	135	S <sup>t</sup> Martin Organs	110	S <sup>t</sup> Peters Pauls White	114
S <sup>t</sup> Bennet Fynch	17	S <sup>t</sup> Lawrence Jewry	94	S <sup>t</sup> Martin Outwich	60	S <sup>t</sup> Peters Parva	79
S <sup>t</sup> Bennet Gracechurch	17	S <sup>t</sup> Lawrence Postney	114	S <sup>t</sup> Martins Vintrey	117	S <sup>t</sup> Stevens Crutand	166
S <sup>t</sup> Bennet Pauls Wharf	155	S <sup>t</sup> Leonard Eastcheap	121	S <sup>t</sup> Matthew Vintry	14	S <sup>t</sup> Stevens Walbrook	14
S <sup>t</sup> Bennet Sberehog	11	S <sup>t</sup> Leonard Postelane	155	S <sup>t</sup> Matthes Milkstree	44	S <sup>t</sup> Swine	91
S <sup>t</sup> Boroloph Bilinggate	51	S <sup>t</sup> Magnus Parishes	101	S <sup>t</sup> Medlins Oldfish	175	S <sup>t</sup> Thomas Arncliffe	61
S <sup>t</sup> Christophers	60	S <sup>t</sup> Margaret Lockbury	100	S <sup>t</sup> Michael Radfild	153	S <sup>t</sup> Trinitie Parishes	115

Summed the 97 Parishes within the wall. 15:07 Whereof of the Plague 9557

S <sup>t</sup> Andrew Holborn	1218	S <sup>t</sup> Andrew Well	98	S <sup>t</sup> Andrew Southwark	10	S <sup>t</sup> Andrew Southwark	10
S <sup>t</sup> Bartholomew Cree	491	S <sup>t</sup> George Southwark	10	S <sup>t</sup> Giles Cryptigates	80	S <sup>t</sup> Thomas Southwark	475
S <sup>t</sup> Bartholomew Little	193	S <sup>t</sup> Giles Cryptigates	80	S <sup>t</sup> Olaves Southwark	470	Trinity Minorcs	168
S <sup>t</sup> Budge	111	S <sup>t</sup> Olaves Southwark	470	S <sup>t</sup> Thomas Southwark	475	At the Pesthouse	159
Summed in the 16 Parishes without the wall	4151	Whereof of the Plague	1835				

S <sup>t</sup> Giles in the Fields	1417	S <sup>t</sup> Katherine Tower	950	S <sup>t</sup> Mary Newington	173	S <sup>t</sup> Mary Whitechappell	476
Huchney Parishes	113	S <sup>t</sup> Mary Newington	173	S <sup>t</sup> Mary Whitechappell	476	S <sup>t</sup> Mary Whitechappell	476
S <sup>t</sup> James Clutenwall	805	S <sup>t</sup> Leonard Shoredich	1669	S <sup>t</sup> Mary Whitechappell	476	S <sup>t</sup> Mary Whitechappell	476
Summed in the 13 Parishes in Middlesex and Surrey	18:54	Whereof of the Plague	11410				

S <sup>t</sup> Clement Danes	1959	S <sup>t</sup> Mary Saury	101	S <sup>t</sup> Margaret Westminster	4710		
S <sup>t</sup> Paul Covent Garden	405	S <sup>t</sup> Margaret Westminster	4710				
S <sup>t</sup> Martins in the Fields	4504	Summed in the Pesthouse	116				
Summed in the 4 Parishes in the City and Liberties of Westminster	12191	Whereof of the Plague	5401				

The Total of all the Christnings. 9957  
The Total of all the Burials this year 97306  
Whereof, of the Plague 68596

