MASTER IN MATHEMATICAL ENGINEERING

GENERATING A CAPLET VOLATILITY SURFACE

IX MODELLING WEEK

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Colaboration: Banco Popular, University Complutense of Madrid, Oxford University

Contents

Presentation

This work has been realized in order to conclude the IX Modelling Week that offers the University Complutense of Madrid.

In this edition, topics of statistic, artificial intelligence, nanotechnology, and finance have been proposed by different companies. We have chosen “Generating a caplet volatility surface” proposed by Banco Popular. This choice has been guided by our interest in finances.

Working on this project has been very gratifying as a personal experience as much as a professional experience. For this reason we want to be grateful to all the people who have made this project possible, especially to Gerardo Oleaga for helping us and Banco Popular for presenting the topic.

Abstract

This project is about how to valuate prices of a Cap with the problem that has arisen these last years. Nowadays, interest rates can be negative. Years ago, this assumption was not questionable.

Specifically, the great extent of quoted interest rates very close to zero and negative quoted forward rates has led to a correction of the assumption of lognormality towards the normal distribution. Here, we study this new assumption and consequences to the economic factors it may imply.

Introduction

That structure that we are going to follow for this work is the one detailed below.

First we are going to comment the description of the problem and the definitions and concepts needed to have a better understanding of the development of the work.

The second section develops in detail Black Scholes formula assuming that the prices follow a normal distribution.

As practice differs from theory, in the third section we propose different models for computing the implied caplet volatility using Normal formula. We start from the model that Banco Popular proposed and develop different models to improve the results. We can compute the implied caplet volatility using linear, exponential, quadratic models… In the same way we can compute the prices of a caplet interpolating the flat volatility or other parameters. The third section presents different models and its results. Some of them are more accurate regarding the price computed with the Black-Scholes formula assuming the normal distribution and other models present more smoothness with his volatilities. In this section, we also study a constrained optimization problem, which allows us to obtain accuracy and smoothness.

In the fourth point, we explain the conclusions that we have been obtained in this project, and the evolution of the models and their results.

At last, to conclude this work we enumerate ideas for improving this valuation model.

1. Description of the problem and basic knowledge

To understand the problem it is necessary to know what a Caplet and a Cap are. A Caplet is an option similar to a Call option but in this case the underling is an interest rate. So, a Caplet is a contract in where we fix an interest called Strike K at time S and if at time T the Forward is higher than K, the Caplet Option pays us the difference. In the other case it pays us nothing.

A Cap is the sum of Caplets. For example if we have a Cap which has a two years maturity that pays each six months the Cap is composed by four Caplets.

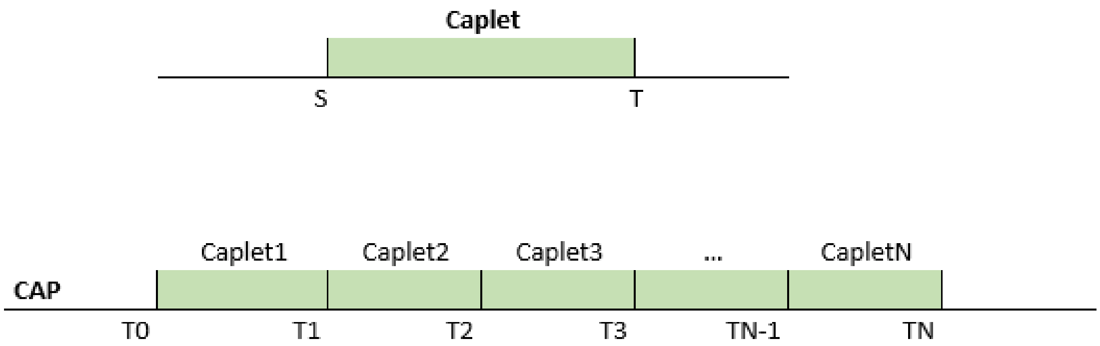


Image 1: Image of a Caplet and a Cap

Years ago, it was possible to compute the prices of a Caplet with Black-Scholes formula:

$$PriceCaplet = N\tau P_0(T)\big(F(S,T)\Phi(d_1) - K\Phi(d_2)\big)$$

$$d_1 = \frac{\log\left(\frac{F}{K}\right) + \frac{\nu^2}{2}}{\nu} \; and \; d_2 = d_1 - \frac{\nu^2}{2}$$

With F = Forward, $\nu = \sigma * \sqrt{T - t_0}$, t_0 date of valuation and T maturity of the Caplet.

We can observe that this valuation formula breaks down, technically speaking, as it contains terms such as $log(F/K)$ that are only defined for a positive strike rates K and for positive forward rates F. This is hardly surprising, as the Black model operates on the core assumption that the underlying values are positive.

As a result, current negative interest rates require modified models as new standards, which should nevertheless be as simple as possible.

To develop the new model we can use any distribution that allows working with negative values. We have taken the Normal one since it is very simple to work with it.

2. Normal distribution

Consider a market variable process X_t for time t starting at 0. We assume the following facts:

1. The existence of a probability measure that gives the price of any contract with pay-off $Pay\_Off(X_T)$ by means of computing:

$$P_0(T)E[Pay\_Off(X_T)],$$

   Where $P_0(T)$ is the zero coupon bond price for maturity T.

2. In this measure, we assume the X_t can be modeled by:

$$dX_t = \nu dW_t$$

   Where W_t represents the Brownian motion (it follows N(0,1)), ν depends on T (maturity), on t_0 (time in which fix the contract) and σ, volatility of X_t. The solution is given by:

$$X_t = X_0 + \nu W_t$$

In our case, X_t is the forward rate F_t . So, $F_t = F_0 + \nu W_t$.

It follows directly that the volatility σ determines the absolute size of the fluctuation around the starting value, in contrast to the lognormal model where the volatility influences the relative deflection. Within a normal model, plain vanilla options also possess a closed form analytical solution.

