RADEMACHER INTEGRABILITY

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Let us agree that a measurable function $f\colon [0,1] \to \mathbb{R}$ is *Rademacher integrable* if

$$\int_0^1 |f(t)| \cdot \left| \sum_{n=1} a_n r_n(t) \right| dt < \infty,$$

for all $(a_n) \in \ell^2$. Denote by $\Lambda(L^1)$ the set of all such functions.

Problem: Give an *intrinsic* description of Rademacher integrable functions.

Comments. 1. It is known that Rademacher integrable functions cannot be described by their distribution functions, that is, the space $\Lambda(L^1)$ is not symmetric (rearrangement invariant), [1, Theorem 2.1].

2. Simple examples of Rademacher integrable functions follow from Khintchine inequalities:

$$f \in \bigcup_{q>1} L^q([0,1]) \quad \Rightarrow f \in \Lambda(L^1).$$

Moreover, the *symmetric kernel* of the space $\Lambda(L^1)$ –i.e., the largest symmetric space continuously embedded into $\Lambda(L^1)$– is known to be Zygmund space $L(\log L)^{1/2}$, [1, Theorem 2.8, Remark 2.10, Corollary 2.11], consisting on all measurable functions $g\colon [0,1] \to \mathbb{R}$ such that

$$\int_0^1 g^*(t) \log^{1/2}(e/t)\, dt.$$

Thus, $L(\log L)^{1/2} \subsetneq \Lambda(L^1)$.

3. It might be of interest for solving of the stated problem to exhibit non-trivial examples of Rademacher integrable functions, that is,

$$f \in \Lambda(L^1) \setminus L(\log L)^{1/2}.$$

References

[1] S. V. Astashkin and G. P. Curbera, Symmetric kernel of Rademacher multiplicator spaces, *J. Funct. Anal.* **226** (2005) 173–192.

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