*The Diseases and Casualties this year.*

Abortive and Scilborne	619	Executed	21	Palfie	30
Aged	1545	Flux and Small Pox	655	Plague	68596
Ague and Fever	5357	Found dead in Streets, fields, &c.	20	Planner	6
Appoplex and Suddenly	116	French Pox	86	Plurisie	15
Bedrid	10	Frighted	25	Poysoned	1
Blasted	5	Gout and Sciatica	27	Quinsie	1
Bleeding	16	Grief	46	Rickets	35
Bloody Flux, Scouring & Flux	185	Gripping in the Guts	1288	Riting of the Leghes	197
Burnt and Scalded	8	Hang'd & made away themselves	7	Rupture	34
Calenture	3	Headmouldthax & Mouldfallen	14	Scurvy	105
Cancer, Gangrene and Fistula	56	Jaundies	110	Shungies and Swine pox	2
Canker, and Thruth	111	Imposume	217	Sores, Ulcers, broken and braised	
Childbed	625	Kild by severall accidents	46	Limbs	83
Chrifomes and Infants	1258	Kings Evill	86	Spleen	14
Cold and Cough	68	Leywofie	2	Spotted Fever and Purples	1959
Collick and Winde	134	Lethargy	14	Stoppin: of the stomach	132
Consumption and Tiffick	4808	Livergrown	20	Stone and Strangury	98
Convulsion and Mother	1036	Meagrom and Headach	12	Surfet	1352
Diftraid	5	Measles	7	Teeth and Worms	1614
Dropse and Timpany	1478	Murthered and Slor	5	Vomiting	52
Drowned	50	Oveislad & Starved	45	VVenn	

Christned	Males 1142	Buried	Males 48569	Of the Plague 68596
	Females 4853		Females 48737	
	In all 5995		In all 97306	

Increased in the Burials in the 130 Parishes and at the Pest-house this year. 78009  
Increased of the Plague in the 130 Parishes and at the Pest-house this year. 68596

## Graunt Contributions

- Whether the Plague is large or small, the City was repopulated after 2 years
- There are 14 males for every 13 females in London
- He produced a table showing how many out of every 10 conceived will die before age 6, how many in the next decade, and so on for every decade up to age 76
- Adam and Eve could have spawned in 5610 years the present population
- Therefore the Scriptures are correct

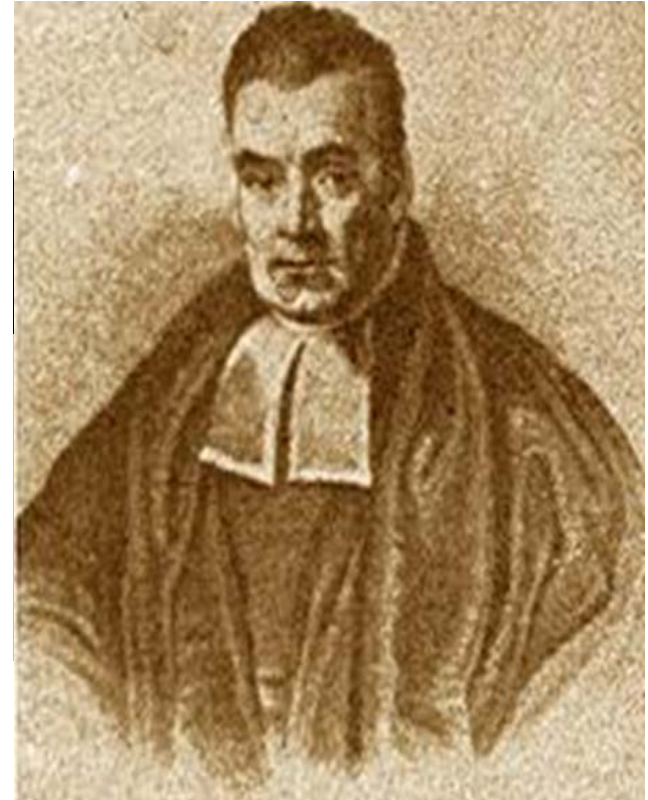
- In every marriage, with each other, 4 children are born
- In England and Wales there are around 6.5 million people
- In London there are 14 men for every 13 women, and in the province 15 men for every 14

### Only with elementary Mathematics

- The art of governing and true politics consist on preserving individuals peace and fullness. Nevertheless men learn how to win by fighting

## Thomas Bayes 1701?-1761

- His father Joshua Bayes was one of the first publicly ordained Protestant ministers
- In 1731 wrote the treatise *“Divine benevolence or an attempt to prove that the principal end of Divine Providence is the happiness of its creatures”*
- In 1736 John Noon published the treatise *“An introduction to the doctrine of fluxions and a defense of mathematicians against the objections of the author of the analyst”*



- In 1764 Richard Price published “*A note on the divergence of the series  $\ln(z!)$* ”
- In 1764 Richard Price published “*An Essay Towards Solving a Problem in the Doctrine of Chances*”
- He died on April 21st 1761 and he was buried in Bunhill Fields the reformist cemetery, where Richard Price and Daniel Defoe were buried ...