We know that the distribution function of the normal distribution and the derivative of the cumulative normal distribution are:

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt$$

$$F^{'}(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

We consider the general case of a vanilla Call on the variableF_t:

$$Pay\_0ff(x) = \max(x - K, 0).$$

We want to compute the expected value of this variable with the assumptions done before. Then:

$$E[Pay\_Off(X_t)] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} \max(X_0 + vz - K, 0)\, e^{-\frac{z^2}{2}} dz =$$

$$= \frac{1}{\sqrt{2\pi}}\int_{\frac{K-X_0}{v}}^{+\infty} (X_0 + vz - K) e^{-\frac{z^2}{2}} dz =$$

$$= \frac{X_0 - K}{\sqrt{2\pi}}\int_{\frac{K-X_0}{v}}^{+\infty} e^{-\frac{z^2}{2}} dz + \frac{v}{\sqrt{2\pi}}\int_{\frac{K-X_0}{v}}^{+\infty} z e^{-\frac{z^2}{2}} dz =$$

$$= (X_0 - K)\Phi(d) + \frac{v}{\sqrt{2\pi}} e^{-\frac{d^2}{2}}$$

where $d = \frac{(X_0 - K)}{v}$.

If we apply this formula to a Caplet with maturity T, starting at time S, interval Δt $(T - S)$, and strike K, we assume that the probability measure is the so-called forward risk measure which has the following property:

$$L_S(T) \equiv F_S(S, T)$$

and it is a martingale with respect to this probability distribution:

$$E[F_S(S, T)] = F_0(S, T)$$

Then

$$X_T = F_S(S, T) = X_0 + vZ$$

$$X_0 = F_0(S, T)$$

So applying the pricing formula to the caplet, we obtain:

$$Caplet(v, K, T) = N\Delta t P_0[(F_0 - K)\Phi(d) + \frac{v}{\sqrt{2\pi}} e^{-\frac{d^2}{2}}$$

where $d = \frac{(F_0 - K)}{v}$ and $v = \sigma\sqrt{T - t_0}$.

As a Cap is composed of Caplets, if we have a Cap for N years with payment each x-time, then the price of a Cap is:

$$PriceCap = \sum_{i}^{\left(\frac{N}{x}\right)} Caplet_i(v_i, K, T)$$

3. Models

In this section, we develop different models to compute the price of a caplet. Like we have said above, in practice, we have the prices computed using normal distribution and we have to compute the implied caplet volatility.

3.1 Banco Popular's Model

The recursive process of calculating volatility caplet from flat volatility is the same as in the case of stripping under a lognormal distribution hypothesis. It is summarized as follows, for each strike K:

- We obtain market prices for caps, for different maturities, using the cap (or flat) normal volatility.

$$PriceFlat = \sum_i PriceCaplet_i = \sum_i G(\sigma_{flat}, K, T_i)$$

- First, caplets volatilities are considered constant up to one year, because that is the first maturity marketable. This hypothesis is questionable.
- Maturity after maturity, we solve the cap price using cap volatilities versus caplet volatilities and minimizing the difference between them. We will need the solutions of the above steps.
- For all the extra unknown variables (for most of the tenors, the equations have several ones), we will assume a certain kind of linear interpolation between adjacent points.

Let's see that with an example:

Let T be 2 years and τ $6\ months$. That means that we have 4 Caplets:

The price for Cap 1 (T=1) is:

$$PriceCap1 = \sum_i PriceCaplet_i = \sum_i G(\sigma_i, K, T) = G(\sigma_1, K, T) = PriceFlat1$$

$$\rightarrow \sigma_1 = \sigma_{flat\,1}$$

The price for Cap 2 (T=2) is:

$$PriceCap2 = \sum_i PriceCaplet_i$$

$$= \sum_i G(\sigma_i, K, T) = G(\sigma_1, K, T) + G(\sigma_2, K, T) + G(\sigma_3, K, T) = PriceFlat2$$

In order to get the values of σ_2 and σ_3, we have to make them depend on each other:

$$\rightarrow \sigma_2 = F(\sigma_3)$$

How do we choose that dependency (F)?

Banco Popular chose a first way to approach that dependency with a simple Linear Interpolation Method. The results that they obtained can be seen on Image 2:

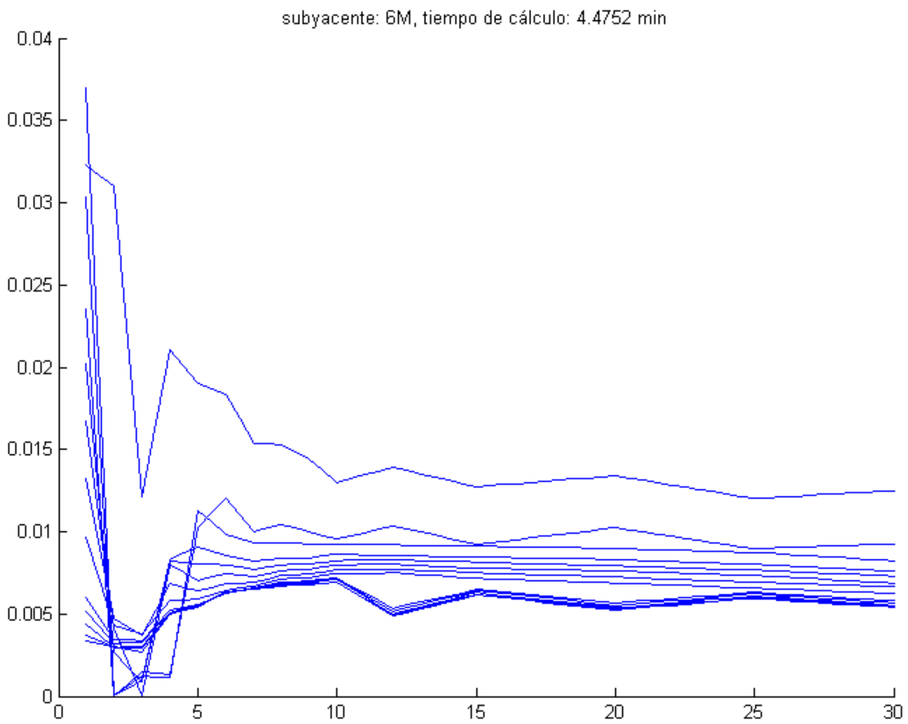


Image 2: Results after solving caplet volatility from cap market prices (underlying Euribor 6M)

We can observe that the graph is not smooth, and this is due to the non smoothness of the volatility function.

As our goal is to get accuracy and smoothness we are going to develop other models with linear and exponential interpolation for unknown variables and other methods.

