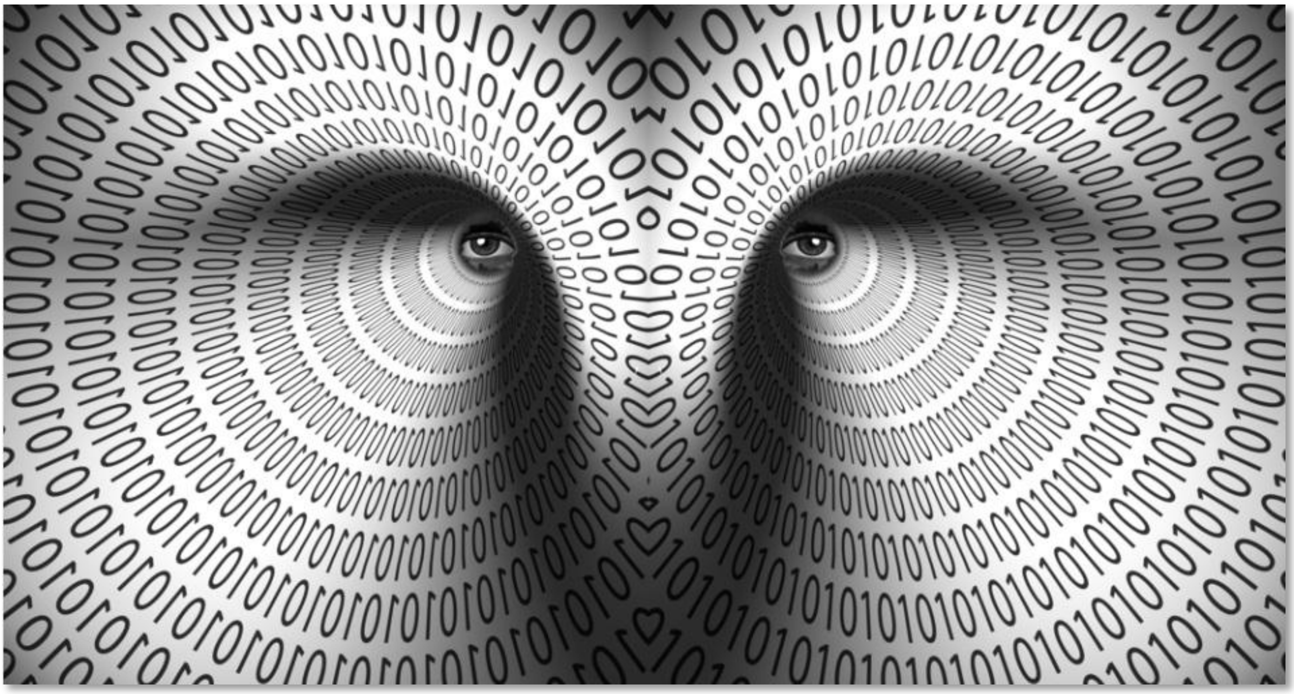
Master in Mathematical Engineering
Modelling Week 2014

Conento



Alberto Martínez, Carlos Bernal, Carlos de Diego, Gabriel Valverde, Andreas Hadjittofis

Building an Optimal Premium Model for an Insurance Company

Table of Content

INTRODUCTION

We are interested in solving a CRM problem for an insurance company. The tasks to be achieved are:

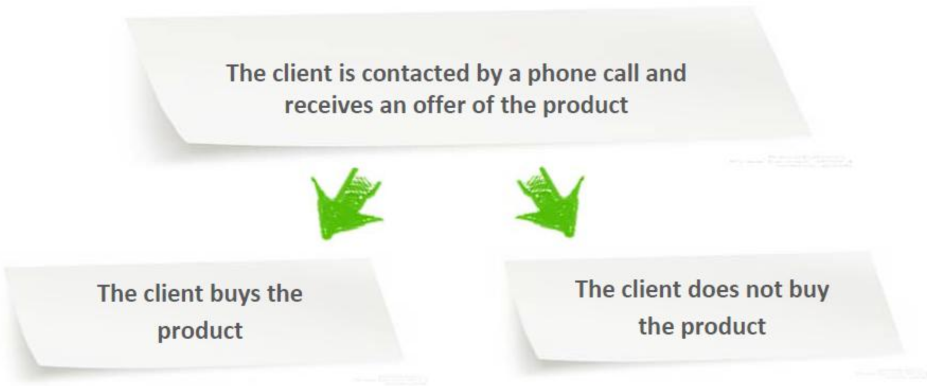
- Finding the ideal target, in this case, people who are more likely to contract their insurance products.
- Identifying the **premium** we should offer to each client, that is to say, the optimal price that should be offered to each client.
- Calculating the difference between offering the premium randomly and optimally, using the information obtained in the model.

Two databases with clients' information are available:

- In the first one we have the information of 20.000 clients which have already been contacted; 9% of them have contracted the product.
- Important data are included such as the premium offered, the number of products that they have already bought, the number of years that they have been clients of the company and the socioeconomic status (an economic and sociological measure combined with the person's work experience and his or his family's economic and social position in relation to others, based on income, education, and work occupation).
- In the second database of non-previously contacted clients, we have the same information about 10.000 clients but only 5.000 are going to be contacted due to practical restrictions.

Is it worthwhile offering the same premium to all clients? Is it better to focus on people with certain characteristics rather than choosing clients randomly?

The objective is, using the first database, find an optimal strategy to be able to contact to those clients who are more likely to buy the product we are offering. This strategy should be applied to the second database to get the ideal target.



INTRODUCTION

The **premium is an important variable** when deciding whether to contract insurance or not. A high one is going to be rejected more frequently by potential clients, and a very low one is not going to maximize the earnings of the company.

Therefore, once the ideal target is defined, **an optimization problem should be formulated to find the optimal premium which maximizes the number of sales, and thus maximizes the amount of money that the company is going to earn.**

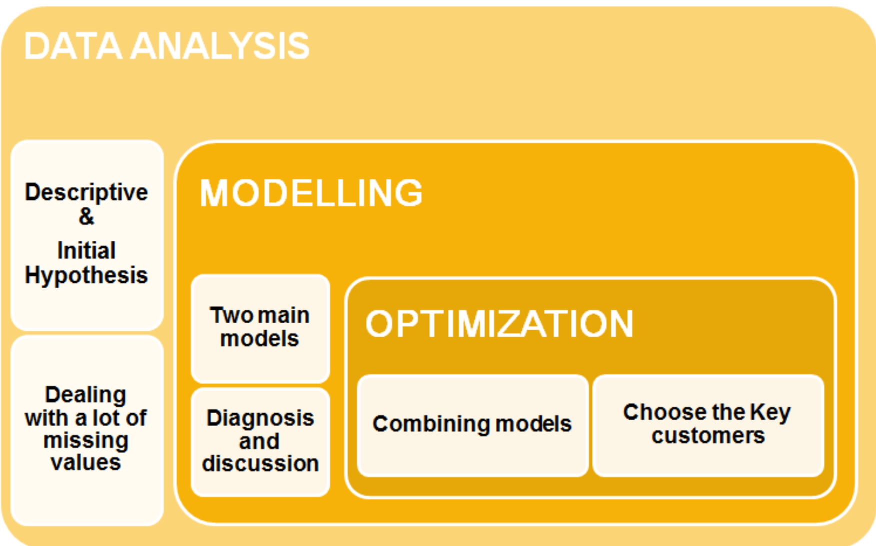
When the optimal premium is calculated, a comparison between the optimal earning and the one that we would get choosing the clients randomly can be calculated to prove the usefulness of the analysis.

SAS, SPSS, Matlab and Excel have been used as software tools: Matlab has been used for the optimization model and also for ROC curves.

