

ESTIMATES FOR COVERING NUMBERS IN SCHAUDER'S THEOREM ABOUT ADJOINTS OF COMPACT OPERATORS

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Abstract. This is joint work with Eliahu Levy and full details are available in our preprint at: <http://arxiv.org/abs/0810.4240>

Let $T : X \rightarrow Y$ be a bounded linear map between Banach spaces X and Y . Let $T^* : Y^* \rightarrow X^*$ be its adjoint. Let \mathcal{B}_X and \mathcal{B}_{Y^*} be the closed unit balls of X and Y^* respectively. We obtain apparently new estimates for the covering numbers of the set $T^*(\mathcal{B}_{Y^*})$. These are expressed in terms of the covering numbers of $T(\mathcal{B}_X)$, or, more generally, in terms of the covering numbers of a “significant” subset of $T(\mathcal{B}_X)$. The latter more general estimates are best possible. These estimates follow from our new quantitative version of an abstract compactness result which generalizes classical theorems of Arzelà–Ascoli and of Schauder. Analogous estimates also hold for the covering numbers of $T(\mathcal{B}_X)$, in terms of the covering numbers of $T^*(\mathcal{B}_{Y^*})$ or in terms of a suitable “significant” subset of $T^*(\mathcal{B}_{Y^*})$.

I mention the possibility of perhaps only briefly surveying the above results so as to also have some time to report upon other joint work with Kyril Tintarev,

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which deals with *cocompact* embeddings with respect to lattice shifts of certain Sobolev and Besov spaces.

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