FLINS Conference, Madrid (Spain), September 22, 2008

Fuzzy Logic—A New Look

Lotfi A. Zadeh*

Extended Abstract

There are many misconceptions about fuzzy logic. To begin with, fuzzy logic is not fuzzy. In large measure, fuzzy logic is precise. Basically, fuzzy logic is concerned with precisiation of imprecision. In fact, one of the most important features of fuzzy logic is its high power of precisiation. It should be noted that the concept of precisiation has a position of centrality in the new look but plays no role in the traditional views of fuzzy logic. In the following, fuzzy logic is looked at in a new perspective. The new look serves to clarify what fuzzy logic is and what it has to offer.

The cornerstones of fuzzy logic are graduation, granulation, precisiation and the concept of a generalized constraint. The point of departure in fuzzy logic is the concept of a fuzzy set. Informally, a fuzzy set, *A*, in a universe of discourse, *U*, is a class with a fuzzy boundary. A set is a class with a crisp boundary. A set is precisiated through association with a characteristic function, c_A: $U \longrightarrow \{0,1\}$. A fuzzy set is precisiated through graduation, that is, through association with a membership function μ_A: $U \longrightarrow [0,1]$ or, more generally, a lattice, with $\mu_A(u)$, $u \varepsilon U$, representing the grade of membership of *u* in *A*. Membership in *A* is a matter of degree. In fuzzy logic everything is or is allowed to be a matter of degree. A concept is fuzzy if its denotation is a fuzzy set. In large measure, concepts—both natural and synthetic—are fuzzy rather than crisp. In fuzzy logic, fuzzy concepts are precisiated through granulation.

With the concept of a fuzzy set as the point of departure, one can move in various directions, leading to various facets of fuzzy logic. A direction which terminates on a theory, *T*, is an instance of what is referred to as FL-generalization. FL-generalization of *T* introduces into *T* the concept of a fuzzy set. The concept of a fuzzy set serves as a point of entry into *T* of other concepts and techniques drawn from fuzzy logic. A facet of *A* is a FL-generalization of a theory, *T,* or FL-generalization of a collection of related theories. The result of FL-generalization of *T* is expressed as fuzzy *T*. Examples: fuzzy topology, fuzzy algebra, fuzzy control, etc.

It should be noted that usually a prefix, *M*, which modifies a suffix, *X*, has the effect of specializing *X*, as in convex set. Unusually, *M* generalizes *X*. As a modifier, fuzzy falls into this category. Many misconceptions about fuzzy logic stem from misinterpretation of fuzzy as a specializer rather than a generalizer.

The principal facets of fuzzy logic are: the logical facet, FLl; the fuzzy-set-theoretic facet, FLs, the epistemic facet, FLe; and the relational facet, FLr. The logical facet of FL, FLl, is fuzzy logic in its narrow sense. FLl may be viewed as a generalization of multivalued logic. The agenda of FLl is similar in spirit to the agenda of classical logic. The fuzzy-set-theoretic facet, FLs, is focused on FL-generalization of set theory. Historically, the theory of fuzzy sets (Zadeh

* Department of EECS, University of California, Berkeley, CA 94720-1776; Telephone: 510-642-4959; Fax: 510-642-1712;
E-Mail: zadeh@eecs.berkeley.edu . Research supported in part by ONR N00014-02-1-0294, BT Grant CT1080028046, Omron Grant, Tekes Grant, Chevron Texaco Grant and the BISC Program of UC Berkeley.

1965) preceded fuzzy logic (Zadeh 1975c). The theory of fuzzy sets may be viewed as an entry to generalizations of various branches of mathematics, among them fuzzy topology, fuzzy measure theory, fuzzy graph theory and fuzzy algebra. The epistemic facet of FL, FLe, is concerned in the main with knowledge representation, semantics of natural languages, possibility theory, fuzzy probability theory, granular computing, computing with words and the computational theory of perceptions. The relational facet, FLr, is focused on fuzzy relations and, more generally, on fuzzy dependencies. The concept of a linguistic variable—and the associated calculi of fuzzy if-then rules—play pivotal roles in almost all applications of fuzzy logic.

