ON UNIFORM NON-ℓ_1^n-NESS FOR DIRECT SUMS OF BANACH SPACES

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Abstract. We shall first discuss the uniform non-ℓ_1^n-ness for the direct sum of Banach spaces X and Y with the norm associated with a convex function ψ on the unit interval. Next, as the extreme cases, the uniform non-ℓ_1^n-ness of the ℓ_1- and ℓ_∞-sums for fintely many Banach spaces will be mentioned.

Let Ψ be the family of all convex functions ψ on $[0,1]$ satisfying

$$\psi(0) = \psi(1) = 1 \text{ and } \max\{1-t, t\} \le \psi(t) \le 1 \ (0 \le t \le 1). \tag{1}$$

For any $\psi \in \Psi$ define

$$\|(z,w)\|_\psi = \begin{cases} (|z|+|w|)\psi\left(\frac{|w|}{|z|+|w|}\right) & \text{if } (z,w) \ne (0,0), \\ 0 & \text{if } (z,w) = (0,0). \end{cases} \tag{2}$$

Then $\|\cdot\|_\psi$ is an absolute normalized norm on $\mathbb{C}^2$, that is,

$$\|(z,w)\| = \|(|z|,|w|)\| \text{ and } \|(1,0)\| = \|(0,1)\| = 1, \tag{3}$$

and $\|\cdot\|_\psi$ satisfies

$$\psi(t) = \|(1-t,t)\|_\psi \ (0 \le t \le 1). \tag{4}$$

Conversely for any absolute normalized norm $\|\cdot\|$ on $\mathbb{C}^2$, let $\psi(t) = \|(1-t,t)\|$ $(0 \le t \le 1)$. Then $\psi \in \Psi$. Thus Ψ and the collection N_a of all absolute normalized norms on $\mathbb{C}^2$ correspond in a one to one way (Bonsall and Duncan, "Numerical ranges II", 1973).

The ℓ_p-norms $\|\cdot\|_p$ are typical examples of this situation. The convex functions ψ_p corresponding to the ℓ_p-norms are given by

$$\psi_p(t) = \begin{cases} \{(1-t)^p + t^p\}^{1/p} & \text{if } 1 \le p < \infty, \\ \max\{1-t, t\} & \text{if } p = \infty. \end{cases} \tag{5}$$

It is immediate to see that for all $\psi \in \Psi$

$$\|\cdot\|_\infty \le \|\cdot\|_\psi \le \|\cdot\|_1 \tag{6}$$

The ψ-direct sum $X \oplus_\psi Y$ of Banach spaces X and Y is the direct sum $X \oplus Y$ equipped with the norm $\|(x,\, y)\|_\psi = \|(\|x\|,\, \|y\|)\|_\psi$. This extends the notion of the ℓ_p-sum $X \oplus_p Y$.

The key result for this talk is the following (Kato-Saito-Tamura, MIA, 2004): $X \oplus_\psi Y$ is uniformly non-square if and only if X and Y are uniformly non-square and neither $\psi = \psi_1$ nor $\psi = \psi_\infty$, where $\psi_1(t) = 1$ and $\psi_\infty(t) = \max\{1-t, t\}$ are the convex functions corresponding to the ℓ_1- and ℓ_∞-norms, respectively.

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