OVERVIEW

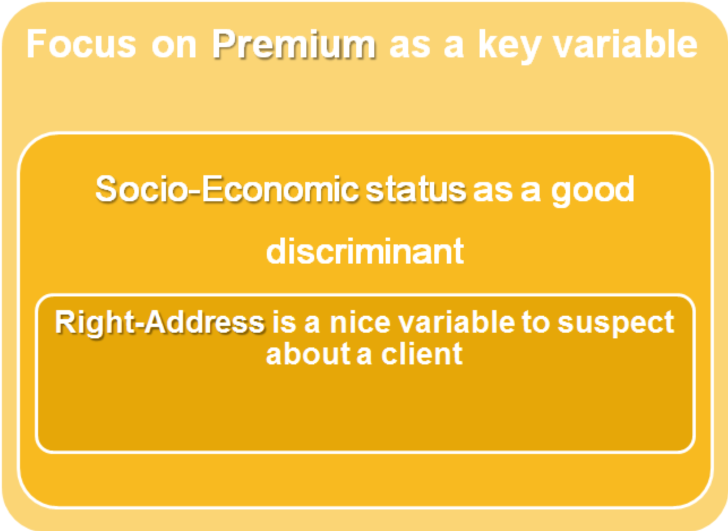
This section provides a brief introduction regarding the roadmap followed and provides a breakdown of the activities considered in the problem resolution.

The study covered is split in three main blocks: **data analysis**, **modelling** and **optimization**. The areas/steps covered in each block are summarized in the following figure:



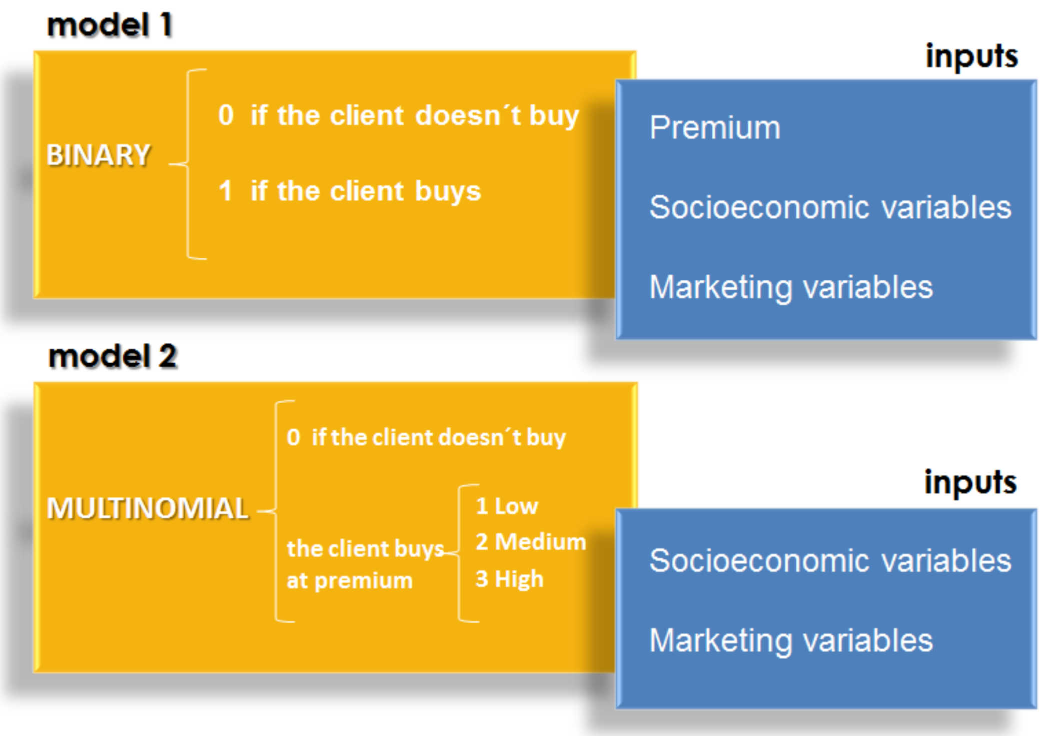
OVERVIEW

Due to the amount of input variables and potential relationships between explanatory variables, several models could be considered. However, the analysis is driven putting the focus in the following initial hypothesis:



Conceptual Framework:

Two different models have been considered to approach the problem according to the class variable defined:



DATA ANALYSIS

DESCRIPTIVE ANALYSIS

The variables considered in the datasets are:

Variable Name	Meaning
Obs	Number of Observations
Sales	It indicates whether the client bought a product: 1 (yes), 0 (otherwise)
Price Sensitivity	It indicates the client's sensitivity to the price: 1 (less sensitive) - 6 (more sensitive)
PhoneType	Client's phone type: Fixed or Mobile
Email	It indicates whether the client's email is available: 1 (yes), 0 (otherwise)
Tenure	Client's tenure (year when the person became a client of the company)
NumberofCampaigns	Number of times the client has been called
ProdActive	Number of active products
ProdBought	Number of different products previously bought
Premium Offered	Premium offered to the client
Phone Call Day	Day the phone call is received
CodeCategory	Category of the phone call answer
Birthdate	Client's birthdate
Product Type	It indicates the type of product that the client buys
Number of Semesters Paid	Number of semesters paid
Socieconomic Status	It indicates the client's socieconomic status
Province	Province where the client lives
Right Address	It indicates whether the client's address is correct: 1 (yes), 0 (otherwise)
Living Area (m^2)	Estimated surface area of house
House Price	Estimated price of the house
Income	Estimated income
yearBuilt	It indicates when the client's house was built
House Insurance	Price of the house insurance
Pension Plan	Estimated amount of money the client would have in a pension plan
Estimated number of cars	Estimation of the number of cars owned by the client
Probability of Second Residence	Probability of having a second residence
Credit	Estimation of the amount of credit that could be offered to the client
Savings	Estimation of the amount of money saved by the client
Number of Mobile Phones	Number of mobile phones
Number of Fixed Lines	Number of Land Lines
ADSL	It indicates whether the client has ADSL:1 (yes), 0 (otherwise)
3G Devices	It indicates whether the client has 3G Devices:1 (yes), 0 (otherwise)
Type of House	Type of house: Urban or Rural

DATA ANALYSIS

DESCRIPTIVE ANALYSIS

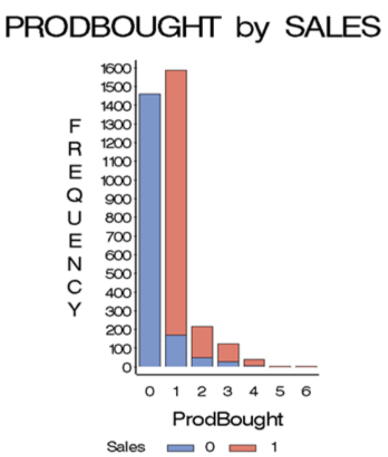
With all the data available in the first database, it is really important to make a complete descriptive analysis of the variables to understand the type of information we are dealing with, which can give us an idea of which variables are relevant to help us solve our problem.

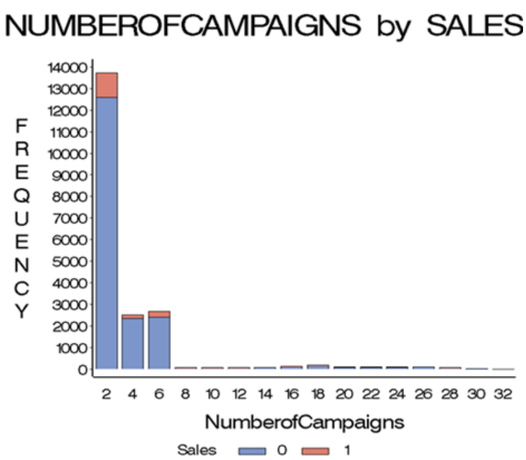
Our dataset is composed by **34 variables**, and we must know which ones are significant and relevant to explain the behaviour of our target variable.