As was noted earlier, the cornerstones of fuzzy logic are graduation, granulation, precisiation and the concept of a generalized constraint. Graduation serves to precisiate the meaning of fuzzy concepts. The concept of granulation is unique to fuzzy logic. The concept of granulation is inspired by the way in which humans deal with complexity and imprecision. Basically, granulation of an object, *A*, partitions *A* into a collection of granules. Informally, a granule is a clump of elements drawn together by indistinguishability, proximity, similarity or functionality. Granulation may be viewed as a generalization of quantization and an instance of the principal of divide and conquer.

A singular variable is a variable which takes singletons as values, while a granular variable is a variable which takes granules as values. A linguistic variable is a granular variable with linguistic labels of granular values. The results of precisiation and granulation of *p* are denoted as *p** and **p*, respectively. Granulation may be applied to objects of arbitrary complexity. In particular, granulation may be applied to variables, functions, relations, dynamical systems, etc. Granulation transforms a singular value of a variable, *X*, into a granular value, **X*, with the granular value, **X*, representing the state of information about the singular value, *X*. Summarization and aggregation of information may be viewed as forms of granulation.

The concept of precisiation has a position of centrality in fuzzy logic. In fuzzy logic, a basic differentiation is made between precision of value (v-precision) and precision of meaning (m-precision). A further differentiation is made between two modalities of m-precisiation: human-oriented m-precisiation (mh-precisiation) and machine-oriented mm-precisiation (mm-precisiation), with the understanding that mm-precisiation is mathematically well-defined. In this perspective, a dictionary definition involves mh-precisiation.

The object of precisiation, *p*, and the result of precisiation, *p**, are referred to as precisiend and precisiand, respectively. Modelization may be viewed as a form of precisiation, with m-precisiend playing the role of the object of modelization and m-precisiand being the result of modelization. In the context of modelization, mh-precisiand and mm-precisiand are referred to as the h-model and m-model, respectively.

In general, a precisiend, *p*, has a multiplicity of precisiands, *p**. An important attribute of *p** is the proximity of the meaning of *p** to the meaning of *p*. A qualitative measure of this proximity is referred to as cointension. The choice of the term cointension to describe proximity of meanings is related to the fact that in logic the term intension is employed to label attribute-based meaning. An m-precisiation of *p* as *p** is cointensive if the cointension of *p** in relation to *p* is high.

In defining a concept, cointensive mm-precisiation is an important desideratum. The same applies to m-modelization. Viewed as an mm-precisiation language, one of the most important contributions of fuzzy logic is its high power of cointensive mm-precisiation.

A m-model of *p*, *p**, is associated with two important metrics: cointension and computational complexity. In general, cointension and computational complexity are covariant in the sense that an increase in cointension is associated with an increase in computational complexity. A good model is one which achieves a desired compromise between high cointension and low computational complexity.

Precision carries a cost. In many real-world settings there is a tolerance for imprecision—a tolerance which may be exploited to achieve lower cost. An effective tool which is provided by fuzzy logic for this purpose is referred to as the fuzzy logic gambit, or FLG, for short. FLG involves a deliberate imprecisiation of a system, *S*, through granulation, followed by mm-precisiation of granular variables which are associated with **S*. A system, *S*, is granulated through representation of its input-output relations as fuzzy if-then rules—rules which involve linguistic variables. In one form or another, FLG is employed extensively in the realm of consumer products—a realm in which cost is an important consideration.

A basic question which arises is: Given *p*, how can *p* be mm-precisiated? A key idea in fuzzy logic is that of employing for this purpose the concept of a generalized constraint (Zadeh l996, l999).