## Bayes' Contributions

- He introduced probability using a bet
- He formalized the continuous expression of Bayes' theorem for the Bernoulli's distribution
- Initial or a priori distribution:  $p(\theta)$

- Likelihood or model:  $f(x_1, \dots, x_n | \theta)$

- Final or a posteriori distribution:

$$p(\theta | x_1, \dots, x_n) = \frac{p(\theta) f(x_1, \dots, x_n | \theta)}{m(x_1, \dots, x_n)}$$

- He introduces as prior distribution the uniform for Bernoulli's distribution

## Laplace 1749-1827

- He was born in Beaumont-en-Auge (Calvados)
- In 1765 he joined the Faculty of Arts in Caen
- In 1772 he was elected to the French Academy
- In 1774 he published *Memoire sur la Probabilité des Causes par les Événements*
- In 1789 he participated in the Statutes Committee
- In 1794 he became professor of the Normal School
- In 1796 he presented the Report on the Progress of Science
- In 1802 he published the third volume of *Celestial Mechanics*
- In 1810 he obtained the Central Limit Theorem
- In 1812 he published the *Théorie Analytique*
- In 1820 he participated in the Commission of Longitudes





## Laplace's contributions

- He defined the probability as a ratio of favorable against possible cases
- He rediscovered the Bayes' Theorem
- He studied the proportion of births
- He introduced the Loss functions for estimations
- He introduced the Characteristic function
- He obtained the first version of the Central Limit Theorem
- He justified, in a Bayesian way, least-squares estimators

A

NAPOLÉON-LE-GRAND.

SIRE,

LA bienveillance avec laquelle VOTRE MAJESTÉ  
a daigné accueillir l'hommage de mon Traité de  
Mécanique Céleste, m'a inspiré le désir de Lui

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A Subjective History of Bayesian  
Statistical Inference

dédier cet Ouvrage sur le Calcul des Probabilités. Ce calcul délicat s'étend aux questions les plus importantes de la vie, qui ne sont en effet, pour la plupart, que des problèmes de probabilité. Il doit, sous ce rapport, intéresser VOTRE MAJESTÉ dont le génie sait si bien apprécier et si dignement encourager tout ce qui peut contribuer au progrès des lumières, et de la prospérité publique. J'ose La supplier d'agréer ce nouvel hommage dicté par la plus vive reconnaissance, et par les sentimens profonds d'admiration et de respect, avec lesquels je suis,

SIRE,

DE VOTRE MAJESTÉ,

Le très-humble et très-obéissant  
serviteur et fidèle sujet,

LAPLACE.

The next stage consists in applying it to Social Arithmetic

- Condorcet (1743-1794), Quetelet (1796-1875) y Arbuthnot (1667-1735)

## Condorcet (1743-1794)

- He was born in Ribemont (France)
- He is educated at Reims' Jesuits school
- In 1758 he entered the school of Navarre in Paris to pass later to the Mazarino school in Paris
- In 1759 he graduated in Philosophy
- In 1765 he publishes the *“Essai sur le Calcul Integral”*
- In 1769 he joined the French Academy
- In 1772 met Turgot, a great economist and minister of Louis XVI
- In 1773 became Secretary of the French Academy
- In 1774 Turgot appointed him for General Inspector of the Currency
- In 1785 appeared his *“Essay on the Application of Analysis to the Probability of Majority Decisions”* a very important work for the development of Probability Theory



- In 1786 he published the “*Life of M. Turgot*” where he supported his ideas
- In 1789 he wrote “*Voltaire`s life*”
- During the French Revolution he supported the liberal cause and he was elected in the Legislative Assembly representing Paris. He was also elected secretary of the Legislative Assembly.
- In 1792 he joined the moderate Girondists and he argued strongly in favor of forgiving Louis XVI dead penalty
- During the terror he is against the new constitution and he retired to write “*Esquisse d'un tableau historique des l'esprit humain*” (1795)
- In 1794 he was imprisoned in Bourg-la-Reine, and 2 days later he was poisoned in his cell

## Condorcet's contributions

- Condorcet paradox: majority preferences may not be transitive
- Difference between **own probabilities** (favorable cases among possible cases) and **probabilities as credibility degrees** (subjective probabilities)
- Applies Probability to the composition of courts, to reach a jury a correct verdict, ...