In order to don´t extend the length of this document we have decided to focus on the modelling step and from a descriptive analysis point of view we have just provided as examples in the document, the barcharts crossing the target variable considered with the most relevant explanatory variables we found.

We haven´t included contigency tables for cualitative variables, neither box-plots nor distribution test for quantitative ones relating different variables and in case, running different contrast to study more deeply the input variables.

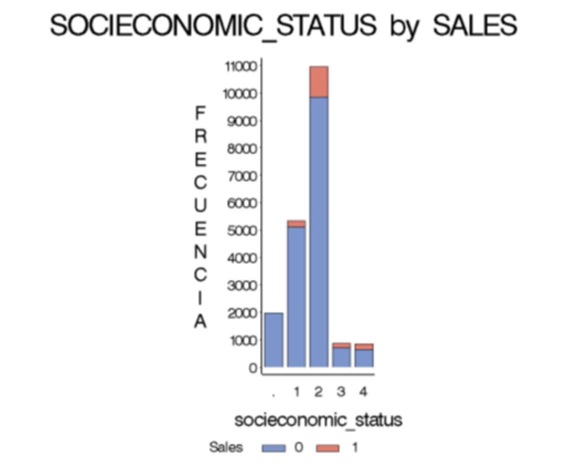
By confronting each of the explicative variable against the target we might get an overall idea of the correlation among them. For this reason we will show some charts:





DATA ANALYSIS

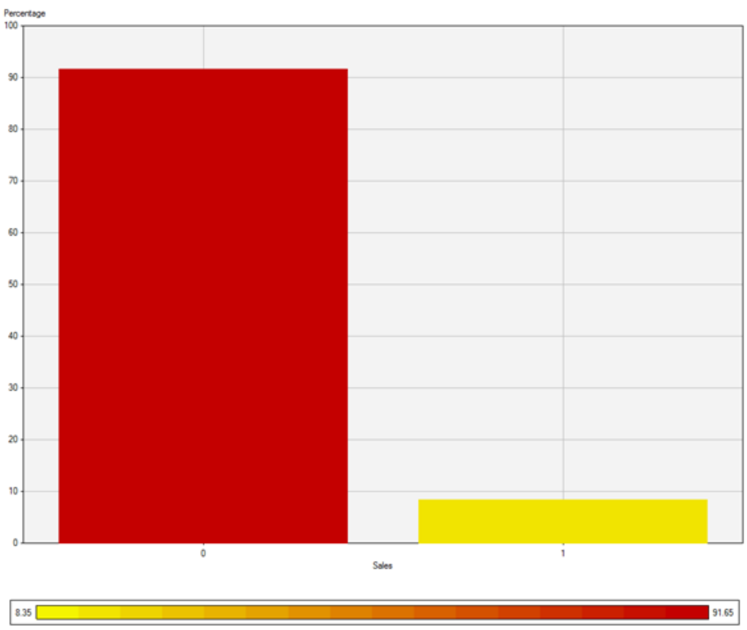
DESCRIPTIVE ANALYSIS



As we can see, [**prodbought**] has a great discriminant power. It is obvious since a costumer that usually buys products tend to accept the offer more frequently.

As for the others, it is not so easy to distinguish whether the sale was carried out or not. However, they are still good discriminant variables regarding of the proportion of 1's and 0's that has each bar of the histogram.

This phase is also fundamental to know how our data is classified in detail. Principally we are interested in the number of 1's and 0's in the sample, that is, how our target is distributed.
If we observe the picture we can see that a 91.43% of the customers don't buy a product while a 8.58% of the sampling buys an insurance product:



This difference between the proportion of 0's and 1's point out that we must do an undersampling in order to balance the sample according to the class variable [Sales].

DATA ANALYSIS

SAMPLING and DATA PROCESSING

Sampling Phase

As we said, the representation of the number of sales is lacking in size. This means an inconvenient for our model since it can happen that our model wasn't able to extract patterns or rules that define this event.

