Popular

Generating a caplet volatility surface

1. Introduction

In order to effectively calculate the outstanding position held by the bank, there is a need of accurate models as well as valuation inputs.

To achieve this objective, it is necessary to invest a considerable team effort into the investigation of many market metrics, models, and techniques used for the estimation of reliable parameters.

During the last few years, financial markets have suffered great disruption and most of the models and assumptions have been questioned. One of the latest examples is the lognormal assumption of the distribution of the interest rates.

Specifically, the great extent of quoted interest rates very close to zero and negative quoted forward rates has led to a correction of the assumption of lognormality towards the normal distribution.

Here, we propose to study this new assumption and consequences to the economic factors it may imply.

2. Objective

Valuation models rely on a parametric distribution of the return of the assets. The most common distribution for returns is the normal one, which implies a lognormal distribution for the asset meaning strictly positive values.

Years ago, the assumption of positive prices was not questionable but the current level of interest rates has made it necessary to use models that accept negative values.
This change of model also implies a change in the volatility input.

The problem, outlined in more detail in the next section, focuses its attention on obtaining a caplet volatility surface that combines both an accurate valuation of liquid products (via mark to market valuation) while maintaining smoothness.

3. Detailed explanation of the problem

Due to their closeness to zero, we will focus on interest rate derivatives. The most common financial products involving interest rates are caps and floors that can be decomposed into caplets and floorlets.

A caplet can be seen as a call option with an interest rate as underlying asset. The payoff associated to a caplet with strike price K and maturity T is:

$$Caplet\_Payoff = Nom \cdot \Delta t \cdot \max(r_T - K;0)$$

Where r_T is the annualized value of the rate in T, Nom is the nominal and Δt is the year fraction for the day count convention.

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This payoff is positive when the value of the interest rate is higher than K because the buyer will pay a fixed rate of K while the actual level of the rate is higher.

Making the same analogy, a floorlet can be seen as a put option. This financial product has the following payoff with strike price K and maturity T:

$$Floorlet\_Payoff = Nom \cdot \Delta t \cdot \max(K - r_t; 0)$$

Knowing the definition of a caplet, the cap (floor) definition is straightforward: a series of caplets (floorlets) with concatenated maturities.

The most common valuation method of interest rate caplets/floorlets is via the lognormal Black 76 model which is broadly explained in the Master's degree (Mathematical Engineering).

Throughout this problem, we will implement normal Black Scholes formula.

Once we have floors correctly valued, we will implement volatility stripping. This is an iterative process to obtain caplet volatility based on cap volatility.

This process could end up in a sharp volatility surface that will be unrealistic on financial markets so some methods will be discussed to smooth it.

Stripping volatility is an essential process because only cap volatility is available in financial platforms such as Thompson Reuters or Bloomberg, so caplet volatility is needed when valuing any non-quoted product.

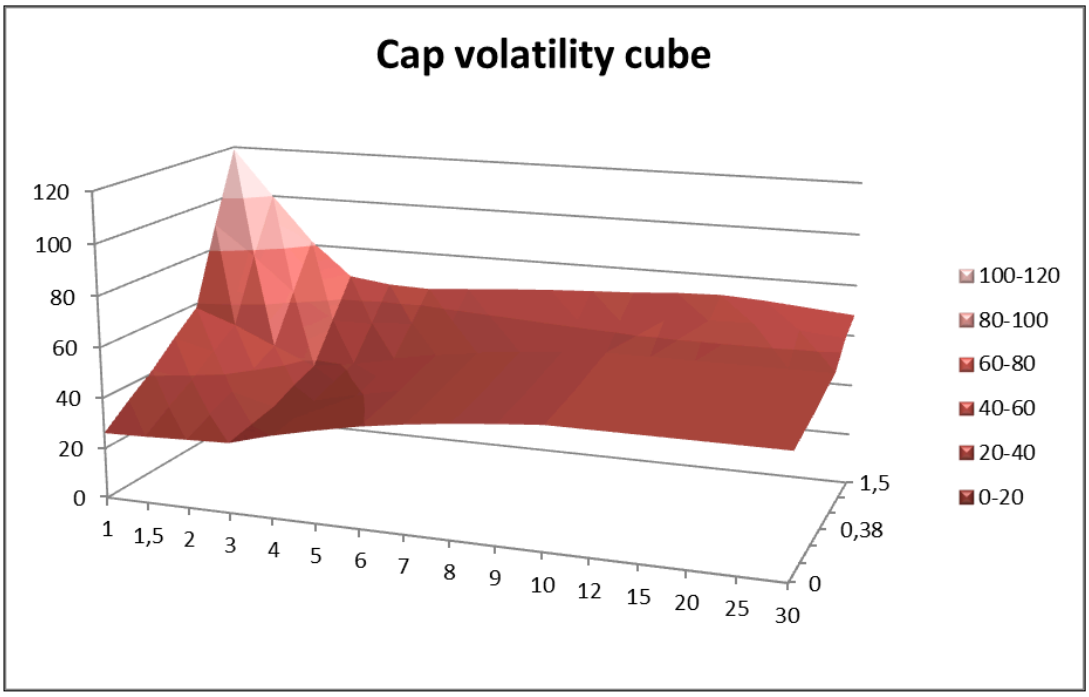

Figure 1

4. Main goals

- Describe the main differences between lognormal and normal Black Scholes valuation formula.
- Implement normal Black Scholes valuation formula for caps and floors.
- Become acquainted with standard formats of data in financial platforms.
- Implement an iterative process to obtain caplet volatility surface from cap volatility surface.
- Discuss smoothness versus accuracy to the issue we are dealing with.
- Evaluate the accuracy of your results comparing them with quoted prices.