3.2 Linear Models With Implied Caplet Volatilities

- Model 1:

First of all, we tried to improve the algorithm that the bank provided us. In order to find all the implied caplet volatilities, we decided to try this algorithm/method using the last caplet volatility (we already know it) and the last value of the caplet volatility (it's fixed).

σ_1	σ_2	σ_3 = ?	...	σ_{n-1}	σ_n
τ_1	τ_2	τ_3	...	τ_{n-1}	τ_n

$$\sigma_3 = \frac{\sigma_2 \tau_n + \sigma_n \tau_2}{\tau_3 + \cdots + \tau_n}$$

The results that we obtained (Image 3) aren't better than theirs so that's not what we're looking for, because even though the prices that we get using these caplet volatilities are pretty similar to the ones that we get using flat volatilities (market prices), we don't have a smooth curve, instead we have a peaky one which doesn't make any sense, financially speaking.

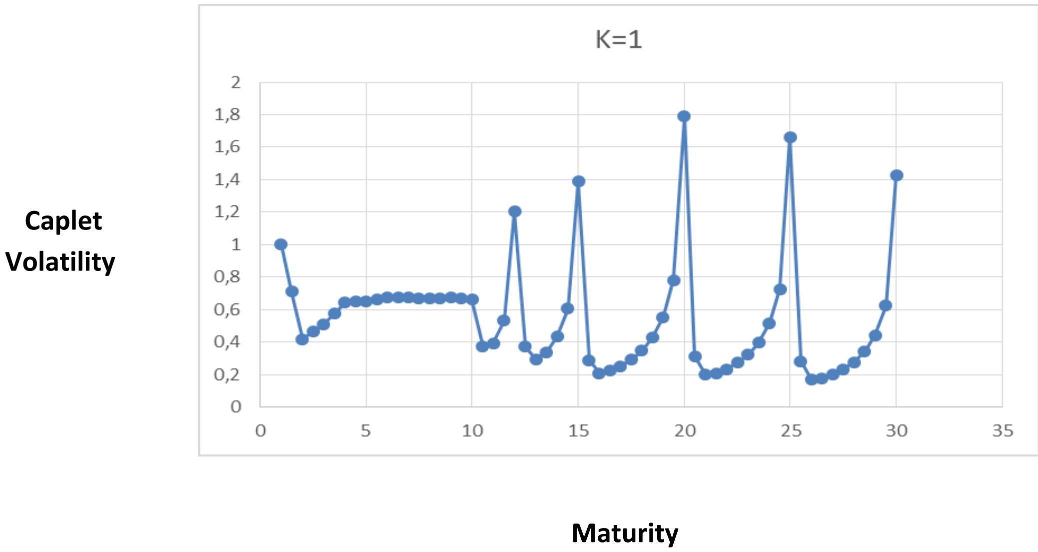
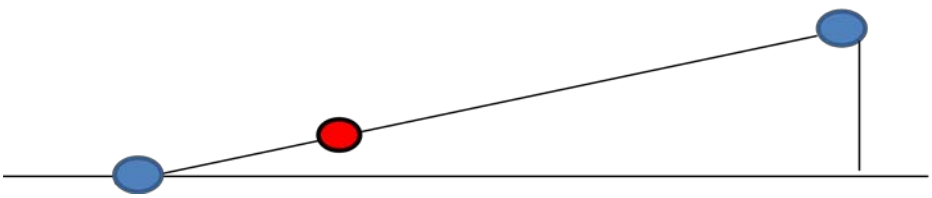


Image 3: Results after solving caplet volatility from cap market prices (underlying Euribor 6M) with Strike 1% using Model 1

Because of these results, we keep working on a new model that gave us what we want which is to find an accurate and smooth caplet volatility curve. Following this path, we came with this new idea for a new model:

- Model 2:

For this model, we'd used, as before, the last caplet volatility that we've known and the last one which it's the one that we're going to fix. To improve the last model that we'd proposed, we use a Root Finding Method. We can see the formula right below:



σ_1	σ_2	σ_3 = ?	…	σ_{n-1}	σ_n
τ_1	τ_2	τ_3	…	τ_{n-1}	τ_n

$$\sigma_3 = \frac{\sigma_n - \sigma_2}{\tau_2 + \tau_3 + \cdots + \tau_n}\tau_3 + \sigma_2$$

Thus, all the implied caplet volatilities (σ_i) are going to depend on the last caplet volatility of the Cap (σ_n). Minimizing the accuracy of the prices, as we did in the Banco Popular's model, is going to give us the last caplet volatility value, therefore all the implied caplet volatilities.

It seems like the results are not going to make a difference with the last ones but let's see what the graphs show (Images 4 and 5):

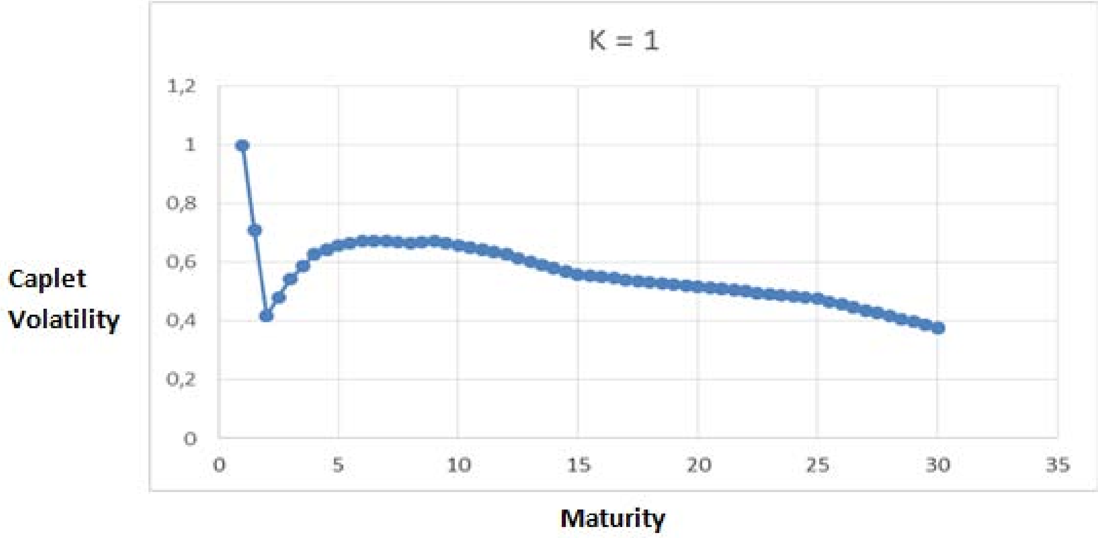


Image 4: Results after solving caplet volatility from cap market prices (underlying Euribor 6M) with Strike 1% using Model 2