To solve this problem, we should equalize the sample so that the percentage of 1's will be same as the number of 0's. This method is known by under-sampling, that is, keep events of interest and reduce the complementary event.

The technique of sampling used is the stratified sampling, so that we must select representative elements from the different populations not homogeneous among themselves. The size of the sample will be of 3600 registers more or less, consisting of 50% of 1's and 50% of 0's.

Data Processing

In this point we will apply certain transformations to our data in order that our model could estimate the parameters correctly. It is possible to identify two types of data treatment: one based on the business and other based on statistics.

First, we need to deal missing values. It is necessary to recall that in our data sets we encountered lots of missing values. In our case, we used three ways to achieve the purpose. On the one hand, we implemented a regression method to predict some of the variables. In concrete, "price sensitivity". This variable has some missing values and we predicted them in order to fill in the empty spaces. We selected correlative variables to predict the "price sensitivity".

On the other hand, for socio economic status variables like "socioeconomic status", "living area", "house price",… we took a by group average having into account the socio economic status. Finally, for variables with lots of missing values ("ADSL", "3G devices", "number of fixed lines" and "number of mobile phones") we decided that the best way to deal with them was by removing the variable itself.

Transformation of categorical variables splitting them into the corresponding binary dummy variables for regression purposes and some numerical transformations for specific attributes (i.e., birthday) has been done. Examples of that are SOCIECONOMIC_STATUS, PRODUCT_TYPE,TYPE_OF_HOUSE,PHONE_TYPE,BIRTHDATE,PRICE_SENSITIVITY.

DATA ANALYSIS

SAMPLING and DATA PROCESSING

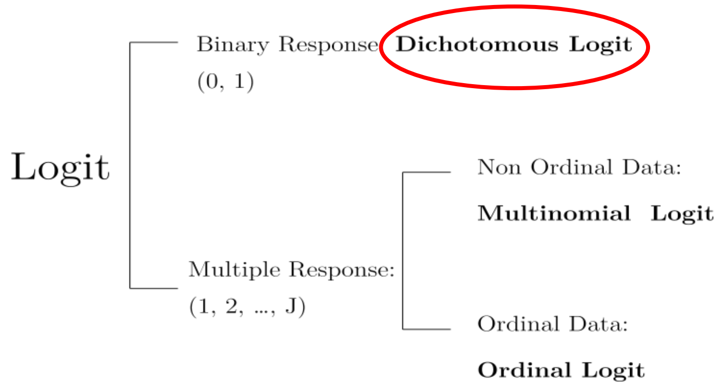
Regarding initial variables selection, logistic regression with stepwise and forward methods has been use as mechanism. Due to the different potential relationships between explanatory variables, several runs/models have been done, forcing in some cases some variables of interest to be present and therefore different results were obtained. Finally, a large enough set of variables has been selected as initial selection (keep in mind that considering most of input variables a regression ends with two or three relevant ones, for example, number of campaigns and product bought).

CREDIT,EMAIL,NUMBEROFCAMPAIGNS,PRODACTIVE,PRODUCTBOUGHT,PROVINCE,RIGHT_ADDRESS,SOCIOECONOMIC_STATUS,SAVINGS have been considered as input variables for model II variants.

MODELLING: Model I (binary)

