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PROMOTION MIX OPTIMIZATION

Accenture

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1. INTRODUCTION

- The problem we have to solve is a Promotion Mix Optimization problem proposed by Accenture.

- The objective in this problem is to define the promotions plan that a company has to apply to its products in order to achieve its goals.

- We will restrict ourselves to a company which sells drinks. Our purpose is to decide what promotion and when it should be applied to every different kind of drink.

2. PROBLEM DESCRIPTION

To perform the objective that we have mentioned before, this company has provided us a dataset with a big number of variables, that they are the following:

- BRAND: Type of drink.
- CATEGORY: Subtype of drink.
- CAPACITY: Capacity of the container in cc.
- PACK UNITS: Number of containers by pack.
- MONTH: Month of the record.
- WEEK: Week of the record. (In this case, week corresponds to a half of a month, so if week = 1, then we are referred to the first half of the month).
- PROMOTION: Type of promotion.
- EXPECTED INCREASE PCT: Percent of litres that we expect to increase sold if we apply the promotion to the brand-category-capacity-pack in the month-week.
- BASELINE: Expected litres sold to the brand-category-capacity-pack in the month-week.
- EXPECTED INCREASE LITRES: Expected increase in litres if we apply the mechanic to the brand-category-capacity-pack in the month-week.

Expected increase litres = (Expected increase pct)*(Baseline)

- TOTAL PROMO EXPECTED INCOME: Expected revenue to the brand-category-capacity-pack in the month-week if we apply the promotion.
- ROI NUMERATOR: Expected revenue (total promo expected income) – cost of the promotion (ROI denominator)
- ROI DENOMINATOR: Cost of the promotion.

Moreover, in this dataset, we can find all the constraints that have to satisfy the solutions that we have obtained. Some of these constraints are mandatory and the remaining is optional.

The last thing we have to pay attention is the budget that the company wants to spend on the promotions.

According to the data mentioned previously, the objectives that this company wants us to keep are the followings:

- Maximize the increase of units (litres) sold.
- Maximize the Return of Investment (ROI).

$$ROI = \frac{Revenue - Cost}{Cost}$$

3. SOLUTION METHOD

To solve this problem, we have applied Mixed Integer Programming (MIP).

As we can see, our last objective function is not a linear function, so if we want to apply MIP, we have to divide it in two different functions as follows:

First of all, we are going to use the ROI definition.

$$ROI = \frac{Revenue - Cost}{Cost} = \frac{Revenue}{Cost} - \frac{Cost}{Cost} = \frac{Revenue}{Cost} - 1$$

As we have proved, at this moment, we can write ROI:

$$ROI = \frac{Revenue}{Cost} - 1$$

For this reason, our new objective function is to maximize the quotient $ROI = \frac{Revenue}{Cost}$, because a number (in this case, we have number 1) doesn't affect to an optimization problem; so for maximize the ROI, we are going to maximize the Revenue and minimize the Costs.

At this moment, we are going to consider three objective functions, so it's recommended the use of lexicographic optimization to solve the problem.

- First of all, we maximize the number of units (litres) sold that is the most important result for the company, and after that we are going to make to different experiments.

    - The first one consists on:
      Maximize revenue, and then, we fix this revenue and minimize costs.

    - The second is option is the opposite of first one, that is, we minimize costs and, after that, we maximize revenue.

      As a conclussion, we can say that our problem is going to consist on:

      (1) Lex max [Units sold, Revenue, -Costs]

      (2) Lex max [Units sold, -Costs, Revenue]

      We have solved the different problems with GAMS, using CPLEX (a commercial solver).

Our first idea was to solve this problem using Mixed Integer Non Linear Programming (MINLP) but it took too long (the solver could not converge in half an hour), so we had to change it.

