ON GENERALIZED K - FUNCTIONALS AND MODULI OF SMOOTHNESS RELATED TO TRIGONOMETRIC APPROXIMATION

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Abstract. The aim of the talk is to present a unified approach to trigonometric approximation in spaces L_p for all $p,\ 0 < p \leq \infty$. We deal with so-called families of linear trigonometric polynomial operators $(\mathcal{L}_n^\varphi)_{n\in\mathbb{N}}$ defined via

$$\mathcal{L}_n^\varphi f(x,\lambda) = \frac{1}{2N+1}\sum_{\nu=0}^{N} f(t_N^\nu+\lambda)W_n^\varphi(x-\lambda-t_N^\nu)$$

$(x,\lambda)\in\mathbb{T}^2$, $N=[rn]$, $t_N^\nu = \frac{2\pi\nu}{2N+1}$, $f\in L_p(\mathbb{T})$, where

$$W_n^\varphi(y) = \sum_{k\in\mathbb{Z}}\varphi(k/n)e^{i\,ky}$$

is a kernel generated by a continuous function with compact support in $[-r,r]$ satisfying the conditions $\varphi(0)=1$, $\varphi(-\cdot)=\overline{\varphi(\cdot)}$. Convergence (if $n\to\infty$) is considered in the the space $L_p(\mathbb{T}^2)$. If $1\leq p\leq\infty$ we recover the convergence of classical approximation processes in the spaces $L_p(\mathbb{T})$ and $C(\mathbb{T})$, respectively. We discuss

- necessary and sufficient conditions for convergence
- equivalences of approximation error and polynomial K-functionals related to appropriate differential operators
- equivalences of approximation error and generalized moduli of smoothness
- special kernels and extensions to the multivariate case

The talk is based on joint work with K. Runovski (Sevastopol).

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