BINOMIAL LOGISTIC REGRESSION MODEL

Model Definition

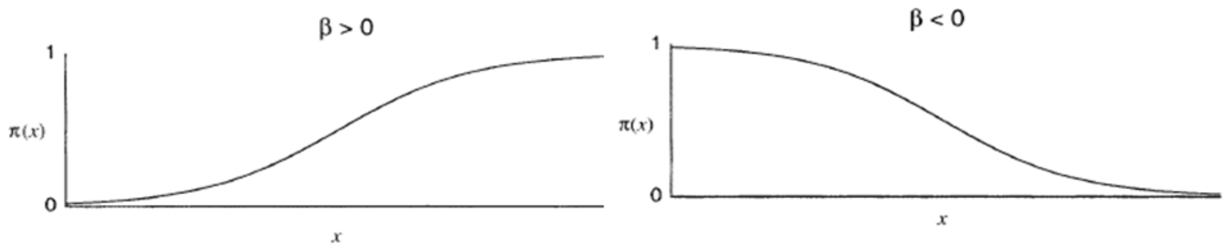


Let Y denote a binary response variable, X a set of explanatory variables and and $\pi(x) = \text{Prob}(Y{=}1|x)$. For a binary response, the regression model $\pi(x) = \alpha + \beta x$ is the lineal probability model. The linear probability model has a major problem: probabilities fall between 0 and 1, but linear functions take values over the entire real line.

Usually, binary data result from a *nonlinear* relationship between $\pi(x)$ and x. A fixed change in x often has less impact when $\pi(x)$ is near 0 or 1 than when $\pi(x)$ is near 0.5.

In practice, nonlinear relationships between $\pi(x)$ and x are often monotonic, with $\pi(x)$ increasing continuously or $\pi(x)$ decreasing continuously as x increases. These relationships provide S-shaped curves. The most important one corresponds with the logistic regression model defined by:

$$\pi(x) = \frac{e^{\alpha+\beta x}}{1 + e^{\alpha+\beta x}}$$



As $x \to \infty, \pi(x) \downarrow 0$ when $\beta < 0$ and $\pi(x) \uparrow 1$ when $\beta > 0$.

MODELLING: Model I (binary)

BINOMIAL LOGISTIC REGRESSION MODEL

If we consider as new response variable π(x) and the new variable $logit(\pi(x)) = \log\left(\frac{\pi(x)}{1-\pi(x)}\right)$ (log for the odds of π(x)) any real value between can be taken and therefore the linear regression makes sense: $logit(\pi(x)) = \alpha + \beta x$, i.e., the log odds has the linear relationship.

General expression for the model:

- **General Logit Equation**:

$$\operatorname{logit}(\pi(\mathbf{x})) = \log\left(\frac{\pi(\mathbf{x})}{1-\pi(\mathbf{x})}\right) = \alpha_j + \beta_j x \text{ where } \pi(\mathbf{x}) = \operatorname{Prob}(Y=1|\mathbf{x}) = 1 - \operatorname{Prob}(Y=0|\mathbf{x})$$

- **Probabilities for response variable**:

$$\pi(x) = \Pr ob(Y_i = 1) = \frac{1}{1+e^{-(\alpha+\beta_1 X_{1i}+\ldots+\beta_k X_{ki})}} = \frac{e^{\alpha+\beta_1 X_{1i}+\ldots+\beta_k X_{ki}}}{1+e^{\alpha+\beta_1 X_{1i}+\ldots+\beta_k X_{ki}}}$$

for a case with two explanatory variables:

$$\pi(x) = \Pr ob(Y_i = 1) = \frac{1}{1+e^{-\alpha-\beta_1 X_{1i}-\beta_2 X_{2i}}} = \frac{e^{\alpha+\beta_1 X_{1i}+\beta_2 X_{2i}}}{1+e^{\alpha+\beta_1 X_{1i}+\beta_2 X_{2i}}}$$

Use of Binomial Logistic Regression Model in the problem

Focusing in our problem, the aim is to predict the value of the binary variable "sales" regarding to the sale or not of the product. Also, we must take into account the three possible type of premium.
In order to achieve this, we are building a logistic regression. It will calculate the influence of the different variables over the probability that the sale is done or not.
In our model we force one of the variables to be "premium". This is because we would like to obtain the probability of a customer to be a 1 conditioned to the premium offer, that is, the probability for each premium. Apart from that variable, other variables will be determined by the model to be included.

