

NEW CALDERON COUPLES OF LORENTZ SPACES WITH SOME APPLICATIONS

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Abstract. We prove that two Lorentz spaces $\Lambda_{\alpha_0}, \Lambda_{\alpha_1}$ form a K -monotone couple if even they are very close or equal to L_1 . This allows us to study the function sets $G = \{f : g(t) = t^{-\theta}(f^{**} - f^*) \in A, f^*(\infty) = 0\}$ for various $\theta \in (0, 1)$ and r.i. spaces A . In particular, we show that if the norms of two r.i. spaces B_1, B_2 are equivalent on the functions $g(t)$, generated by all f from some G , then $B_1 = B_2$. And if A is interpolation in the couple L_1, Λ_{t^θ} , then the set G becomes linear and normable with the norm $\|f\|_G \sim \|g\|_A$. Moreover, if the space A itself is not interpolation in the couple L_1, Λ_{t^θ} and the space \tilde{A} is its smallest interpolation extension, then the space \tilde{G} with the norm $\|g\|_{\tilde{A}}$ is the smallest r.i. extension of the set G . As a simple example we obtain that the space $\exp L$ is the smallest r.i. space containing BMO .

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