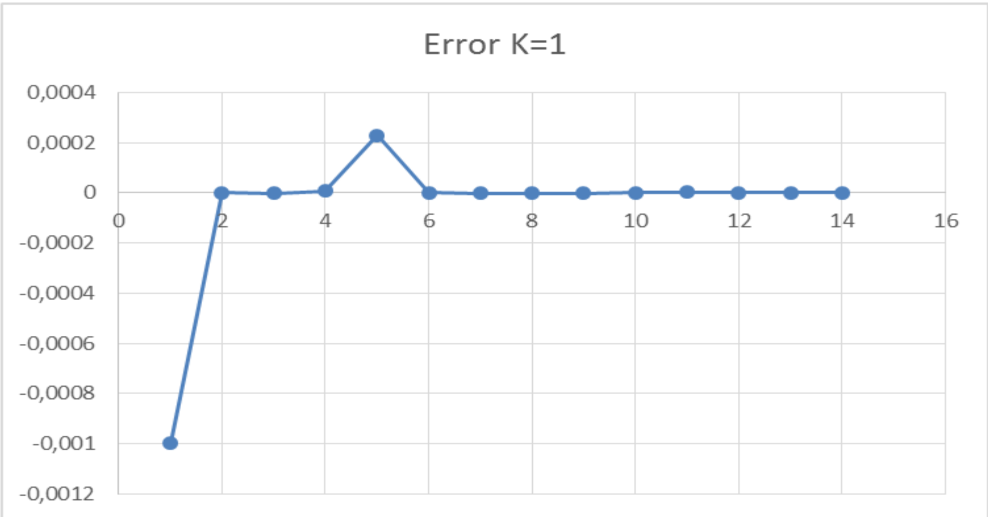


Image 5: Difference error between prices with implied caplet volatilities and prices with flat volatilities (underlying Euribor 6M) with Strike 1% using Model 2

When we look at the error graph, we can see how they're really low in the order of 10^{-3} which means that the prices we are getting using implied caplet volatilities are practically the same as the market prices.

But, what really makes a difference between the other idea is the caplet volatility curve. We can see how at the beginning of it, it makes a peak but that's normal because of the way that we fixed the first caplet volatility σ_1 which it's the same as the flat volatility that we'd used for the First Cap, but then we can see how the curve stabilizes at the end and that's what makes that we have a smooth curve which financially speaking has more sense than a peaky one.

Here it is the Caplet Volatility Surface which it's actually a pretty good result using Linear Interpolation with the Caplet Volatilities (Image 6):

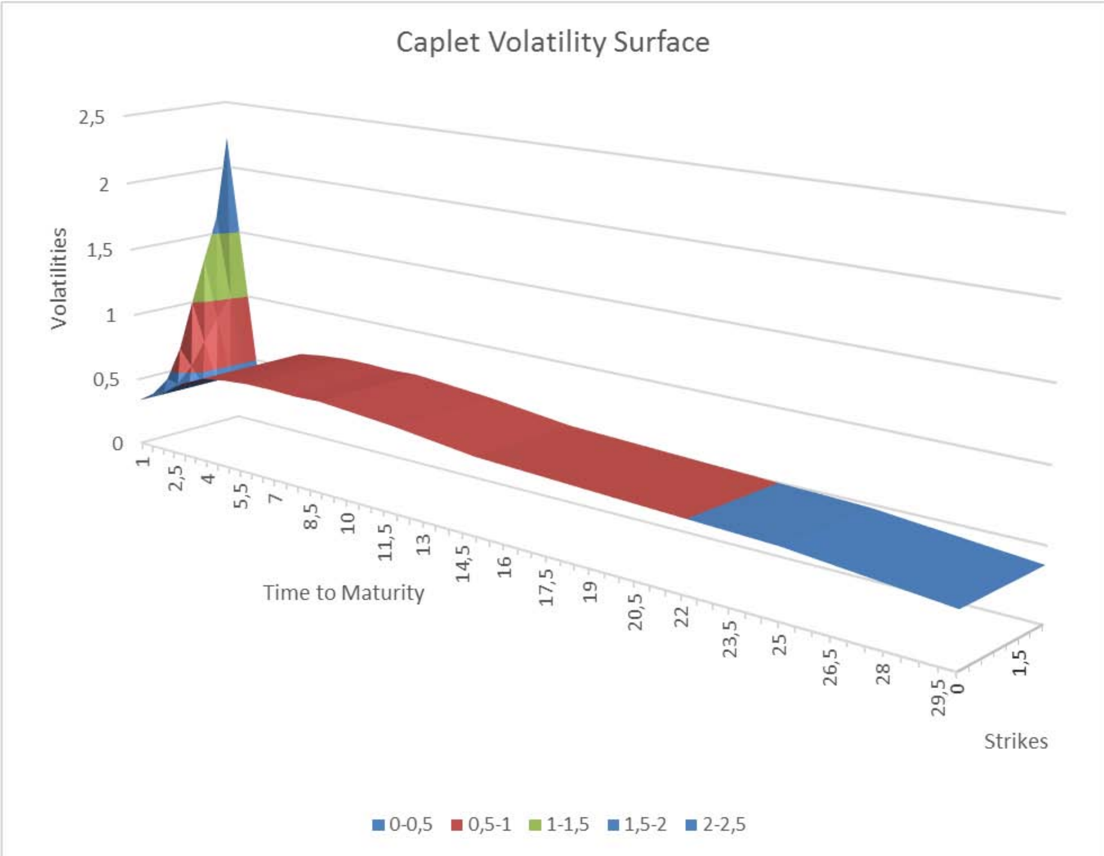


Image 6: Caplet Volatility Surface for different Strikes and Maturities using Model 2

Even though we've already achieve our goal, we want to check if we could get better results by changing the way of interpolation or maybe using some other kinds of methods.