## Quetelet (1796-1874)

- He was born in Ghent
- He taught mathematics at Ghent's Grammar School
- In 1819 he obtained a PhD in Mathematics (Geometry)
- In 1819 he taught mathematics at Brussel's Ateneo
- In 1823 he was awarded in Paris with a scholarship to open an observatory in Brussels
  - Arago
  - Fourier, Poisson and Laplace
- In 1827 he traveled to England
- In 1828 he became director of the Royal Observatory of Brussels
- In 1832 he was appointed by the Belgian government to attend to the meeting of the British Society for Science Advancement





- In 1834 he rejected a chair at the Free University of Brussels
- In 1851 he launched in Brussels the idea of a Universal Conference on Statistics to unite and to develop works in this science
- In 1853 he gave the opening lecture and is appointed President of the International Statistical Institute
- Until his death in 1874 he dedicated himself to the developments of the "average man" and his applications to compare different populations
- He co-founded:
  - The Royal Statistical Society of London
  - The International Congresses of Statistics
  - The Statistics Section of the British Association for the Advancement of Science
- He was a broad-spectrum man
- He wrote numerous poems
- He composed an opera
- He wrote an History of the novel

MESURES de la POITRAIL.	NOMBRE d'hommes.	NOMBRE PROPORTIONNEL.	PROBABILITÉ d'après L'OBSERVATION.	RANG dans LA TABLE.	RANG d'après le CALCUL.	PROBABILITÉ d'après LA TABLE.	NOMBRE d'OBSERVATIONS calculé.
Pouces.							
55	3	5	0,5000			0,5000	7
54	18	51	0,4995	52	50	0,4993	29
55	81	141	0,4964	42,5	42,5	0,4964	110
56	185	322	0,4825	35,5	34,5	0,4854	323
57	420	732	0,4501	26,0	20,5	0,4531	732
58	740	1305	0,3769	18,0	18,5	0,3799	1333
59	1075	1867	0,2464	10,5	10,5	0,2466	1838
			0,0597	2,5	2,5	0,0628	
40	1070	1882	0,1285	5,5	5,5	0,1330	1987
41	934	1628	0,2913	13	13,5	0,3034	1675
42	658	1148	0,4061	21	21,5	0,4130	1096
43	370	645	0,4706	30	29,5	0,4690	560
44	92	160	0,4866	35	37,5	0,4911	221
45	50	87	0,4935	41	45,5	0,4980	69
46	21	38	0,4991	49,5	55,5	0,4996	16
47	4	7	0,4998	56	61,8	0,4999	3
48	1	2	0,5000			0,5000	1
	5758 <del>5729</del>	1,0000					1,0000

Primer ajuste de una nube de puntos a una  $N(\mu, \sigma)$   
 6el 1 datos, 6els 2 y 3 frecuencias, 6els 4 a 7 auxiliares  
 6el 8 datos ajustados.

(1846) Lettres à S.A.R le Duc...  
 pag 400.

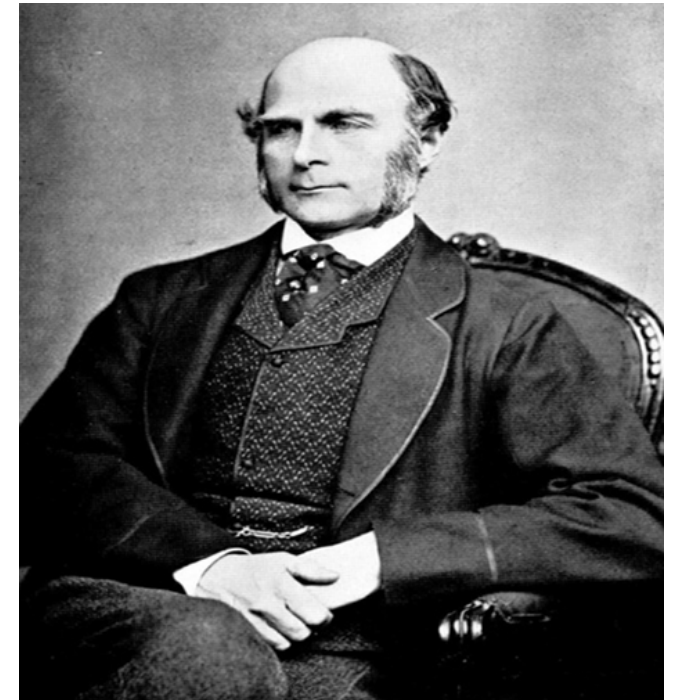
## The next stage is the application to Biology and Social Sciences

