Complex Interpolation and Entropy Explosion

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Abstract.

Let $\bar{A} = (A_0, A_1)$ and $\bar{B} = (B_0, B_1)$ be Banach couples, let $T : \bar{A} \to \bar{B}$ be a bounded linear operator and let $\mathcal{F}$ be a real interpolation functor of exact type θ. Recently D.E. Edmunds and Yu. Netrusov* proved that there cannot exist a general simple formula relating the entropy numbers of $\mathcal{F}(T) : \mathcal{F}(\bar{A}) \to \mathcal{F}(\bar{B})$ to those of T_0, T_1. We explore the same phenomenon in the case of complex interpolation showing that such a formula fails to exist even in cases in which it has been established that the interpolated operator is compact if the restriction of T to one of A_0 or A_1 is compact. We call this phenomenon the entropy explosion.

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