3.3 Exponential Models

The linear solution to the problem has many benefits and it seems suitable for testing phase. However, it has some difficulties to apply to high strikes. Another setback for the linear model is its stiffness for very early maturity. Linear model does not have into account second derivative so the surfaces generated are sharp-edged.

The behavior of volatility as a function of time and strike is expected to be continuous and smooth. We want to find a model with those properties that resembles the data provided. The next graph shows the volatility surface generated with Bloomberg's data. Exponential functions can generate very similar surfaces, thus it deserves to be studied.

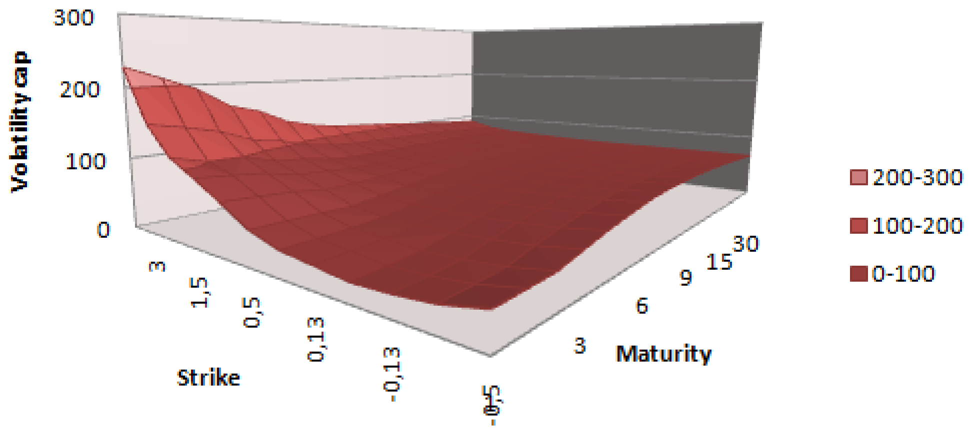


Image 7: Cap Volatility surface Bloomberg

We will take a family of exponential functions parametrized with four terms, as follows

$$\sigma(t) = (a + bt)e^{-dt} + c$$

being c as the constant term, d as the coefficient in the exponent and a y b as the coefficients for the linear exponential.

The first requirement is fulfilled, surfaces will be continuous and derivable. But there are more reasons to choose this family of functions:

- They are in concordance with the model. Terms in Normal formula belongs to this family and probability distributions.
- It avoids unbounded results which could have come up using asymptotic or polynomial families.
- The functions are stable and avoid stiffness so they are suitable for parameters variation.
- With just four parameters the family can be adapted to many shapes.

One of the most classical ways to fix the parameters is setting a cost function and tries to minimize it following an iterative method. The cost function has to be the norm 2 difference with the flat volatilities, in order to resemble the actual data.

$$\min_{a,b,c,d}\left(\sum_{t=1}^{T}\left\|(a+bt)e^{-d\cdot t}+c-\ \sigma_{flat}\,(k,t)\right\|^{2}\right)$$
$$with\ a, b, c, d\ \in \boldsymbol{R}\,, \qquad for\ each\ k\ in\ Strikes\ set.$$

This problem has been solved using the Matlab function ***lsqcurvefit***. It is an iterative process that solves nonlinear curve-fitting or data-fitting problems in least-squares sense. It is based in a trust region reflective algorithm.

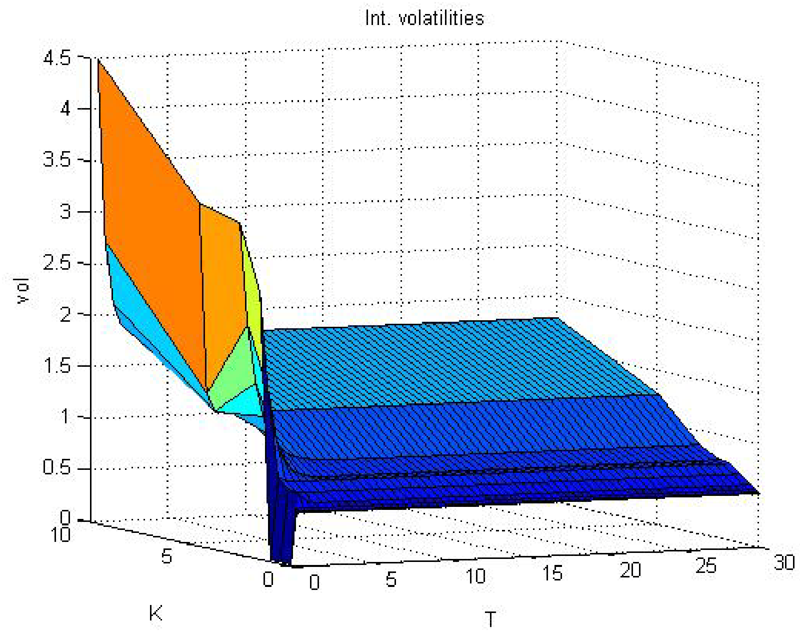


Image 8: Volatilities for different Strikes K and Maturities T after applying the Exponential Model previously explained

The previous diagram plots the results after solving. The analysis has been done between a Strike-Volatility Relationship. All the solutions were reached under the same stop criterion based on number of iterations. Thus, spending more time leads to better approximations.

Once we have the model for volatilities we can select the data for intermediate periods and compute the prices using the Normal formula for valuation. The result can be seen in the [Price Surface] graph. At first sight we get the smooth surface that we expected. But when it is compared with actual market prices (red dots in the graph) the model does not resemble real data. Specially, there is an important gap for big values of strike and maturity.