- English school: Galton (1822-1911), K. Pearson (1857-1936), Student (1876-1937), Neyman (1894-1981) y Fisher (1890-1962)
- The Bayesian approximation shows a lack of results, until the appearance of names like: Jeffreys (1891-1989), De Finetti (1906-1985) y L. Savage (1917-1971)

## Galton (1822-1911)

- He was born in Birmingham
- He was Darwin's nephew
- He studied mathematics in Cambridge and medicine in London
- In 1850 his father dies, inherits a great fortune and dedicated himself to explore the African continent
- His geographic discoveries earned him the gold medal of the Geographical Society
- In 1860 he is elected Fellow of the Royal Society
- In 1862 he wrote “*Meteorographica, or methods of mapping the weather*”

- He introduced the term anticyclone
- He did not hold any chair
- In 1869 he wrote *“Hereditary Genius an Inquiry into its Laws and Consequences”*
- In 1886 wrote *“Regression towards mediocrity in hereditary stature”*
- From 1890 onwards **he passed the relay** to his collaborators, specifically to K. Pearson

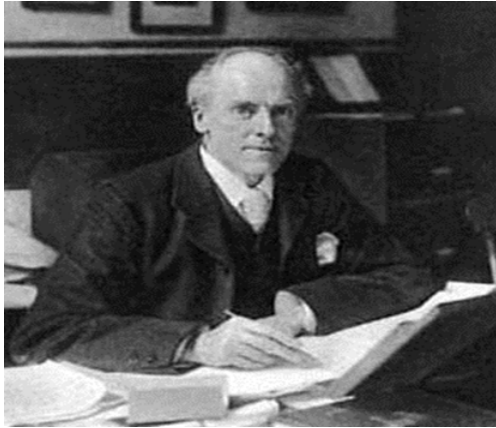


# Galton's contributions

- He published on Psychology, Anthropology, Sociology, Education, Fingerprints...
- He coined the term Eugenics
- He departed from Quetelet average man
- He studied the intelligence in politicians, poets, and scientist families... he studied the fit by means of points diagrams
- He assumed that the inheritance caudal is a geometric progression or rate 0.5 that starts with 0.5 for the parents and goes back to the grandparents and so forth until it sums up to 1.
- Quincunx designed to prove that if

$$X_i \sim \text{Bernoulli}(0.5) \text{ then } \sum_i^n X_i \sim \text{Normal}$$

## Karl Pearson (1857-1936)



- He was born in London. His father was a lawyer
- He studied at University College School
- In 1875 he studied mathematics at the King College of Cambridge
- At age 22, he went to Germany to study Law, Physics and Metaphysics
- Since 1880 to 1884 he was Professor of Mathematics at King College and University College
- In 1911 he was the first Galton Professor of Eugenics
- He was a convinced Darwinist and a fervent socialist
- Since 1891 to 1892 he gave conferences at Gresham College on Geometry of Statistics

- These conferences marked the beginning of a new era in the theory and practice of statistics
- Between 1893 and 1906 he published 100 articles on statistics and their applications
- In 1901 he founded the magazine "Biometrika" to publish works of applied statistics to the biology and he published the Tables for Statisticians and Biometricians
- In 1905 he published “On the general theory of skew correlation and non linear regression”
- In 1914 he began his controversy with Fisher



- In 1925 he founded “[Annals of Eugenics](#)”
- In 1932 he retired from the University College that divided the Statistics Department in the Galton chair of Eugenesis, occupied by Fisher, and the chair of Statistics, that was occupied by E. Pearson
- In 1934 he published “[The Tables of the Incomplete Beta-Function](#)”
- In his first period he discovered that the values of the roulette are not random. He wrote to the French government asking to close the casinos and send their funds to the [Academie des Sciences](#) for the foundation of a laboratory of probabilities with the task of applying probability theory to the problem of biological evolution

## K. Pearson's contributions

- He founded the statistics laboratory of The University College
- He introduces the family of curves (beta symmetric, asymmetric, gamma and normal) and adjusts its parameters by the method of moments
- He develop the correlation
- In 1990 he introduced the Pearson's  $\chi^2$  as a model measure of fit
- **The History of Statistics in the 17th and 18th Centuries**  
(conferences given in the academic sessions at University College of London between 1921-1933)

