Structure of the set of hypercyclic functions for some classical hypercyclic operators

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The first two examples of hypercyclic operators were given in the space $\mathcal{H}(\mathbb{C})$ of entire functions endowed with the compact topology. In 1929, Birkhoff saw that the translation operator is hypercyclic on $\mathcal{H}(\mathbb{C})$. Later, MacLane proved that the derivative operator is also hypercyclic. In both cases, the authors provide the construction of a hypercyclic function. The Baire Category Theorem provides a G_{δ} dense set of hypercyclic vectors for both of them. Besides, they also share a common dense manifold of hypercyclic vectors. In this talk we will show that no power of any entire function is hypercyclic for Birkhoff's translation operator on $\mathcal{H}(\mathbb{C})$. On the other hand, we see that the set of functions whose powers are all hypercyclic for MacLane's differentiation operator is a G_{δ} -dense subset of $\mathcal{H}(\mathbb{C})$. Related results on the linear structure of certain subsets of entire functions will also be presented.