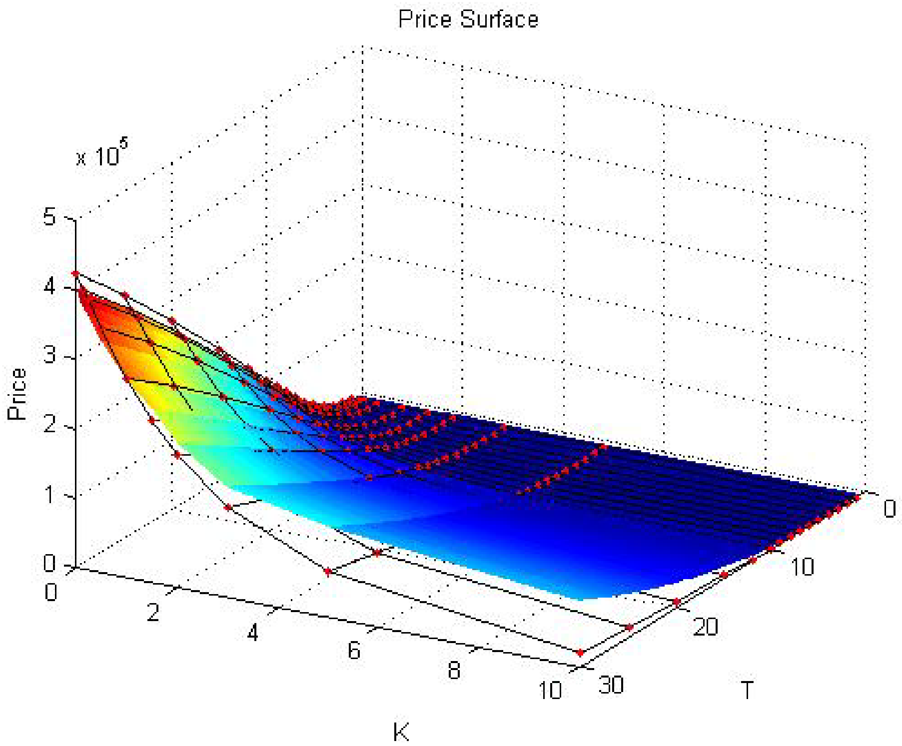


Image 9: Prices for different Strikes K and Maturities T with the volatility computed above and Prices with flat volatility.

The nominal we are using is 1 million €, so one may think that small errors computing flat volatilities can lead to huge errors in prices. Another way of proceed consists in going backwards. First we adapt the prices with the exponential functions and select the intermediate values. Volatilities can be computed by using an iterative method (normally a root method or a fixed-point algorithm) in Normal formula. Finally caplets can be valued by bootstrapping.

But the problem now is similar to the previous approach. In the following graph is represented the price surface computed directly with the actual market prices (red dots). The exponential functions adaptation makes big errors in the initial step. Better results may be obtained if we drop the global approach and focus in a local treatment.

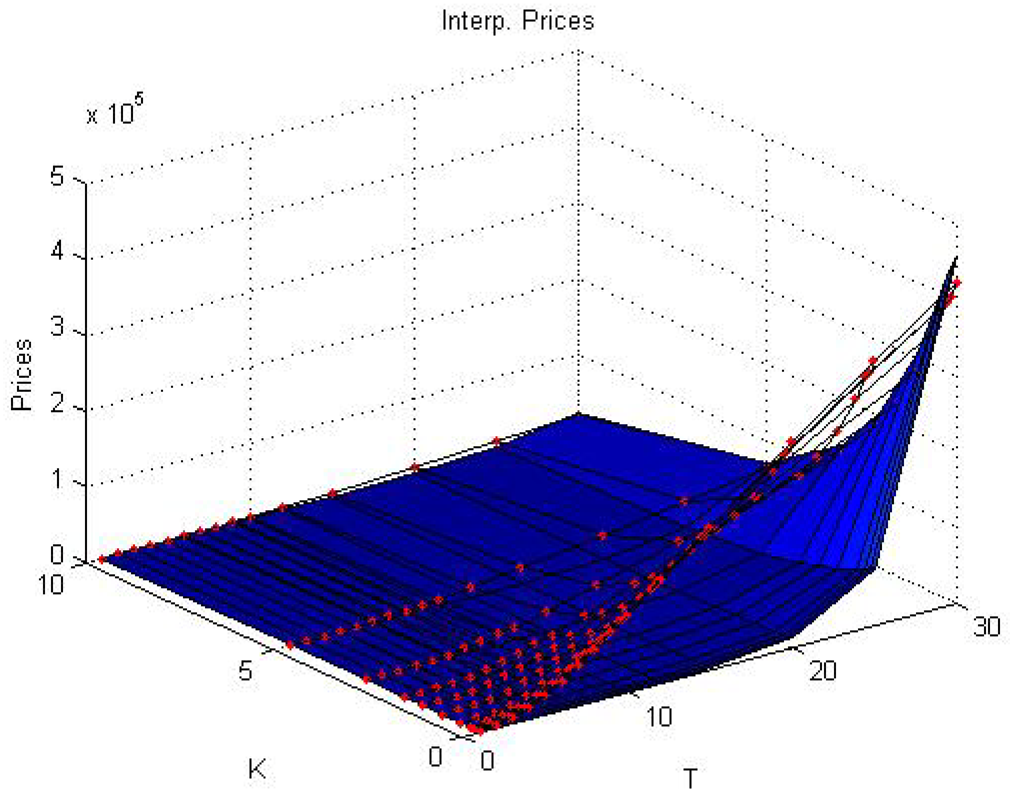


Image 10: Prices for different Strikes K and Maturities T using the backwards procedure detailed above

3.4 Flat Volatilities

We've already seen few methods using Linear and Exponential Interpolation with the caplet volatilities. In this section, we are going to do the same but instead of using caplet volatilities, we use flat volatilities. With this new approach, we want to improve our previous results if possible.

As before, we want to minimize the error between the prices that we have obtained with the flat volatilities and the ones that we have obtained with the "exact" caplets' valuation.

In order to achieve our new goal, we'd used the methods that we'd explained before based on the two nearest points. We remember these three methods below:

- Linear interpolation
- Quadratic interpolation
- Exponential interpolation

All of the methods are very interesting to study but we are going to focus only in one of them: Linear Interpolation using flat volatilities.

At the table below, we can see the flat volatilities values in a volatility curve which are given for different strikes (columns) and maturities (rows). If we look closer at the table, we don't have the flat volatilities values for all maturities and we need them in order to print the volatility curve. We are going to obtain those values interpolating the two nearest points to them lineally.