4. DATASET

Because of the provided dataset has a lot of variables and we don't need all of them to solve the problem, the final variables we

- **BRAND**: There are five types of brand (SW, HO, MI, SUB and BF).
- **CATEGORY**: There are ten types of drinks. SWINE belongs to brand SW; HO CITRUS, HO COLA and HO SPARKLING belong to brand HO; MI CITRUS, MI TONIC and MI TONIC PM to MI; SUB OG to SUB; and finally, BF REG and BF LIGHT to BF.
- **CAPACITY**: The nine types of capacities are: 200, 250, 310, 330, 500, 1000, 1250, 1500 and 2000.
- **PACK UNITS:** 1CT, 2CT, 4CT and 6CT.
- **MONTH:** Our study is going to be applied in April(4), May(5) and June(6).
- **WEEK:** Number 1 is referred to the first half of the month and number 3 is referred to the second one.

- **PROMOTION:** The available promotions are the followings:

Promotion(Mechanic)	Description
ExternalAddedValue	Small value gift sold together with the product (For example, an inflatable balloon during summer)
MB2u50	Multibuy: Second unit with 50% discount
MB2u70	Multibuy: Second unit with 70% discount
MB3x2	Multibuy: 3 units for the price of 2
MBFixedPrice	Multibuy: At a fixed price. For example: 3 units for 2 euros
MBMxN	Multibuy: M units for the price of N (excluding 3x2, usually N=M-1, for example 8x7)
MBSets	Multibuy: Sets. Consumer can make his own set of a fixed number of units of the same brand. For example: 1 BF REG + 2 BF LIGHT for 2 euros
TPRFixedPrice	Temporary price reduction: Fixed price, for example: 1,69 euros
TPRRoundPrice	Temporary price reduction: Round Price, 1 euro, 2 euros...

- **EXPECTED INCREASE LITRES:** Expected increase in litres if we apply the mechanic to the brand-category-capacity-pack in the month-week.
- **TOTAL PROMO EXPECTED INCOME**: Expected revenue to the brand-category-capacity-pack in the month-week if we apply the promotion.
- **ROI NUMERATOR**: Expected revenue (total promo expected income) – cost of the promotion (ROI denominator)
- **ROI DENOMINATOR**: Cost of the promotion.

To continue, we are going to formulate our problem.

5. FORMULATION

In this section, we are going to explain how to formulate the different problems that we want to solve.

5.1. SETS

Here, we define the notation that we are using when we formulate the problem.

- **B**: set of brands.
- **D**: set of drinks (category).
- **P**: set of packagins.
- **C**: set of capacities.
- **M**: set of months.
- **W**: set of couples of weeks.
- **R**: set of promotions.
- **T**: set of time step.
- **ID**: set of identifiers for each possible promotion.

As we can see, we have defined notation for each variable that we use in the model. Moreover, we have defined two sets more.

The first one's values are 1,2,3,4,5 and 6; number 1 means that we are in the first half of the April, number 2 means that we are in the second half of April,...

The second one is a set that we are going to use to identify each combination of brand-category-capacity-pack-month-week-promotion.

5.2. PARAMETERS

The parameters that we need to determinate the model are the followings:

- es_i : expected sales increase (litres).
- er_i : expected revenues (€).
- ec_i : expected costs (€).
- Budget: 144.000€

After defining the parameters that we can deduce because of the dataset, we are going to define some parameters we need to set out the objective functions. They are the followings:

- $\beta_{ib} = \begin{cases} 1, & if\ i\ corresponds\ to\ a\ Brand\ of\ type\ b \\ 0, & otherwise \end{cases}$

- $\delta_{id} = \begin{cases} 1, & if\ i\ corresponds\ to\ a\ Drink\ of\ type\ d \\ 0, & otherwise \end{cases}$

- $\pi_{ip} = \begin{cases} 1, & if\ i\ corresponds\ to\ a\ Packaging\ of\ type\ p \\ 0, & otherwise \end{cases}$

- $\gamma_{ic} = \begin{cases} 1, & if\ i\ corresponds\ to\ a\ Capacity\ of\ type\ c \\ 0, & otherwise \end{cases}$

- $\mu_{im} = \begin{cases} 1, & if\ i\ corresponds\ to\ a\ Month\ of\ type\ m \\ 0, & otherwise \end{cases}$

- $\omega_{iw} = \begin{cases} 1, & if\ i\ corresponds\ to\ a\ Week\ of\ type\ w \\ 0, & otherwise \end{cases}$