The concept of a constraint is high on the list of basic concepts in science. There is an extensive literature. A basic assumption which is commonly made in the literature is that constraints are hard (inelastic) and are precisely defined. This assumption is not a good fit to reality. In most realistic settings, constraints have some elasticity and are not precisely defined. This applies, in particular, to the meaning of words, predicates and propositions drawn from natural language. In large measure, the concept of a generalized constraint is intended to provide a basis for construction of a maximally expressive meaning precisiation language for natural languages. A generalized constraint is an expression of the form *X* isr *R*, where *X* is the constrained variable, *R* is the constraining relation and *r* is an indexical variable which defines the modality of the constraint, that is, its semantics. The principal modalities are: possibilistic (r = blank), probabilistic ($r = p$), veristic ($r = v$), usuality ($r = u$) and group ($r = g$). The primary constraints are possibilistic, probabilistic and veristic. The standard constraints are bivalent possibilistic, probabilistic and bivalent veristic. In large measure, scientific theories are based on standard constraints.

Generalized constraints may be combined, projected, qualified, propagated and counterpropagated. The set of all generalized constraints, together with the rules which govern generation of generalized constraints from other generalized constraints, constitute the Generalized Constraint Language (GCL). Actually, GCL is more than a language—it is a language system. A language has descriptive capability. A language system has descriptive capability as well as deductive capability.

The concept of a generalized constraint plays a key role in fuzzy logic. In particular, it serves two major functions. First, as a means of representing the meaning of words, predicates and propositions as generalized constraints; and second, through representation of words, predicates and propositions as generalized constraints it serves as a means of dealing with words, predicates and propositions as objects of computation. A linkage between the concept of a generalized constraint and mm-precisiation derives from the fundamental thesis of fuzzy logic: information = generalized constraint. A consequence of the fundamental thesis is the meaning postulate: mm-precisiated meaning = generalized constraint. What this implies is that the Generalized Constraint Language, GCL, may be viewed as a meaning precisiation language. More importantly, GCL provides a basis for computation with information described in natural language. This is the province of computing with words, CW. The point of departure in CW is an information set, *I*, and a question, *q*. The information set consists of a system of propositions drawn from a natural language, $I = (p_1, \ldots, p_n, p_{wk})$, where $p_1, \ldots, p_n$ are given and p_{wk} is drawn from world knowledge. The problem is to derive an answer to q, ans(I/q), given I. Computation of q involves two phases. In Phase 1, the p_i are precisiated as generalized constraints. In Phase 2, the generalized constraints serve as objects of computation and deduction. The rules governing computation/deduction are, in the main, the rules which govern propagation and counterpropagation of generalized constraints. The principal computation/deduction rule is the extension principle (Zadeh 1965, 1975). There are many versions of the extension principle. In the basic version of the extension principle, the premise is a possibilistic constraint, $f(X)$ is A, where f is a given function from U to V and A is a fuzzy set in V. The problem is to find the induced possibilistic constraint on $g(X)$, $g(X)$ is B, where g is a given function from U to W and B, a fuzzy set in W, is the result of computation. Computation of B is reduced to the solution of a variational problem. The methodology of computing with words and the related methodology of granular computing are among the major contributions of fuzzy logic.

In summary, underlying the new look is a simple idea—using the concept of a fuzzy set as the nucleus of fuzzy logic. This idea is the point of departure for FL-generalization. FL-generation of a theory, T, involves introduction of the concept of a fuzzy set into T, followed by entry into T of other concepts and techniques drawn from fuzzy logic. In coming years, FL-generalization is likely to be applied to a growing number of scientific theories, including various branches of mathematics.

A major contribution of fuzzy logic is its high power of cointensive precisiation. The high power of cointensive precisiation enhances the capability of fuzzy logic to serve as a basis for the analysis and design of systems in which imprecision, uncertainty, human judgment and perceptions play important roles. This applies in particular to systems which do not lend themselves to realistic analysis through the use of methods based on bivalent logic. Such systems are common in the realms of economics, medicine, law, linguistics and decision analysis.

The fuzzy-logic-based methodologies of computing with words and granular computing open the door to computation with information described in natural language. Much of human knowledge is described in natural language. In coming years, computing with words and granular computing are likely to lead to a wide-ranging enlargement of the role of natural languages in scientific theories.