	0	0,125	0,25	0,5	1	1,5	2	3
1Y	0,33568	0,36065	0,44533	0,66173	0,99931	1,31356	1,62195	2,22692
2Y	0,29477	0,30966	0,35024	0,45209	0,62309	0,79433	0,95677	1,28959
3Y	0,38400	0,39400	0,40500	0,44900	0,55300	0,66300	0,77200	0,98700
4Y	0,44200	0,44900	0,45800	0,49500	0,58300	0,67900	0,77800	0,97600
5Y	0,49100	0,49600	0,50400	0,53600	0,61000	0,69000	0,77300	0,94300
6Y	0,52700	0,53200	0,53800	0,56500	0,63000	0,70000	0,77600	0,93300
7Y	0,55700	0,56000	0,56500	0,58900	0,64500	0,70800	0,77700	0,92500
8Y	0,57700	0,58000	0,58500	0,60600	0,65600	0,71300	0,77500	0,91100
9Y	0,59200	0,59500	0,60000	0,62000	0,66700	0,71800	0,77200	0,88600
10Y	0,60500	0,60800	0,61300	0,63200	0,67500	0,72200	0,77000	0,86900
12Y	0,60500	0,60800	0,63000	0,64600	0,68500	0,72500	0,76600	0,84900
15Y	0,60500	0,60800	0,64500	0,65800	0,69000	0,72500	0,76100	0,83700
20Y	0,60500	0,60800	0,65200	0,66300	0,68800	0,71600	0,74600	0,81000
25Y	0,60500	0,60800	0,64900	0,65800	0,68000	0,70500	0,73100	0,78800
30Y	0,60500	0,60800	0,64100	0,64900	0,66900	0,69100	0,71600	0,76800

Let's see all this with an example: If we need the volatility with Strike 2.5% and maturity 10 years, then we have to interpolate the two flat volatilities values corresponding to the ones with Strikes 2% and 3%. We can also do this using the rows instead of the columns which mean that if we want to get the volatility value for maturity 11 years with a Strike 0.5%, we have to interpolate the flat volatilities corresponding to the ones with Maturities 10 years and 12 years.

Now, imagine that the volatility that we want is for a Strike 2.5% and Maturity 11 years, this is a bit more complicate because the two flat volatilities values that we need to interpolate are the one with Strike 2% and Maturity 10 years and the other one is the flat volatility with a Strike 3% and Maturity 12 years.

We solve the linear interpolation using excel for Strike = 1% and we can see the results on the following image:

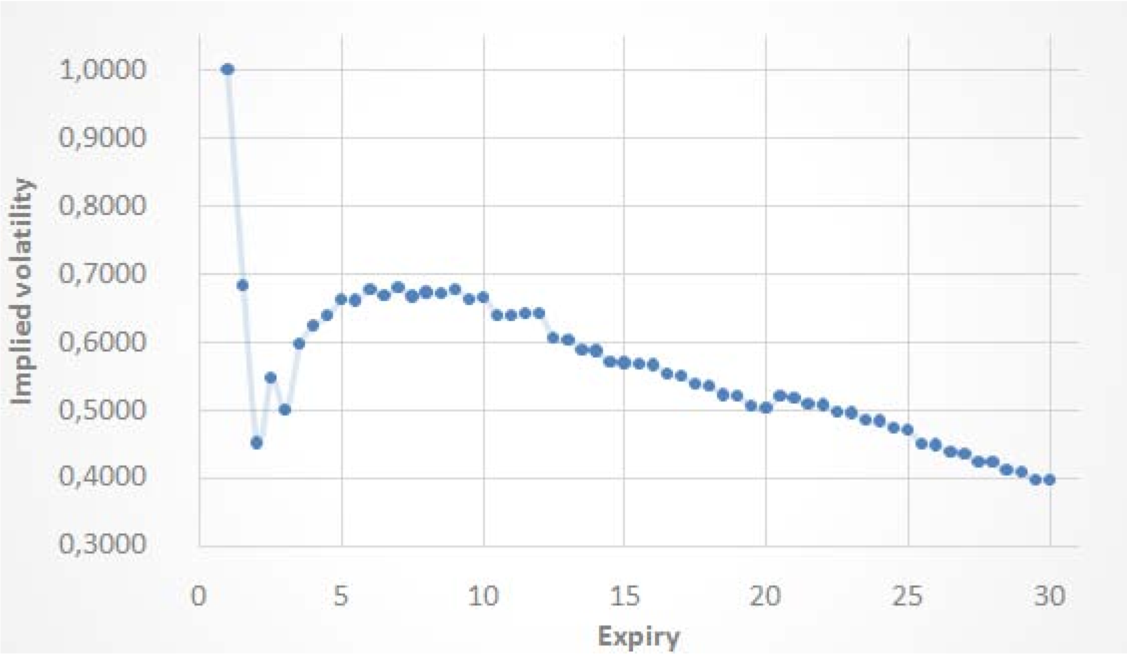


Image 11: Results after solving caplet volatility from cap market prices (underlying Euribor 6M) with Strike 1% using Linear Interpolation on flat volatilities

As we can see, the shape of the curve that we obtained is pretty similar as the one that we obtained using the Linear Model 2.

We can see that this method already works because it gives us what we'd expected for, but it has its limitations:

- All methods that we had seen before fail for higher strike price.
- Methods do not capture initial variations accurately.
- These methods lead us to use better interpolation methods and obtain more accurate flat volatility values in the first interval.
- Interpolation done at the start on "exact" values make less computational errors when they are computed.

The disadvantage of this method, besides the failing that we'd discussed before, is that we are solving a non-linear minimization problem at each step.

3.5 Constraint Model

We have already seen several methods, some of which gives the exact prices but the graphic of the volatility has peaks which means that it is rough as well as some others methods with a smooth surface but with very high errors when we compute the prices.

Our main goal it is to obtain prices of the cap with implied volatilities which are the same as the prices computed with flat volatilities (accuracy), and at the same time to have a smooth surface, without peaks (smoothness). That's what we thought about using a penalty problem/method.

This method is explained as it follows:

$$\min(|PricesCap(Implied\ \sigma) - PricesCap(Flat\ \sigma)|^2 + \lambda\sigma D^2\sigma)$$

$$\text{with}\quad D = \begin{pmatrix} 2 & 1 & 0 & \dots & 0 \\ 1 & 2 & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 2 & 1 \\ 0 & \dots & 0 & 1 & 2 \end{pmatrix} \in M_{60x60}$$

The prices of the cap with implied volatility and flat volatility are computed with the Gaussian model, presented earlier.