- $\rho_{ir} = \begin{cases} 1, & if\ i\ corresponds\ to\ a\ Promotion\ of\ type\ r \\ 0, & otherwise \end{cases}$

- $\tau_{it} = \begin{cases} 1, & if\ i\ corresponds\ to\ a\ Time\ step\ of\ type\ t \\ 0, & otherwise \end{cases}$

5.3. VARIABLES

The variables for the model are the followings:

- X_i: Binary variable which says if the combination i is included in the mix (1) or not (0).
- LIT: The amount of litres sold.
- NUM: The sum of the revenues.
- DEN: The sum of the costs.

Variables **LIT**, **NUM** and **DEN** will be the solution of the optimization problem.

5.4. CONSTRAINTS

As we explained at the beginning, we have two kinds of constraints:

I. **Budget constraints:** Related to the total Budget. We have analyzed how little changes in the budget of the marketing actions can increase the objectives.
II. **Business constraints**: Concerning product combinations, type of promotion and/or time periods.

DESCRIPTION OF CONSTRAINTS:

The list of constraints given by the company is the following:

- MI TONIC-250-4CT must be promoted during April (optional).
- Just one promotion is permitted for each combination of category-capacity-pack units on each two weeks period (mandatory).
- Just one promotion is permitted for each combination of category-capacity-pack units on each month (this one implies the previous one) (mandatory).
- Do not promote the same product during two consecutive two weeks periods (mandatory).
- A maximum of 2 promotions are allowed during the three months period for each combination of category-capacity-pack units (mandatory).
- At least one MI TONIC product must be promoted during April (optional).
- MI TONIC Capacity: 330, Pack Units: 1CT must be promoted during the second half of April because it's a product specially consumed during Easter (optional).
- No more than one promotion is allowed during the three months period for each combination of category-capacity-pack units for SUN products (mandatory).

- During the second half of May and the first half of June only SWINE and SUB OG products can be promoted (optional).
- A maximum of 2 promotions are allowed for the same brand during each two weeks period (mandatory).
- Try to spread promotion pressure throughout the month (not too many promotions on the same half of the same month) (optional).
- Pairs of products not to be promoted simultaneously. Below is a list of products that cannot be promoted simultaneously (suggested).

Pairs of products not to be promoted simultaneously, Manufacturer suspects they can cannibilize each other

Product A	Product B
SWINE 1500 1CT	SWINE 1500 4CT
HO CITRUS 1500 1CT	HO CITRUS 1500 2CT
HO COLA 1500 1CT	HO COLA 1500 2CT
MI CITRUS 330 1CT	MI CITRUS 1000 1CT
MI TONIC 330 1CT	MI TONIC 1000 1CT
BF REG 330 1CT	BF REG 1000 1CT
BF LIGHT 330 1CT	BF LIGHT 1000 1CT

- The total cost of promotions must be less than or equal to Budget.

