MAXIMAL FUNCTION AND RELATED OPERATORS ON L^1

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Abstract. One of the puzzling features of many important integral operators in analysis is their behaviour near L^1. A prominent example is the maximal function M, its relations with extrapolation of L^p spaces to logarithmic L^1 spaces (Stein's and Yano's celebrated results) for $p > 1$, and the behaviour of M on $L^1(\log L)^\alpha$. Passing to L^1 as the domain of M causes certain problems since the defect of the integrability of M changes and does not follows the usual rule—it is not generally true that $Mf \in L^1(\log L)^{-1}$ locally if $f \in L^1$. We will look at the role of the grand Lebesgue spaces here and links to the Kolmogorov inequality. We will show, via appropriate extrapolation procedure, that the grand Lebesgue spaces allow for improving the range of M considered on L^1 and show that this is the behaviour pattern for various clones of M and for singular integrals in the general setup of homogeneous spaces, even with non-doubling measures.

The contents of the talk is an amalgam of joint results with A. Fiorenza and V. Kokilashvili.

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