## Emile Borel (1871-1956)

- He funded *Paris Institute of Statistics* the oldest French school for statistics
- He was among the pioneers **measure theory** and **probability theory**
- A member of the French National Assembly
- A member of the French Resistance
- He was the guilty to forbid the prior distribution



## Fisher (1890-1962)

- Ronald Aylmar Fisher was born in East Finchley (London)
- He attended school in Stanmore and studied at Harrow
- In his youth he was forbidden to read with artificial light
- Thanks to a scholarship he studied at Casius College in Cambridge where he graduated between 1909 and 1912
- In 1913 he was a reader of Mathematical Physics while at the same time he was studying biometry and genetics
- Between 1913 and 1915 he worked in an investment company
- In 1916 he wrote an article showing that Mendel's theories were not rejected by the data.
- K. Pearson referred to him as an statistician and Punnet as a genetic

- In 1917 he married Ruth E. Guinness and had two sons and six daughters
- In 1919 he joined Rothamsted where he developed the [Analysis of variance \(1921\)](#) and [The design of experiments \(1923, 1924\)](#)
- In 1924 he was elected a member of the Royal Society. He thought that it was a mistake to choose people over 50 years old
- In 1930 he published [The Genetical Theory of Natural Selection](#) where he supported and updated Darwin's theory of the Evolution of the Species
- In 1933 he accepted the chair of Eugenics at University College London working in genetics
- In 1938 he traveled to India invited by Mahalanobis
- In 1943 he traveled to the USA becoming a visiting professor of the U. of North Carolina
- Between 1938 and 1954 he dedicated himself to statistical inference

- Between 1952 and 1954 he was the president of the Royal Statistical Society and dedicated his interventions to gloss the contributions of the first statisticians
- In 1956 (with 66 years old) he published **Statistical Methods and Scientific Inference**
- In 1957 he retired and went to Australia as a senior researcher at the University of Adelaide
- He died from a mouth cancer in Adelaide at the age of 72 years old

## Fisher's contributions

- The difference between Population and Sample
- The Maximum Likelihood Method
- The correct determination of Degrees of Freedom
- The Analysis of the Variance
- Design of Experiments (1935, 37, 42, 47, 49, 51, 53, 60, 1966)
- The concepts of sufficiency, verisimilitude, ancillarity, consistency, efficiency



A History of the Bayesian  
Statistical Inference

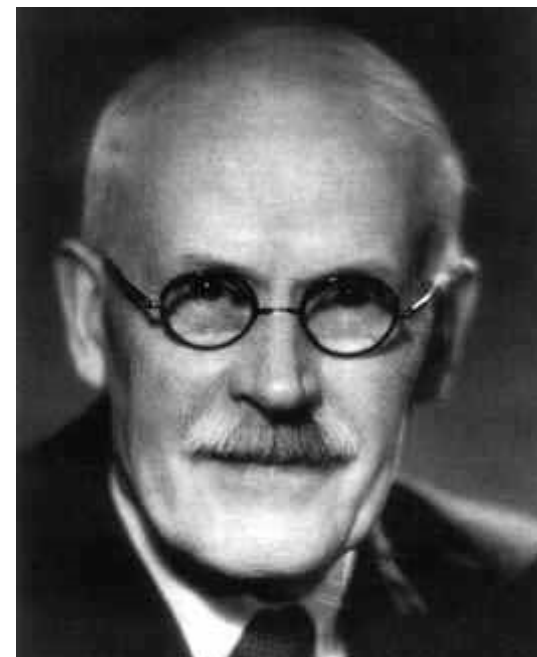


## In the Bayesian approximation

- Final distribution
- Credible regions
- Bayesian contrasts
- Example:
  - a) A friend claims to distinguish a Hayden from a Mozart music sheet.
  - b) A friend says that he can predicts the results of flipping a coin.
  - c) A friend can ensures that distinguishes whether the tea has been made by first pouring the water and then the milk cloud or the other way around

## Harold Jeffreys (1891-1989)

- He spent virtually his entire life in Cambridge
- The probability is the only suitable mechanism to measure uncertainty
- Developed the prior distribution non informative **Jeffreys prior**
- The **“Theory of Probability”** (the “Theory” as he liked to call his book) is now widely cited and will live as one of the truly important scientific works of the twentieth century (Lindley dixit)
- He did the right of the Bayesian testing of hypothesis