In GAMS, we can write the constraints as we can see in the next picture:

```
CON2(d,c,p,m,w) .. SUM[i,X(i)*delta(i,d)*gamma(i,c)*pi(i,p)*mu(i,m)*omega(i,w)] =L= 1;
CON3(d,c,p,m) .. SUM[i,X(i)*delta(i,d)*gamma(i,c)*pi(i,p)*mu(i,m)] =L= 1;
CON4(d,c,p,t)$(ord(t) < 6) .. SUM[i,X(i)*delta(i,d)*gamma(i,c)*pi(i,p)*(tau(i,t)+tau(i,t+1))] =L= 1;
CON5(d,c,p) .. SUM[i,X(i)*delta(i,d)*gamma(i,c)*pi(i,p)] =L= 2;
CON6.. SUM[i,X(i)*delta(i,'MI_TONIC')*mu(i,'APRIL')] =G= 1;
CON7.. SUM[i,X(i)*delta(i,'MI_TONIC')*mu(i,'APRIL')*gamma(i,'330')*pi(i,'1CT')*omega(i,'II')] =G= 1;
CON8(d,c,p) .. SUM[i,X(i)*delta(i,d)*gamma(i,c)*pi(i,p)*beta(i,'SUB')] =L= 1;
CON9(d)$(ord(d) ne 1 and ord(d) ne 8)  .. SUM[i,X(i)*delta(i,d)*omega(i,'II')*mu(i,'MAY')] + SUM[i,X(i)*delta(i,d)*omega(i,'I')*mu(i,'JUNE')] =E= 0;
CON10(b,m,w).. SUM[i,X(i)*beta(i,b)*mu(i,m)*omega(i,w)] =L= 2;
CON11(t).. CARD(t)*(SUM[i,X(i)*tau(i,t)] - UB) - SUM[i,X(i)] =L= 0;
CON12_1(m,w,r).. SUM[i,X(i)*delta(i,'SWINE')*mu(i,m)*gamma(i,'1500')*(pi(i,'1CT') + pi(i,'4CT'))*omega(i,w)*rho(i,r)] =L= 1;
CON12_2(m,w,d,r)$(ord(d) eq 2 or ord(d) eq 3).. SUM[i,X(i)*delta(i,d)*mu(i,m)*gamma(i,'1500')*(pi(i,'1CT') + pi(i,'2CT'))*omega(i,w)*rho(i,r)] =L= 1;
CON12_3(m,w,d,r)$(ord(d) eq 5 or ord(d) = 6 or ord(d) = 9 or ord(d) = 10).. SUM[i,X(i)*delta(i,d)*mu(i,m)*(gamma(i,'1000') +
                                       gamma(i,'330'))*pi(i,'1CT')*omega(i,w)*rho(i,r)] =L= 1;
CBUDGET.. SUM[i,X(i)*ec(i)] =L= BUDG2;
```

While we were doing experiments, we noticed that constraint 1 must be avoided, because it makes impossible to find any solution to the problem, so it would be an infeasible problem.

5.5. OBJECTIVE FUNCTIONS

The functions to maximize are the followings:

- **MIS** *(total increase of the units sold):*

$$\sum_{i \in ID} X_i \cdot es_i$$

- **ROI:** Now, we are considering two objective functions to maximize ROI.
  - *MROINUM:*

$$\sum_{i \in ID} X_i \cdot er_i$$

  - *-MROIDEN*

$$-\sum_{i \in ID} X_i \cdot ec_i$$

6. EXPERIMENTAL RESULTS

Observing the solutions that we have obtained, we see a clear trade-off between the objectives. Actually, when the MIS increases, the ROI decreases and vice versa.

After analyzing all of these solutions, we are going to show non-dominated solutions:

SLACK	BUDGET	SALES(L)	SALES(L) %	ROI	REVENUES
1%	110%	510,209.517	111.28%	0.899	300,675.008
5%	110%	491,316.693	107.16%	0.946	308,248.227
5%	105%	477,417.165	104.13%	0.965	297,085.263
5%	101%	462,421.908	100.86%	0.976	287,285.111
5%	**100%**	**458,477.797**	**100.00%**	**0.983**	**285,542.612**
5%	95%	440,025.277	95.98%	1.002	273,819.497
5%	90%	423,108.438	92.29%	1.004	259,687.751

- **SLACK**: After calculate the number of litres sold, we have to fix it to calculate the ROI numerator and ROI denominator, so we decided to increase or decrease this value in a 1% or 5% and compare all the solutions.

- **BUDGET**: We have proved with different values to the budget, increasing or decreasing it in a 1%, 5% and 10%, and we have studied all the solutions we have obtained.

Now, we cannot decide which is the best solution; it depends on the goal to the company.

We are going to explain some of these best solutions:

- Best solutions for maximizing the units (litres) sold, MIS, using all the constraints or considering only the mandatory ones are:

ALL CONSTRAINTS (SOL 1)

MAXIMIZE SALES	
SLACK	1%
BUDGET	110%
SALES(L)	510209.517
ROI	0.899
PROFIT	142348.265

MANDATORY CONSTRAINTS

MAXIMIZE SALES	
SLACK	1%
BUDGET	110%
SALES(L)	518466.921
ROI	0.897
PROFIT	142145.42

We can see that the SLACK and the BUDGET is the same in both solutions and the ROI is very similar, but we can notice that there is an important observation. If we compare the two results, the increase in sales (litres) if we use all the constraints is lower than the increase in sales if we only use mandatory constraints. Moreover, the profit is greater in the solution with all constraints.