The first add represents the accuracy of prices and the second add it represents the smoothness part because it is the approximation of the second derivative of volatility.

λ is the penalty term which multiplies the smooth part. We will change the value of λ according to the importance that we want to give to the smoothness.

This idea is very interesting because combine both desired parts; on one hand we have the exact prices and on the other hand we have smoothness, and with λ changing the importance that we give to one or the other.

Unfortunately, in practice we didn't get good results because the function has so many local minimums and it takes too much time to compute. It would be necessary and interesting to dedicate an extra time to study the implementation of this model as the theoretical study proves that we could achieve very good results.

So, we need to convert this problem in an easier one. That's the reason why we are going to force prices to be equal with an equality constraint and we only minimize the approximation of the second derivative of the volatility.

To be clear, we want to get equal prices and minimize the smooth part as much as possible, so that's the reason why we convert the penalty problem/method in a constraint problem. This method is explained as it follows:

$$\min \sigma D^2\sigma$$

$$s.a\ |PricesCap(Implied\ \sigma) - PricesCap(Flat\ \sigma)| = 0 \qquad (*)$$

where D corresponds to the classic second difference operator, defined above.

$$\frac{\partial^2 \sigma}{\partial T^2} = \frac{\sigma(T + \Delta t) + 2\sigma(T) + \sigma(T - \Delta t)}{\Delta t^2}$$

and T is time to maturity.

This is the discretization of the volatility, which is a continuous function.

We solve that in Matlab with strike = 1 % and Δt = 6 months (0.5).

This next graphs show the results that we obtained:

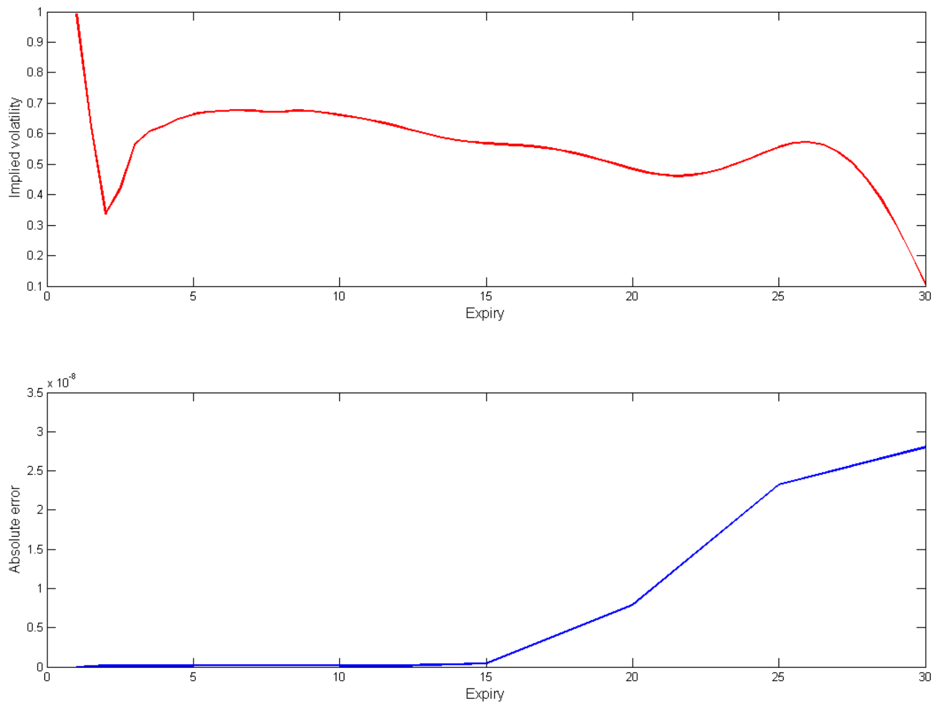


Image 12: First graphic shows the Implied Volatility of Maturity T computed with the model previously explained and the second graphic shows the Error in prices of maturity T.

In the first graphic we can see that the shape is similar to the methods that we had seen before. The graphs of implied volatility is quite smooth, it has only one peak at first which is normal because of the reason that we had explained in the course of this report.
At the end, we observe that the method doesn't work well because the results are not realistic, the volatility approaches to zero which is not true. This is a problem caused by the algorithm implementation.

We can also observe that the errors are low in the order of 10^{-8} which is very good. If we compare these errors with the ones that we've obtained before, we can say that it's a very good improvement. We also observe that the errors are increasing, but there are still very small.

We'd concluded that this improvement gives us very good results.

4. Conclusions

In order to solve the problem that we have been facing in the last years, we have done the prices valuation assuming that they follow a Normal distribution. Based on how the financial market works, we have been proposing different models using that distribution to compute the implied caplet volatilities.

We'd started with the model that the Banco Popular provided us which forces the prices to be equal and computes the implied caplet volatilities using a simple linear interpolation. Their results are pretty accurate because the price matches the market price but the surface shows no smoothness at all, only peaks.

In order to get this smoothness that we want, we had tried an exponential model. As we wanted it to be, we'd obtained a smooth surface, however we also get a huge prices error.

We'd tried some other linear interpolations using flat volatilities and with some of them we'd obtained pretty accurate results next to our main goal.

At last, we'd studied a theoretical model which is true to the constraints and expectations that we'd been looking for the beginning. However, the results are not what we'd been expected, but we believe it is caused by an implementation fail.

Due to the time constraint, we'd simplified this last model setting as constraint the prices accuracy and tried to minimize the caplet volatilities. That's how we'd achieve a pretty valid model.

5. Future work

This problem was unthinkable years ago. It's for this reason that there's a long way to go and we have to keep the research in order to achieve a good model. Some of the proposals that we introduce are:

- Using a Spline method.
- Thinking how we can solve this problem assuming that the prices follow other distribution different to Normal distribution and the prices can be negative.
- Gather more data. In this project, we have been working with 15 values per strike to compute 60 parameters. With more data values, we will get more accurate models and could extend the analysis to negative Strike values.
- Continuing the adaptive analysis with another family of functions or using different kinds of functions in a partition of the mesh.
- Study the Maturity-Volatility Relationship.
- Implementing the model that we had explained on section 3.5 because we didn't have enough time to compute its right implementation.