HARDY'S INEQUALITY AND CURVATURE

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Abstract. A Hardy inequality of the form

$$\int_{\tilde{\Omega}} |\nabla f(\mathbf{x})|^p d\mathbf{x} \geq \left(\frac{p-1}{p}\right)^p \int_{\tilde{\Omega}} \{1 + a(\delta, \partial\tilde{\Omega})(\mathbf{x})\} \frac{|\mathbf{f}(\mathbf{x})|^{\mathbf{p}}}{\delta(\mathbf{x})^{\mathbf{p}}} \mathbf{dx},$$

for all $f \in C_0^\infty(\tilde{\Omega})$, is considered for $p \in (1, \infty)$, where $\tilde{\Omega}$ can be either Ω or $\mathbb{R}^n \setminus \Omega$ with Ω a domain in $\mathbb{R}^n$, $n \geq 2$, and $\delta(\mathbf{x})$ is the distance from $\mathbf{x} \in \tilde{\Omega}$ to the boundary $\partial\tilde{\Omega}$. The main emphasis is on determining the dependance of $a(\delta, \partial\tilde{\Omega})$ on the geometric properties of $\partial\tilde{\Omega}$. A Hardy inequality is also established for any doubly connected domain Ω in $\mathbb{R}^2$ in terms of a uniformisation of Ω, that is, any conformal univalent map of Ω onto an annulus.

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