- Best solutions for maximizing the ROI using all the constraints or considering only the mandatory ones are:

ALL CONSTRAINTS (SOL 2)

MAXIMAZE ROI	
SLACK	5%
BUDGET	90%
SALES(L)	423,108.438
ROI	1.004
PROFIT	130,092.628

MANDATORY CONSTRAINTS

MAXIMAZE ROI	
SLACK	5%
BUDGET	90%
SALES(L)	428,224.197
ROI	1.017
PROFIT	131,862.082

In this case, it occurs the same thing than before. The SLACK and the BUDGET are the same in both solutions. If we use all the constraints, SALES and PROFIT's values are lower than the values if we only use mandatory constraints, and with the ROI it occurs the same. In this case, we obtain the best ROI with only the mandatory constraints.

- After analyzing all the solutions, we propose the next solution that it's a mix of the two previous objectives. It is the best combination obtained using the Budget suggested by the Company and all constraints (SOL 3).

SOLUTION	
SLACK	5%
BUDGET	100%
SALES(L)	458,477.797
ROI	0.983
PROFIT	141,552.730


This solution is a middle ground of the two previous solutions (MAX MIS and MAX ROI).

Now, we are going to compare the last solution with the two solutions we have explained before.

- If we compare this solution (SOL 3) with the results obtained before when we maximize MIS (increase of sales) using all constraints (SOL 1), we can see that the increase in sales (in litres) and the profit in SOL 3 are lower than the values in SOL 1. This is very logical because we spend more money when we maximize MIS, but however, the ROI is better in the last solution (SOL 3).

MAXIMIZE SALES(SOL1)		**MAXIMIZE BOTH (SOL3)**	
SLACK	1%	**SLACK**	5%
BUDGET	110%	**BUDGET**	100%
SALES(L)	510209.517	**SALES(L)**	458,477.797
ROI	0.899	**ROI**	0.983
PROFIT	142348.265	**PROFIT**	141,552.730

- Finally, if we compare the last solution (SOL3) with the results that we obtained when we maximize ROI using all constraints (SOL 2), it is logical that in the third case, the ROI is worse than the ROI in SOL 2, but the increase in sales and the profit in SOL 2 are lower that these values in SOL 3.

MAXIMAZE ROI(SOL2)		MAXIMIZE BOTH(SOL3)	
SLACK	5%	**SLACK**	5%
BUDGET	90%	**BUDGET**	100%
SALES(L)	423,108.438	**SALES(L)**	458,477.797
ROI	1.004	**ROI**	0.983
PROFIT	130,092.628	**PROFIT**	141,552.730

To sum up, we are going to show the interpretation for the last solution we have obtained:

		APRIL				MAY				JUNE			
		I		II		I			II	I	II		
		1CT	2CT	1CT	6CT	1CT	2CT	4CT	1CT	1CT	1CT	2CT	4CT
SWINE	330			TPRFixedPrice					MBFixedPrice				
	1500			MB2u70				MBMxN	MB2u70				MBMxN
HO CITRUS	1500						TPRRoundPrice					TPRRoundPrice	
HO COLA	1500		TPRRoundPrice				TPRRoundPrice						
HO SPARKLING	500				MB3x2								
	1500										MB3x2		
MI CITRUS	330					MB3x2							
	1000	TPRRoundPrice				TPRRoundPrice							
MI TONIC	250										MBFixedPrice		
	330			MBMxN							MBMxN		
	1000			MB2u70									
SUB OG	500									MB2u50			
BF REG	330	MBFixedPrice									TPRFixedPrice		
	1000			MB3x2									
	1500	MB2u50				MBFixedPrice							

This table displays, for example, that the drink SWINE with capacity 330 CC and the pack unit 1CT is going to be promoted during the second half of April with the promotion “TPRFixedPrice” and during the second half of May with the promotion “MBFixedPrice”. The same can be said about all the drinks.