## Bruno de Finetti (1906-1985)

- Was born in Innsbruck “Italian parents and Austrian citizens” as he himself wrote
- He was an engineer enrolled at Milan Polytechnic
- Inspired by the biologist Carlo Foà he started research work in population genetics (the first paper of more than three hundred)
- He met many important mathematicians as: Cantelli, Fréchet, Khinchin, Paul Levy, Neyman, Fisher, Polya and L.J. Savage
- He was the author of the aphorism **Probability does not exist** and the notion of **exchangeable variables** and **scoring rule**



## Leonard (Jimmie) Savage (1917-1971)

- Nace en Detroit, he belong to group of Jewish emigrated to US from Russia
- He was, in **Lindley's opinión** the most original statistics in America during the period from the end of World War II until his premature death in 1971
- In **The foundation of Statistics (1954)** start with six axioms and deduce the existence of a **subjective “personal” probability** and a **utility function**
- He introduced Bayesian hypothesis tests and Bayesian estimation



## Leonard Savage

- Recommended by von Neumann to go at Columbia Univ. In constructing a general system of inference
- He published with the economist Milton Friedman : about utility and decisions
- He published with Paul Halmos about of the Radon-Nikodym theorem
- With Savage is clear that the only sensible decision procedure it is maximization of expected utility



## The next step

- The symposia of Neyman, 5 from 1945 to 1970 and
- The symposia of Bernardo on Bayesian methods, 9 from 1979 to 2011
- The statistics can already be applied to a large number of fields

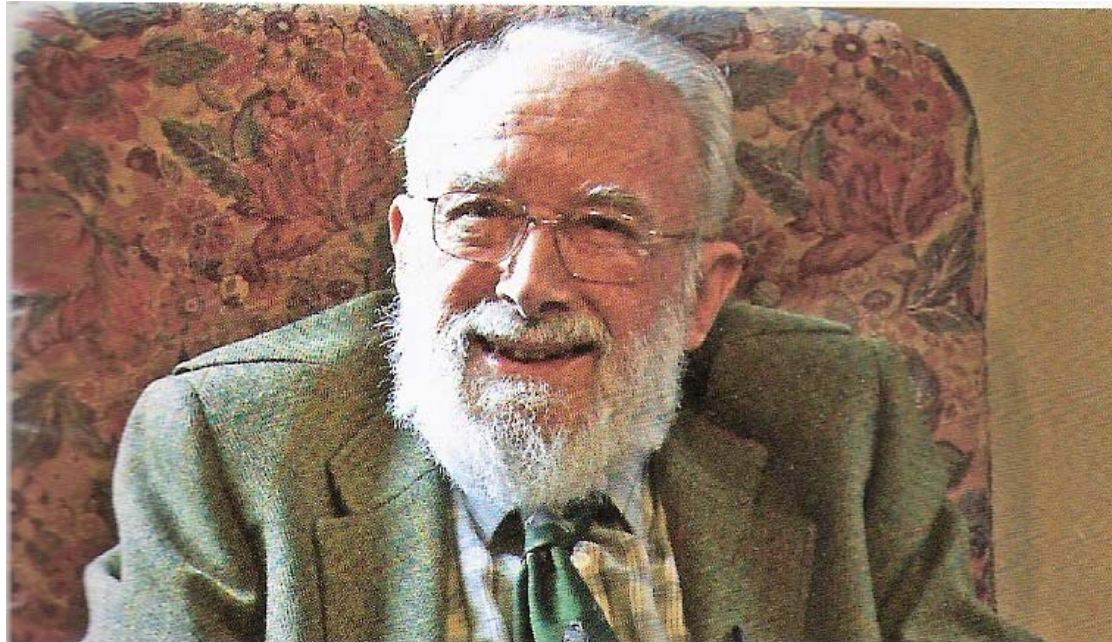


## Some applications

- Large data statistics
- Epidemiology (Microarrays)
- Political Methodology
- Seismology
- Internet data traffic
- Measurement errors
- Obtaining significant elements in catastrophes
- **Approximate methods may allow agreement between Bayesian and frequentist methods**

## Allow me to pay tribute to Professor Lindley

Gómez-Villegas, M.A. (2015) Denis Lindley (1923-2014)  
Obituario, *Boletín de Estadística e Investigación Operativa*, **31**, 1,  
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A History of the Bayesian  
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Thanks for your attention!!