MODELLING: Model I (binary)

BINOMIAL LOGISTIC REGRESSION MODEL

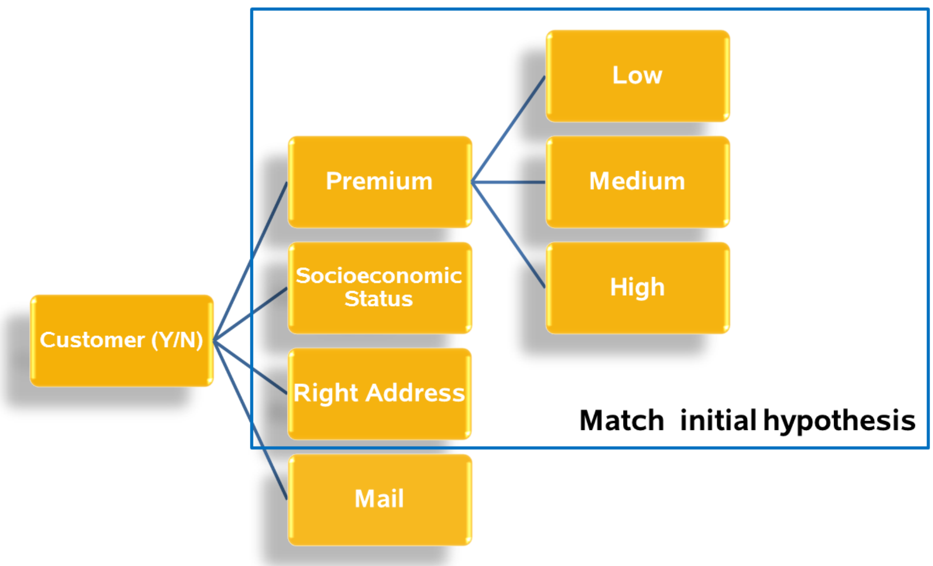
To sum up, the relevant variables are: Premium, Socioeconomic status, Right_address and eMail.

Analysis of Maximum Likelihood Estimates

Parameter		DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq	Exp(Est)
Intercept		1	1.6251	0.1328	149.64	<.0001	5.079
P1	0	1	0.1542	0.0914	2.85	0.0915	1.167
P2	0	1	0.2395	0.0889	7.27	0.0070	1.271
socieconomic_status	1	1	-0.9998	0.1042	92.01	<.0001	0.368
socieconomic_status	2	1	-0.4658	0.0832	31.33	<.0001	0.628
socieconomic_status	3	1	0.6237	0.1574	15.70	<.0001	1.866
right_address	1	1	-0.4819	0.1010	22.78	<.0001	0.618
Email	0	1	-1.3009	0.0739	310.02	<.0001	0.272

The link function used for this model of regression model is the function logit, so that the expression will have the next form:

$$\pi(x) = \frac{1}{1 + e^{-(1.6251+0.1542x_1+0.2395x_2-0.9998x_3-0.4658x_4+0.6237x_4-0.4819x_5\beta-1.3009x_n)}}$$



MODELLING: Model II (multinomial)

MULTINOMIAL LOGISTIC REGRESSION MODEL

Model Definition

Logit
- Binary Response: **Dichotomous Logit** (0, 1)
- Multiple Response: (1, 2, ..., J)
  - Non Ordinal Data: **Multinomial Logit**
  - Ordinal Data: **Ordinal Logit**

Let J describe the number of categories (levels) for response variable Y and $\{\pi_1, \ldots, \pi_j\}$ the probabilities for the different responses satisfying $\sum_j \pi_j = 1$.

The probability distribution for the number of observations falling in the different J categories follow a multinomial distribution. This distribution models the probability of the different ways by which *n* independent observations can be spread out between the J categories.

Given a nominal measure scale, the order betweeen categories is not relevant. A category is taken as base response and a logit model is defined with respect to it.

$$\text{logit}\left(\frac{\pi_j}{\pi_J}\right) = \alpha_j + \beta_j x$$

where *j = 1,...,J-1*. The model has J-1 equations with their own parameters and the **effects** vary with respect to the **base category**. When J=2, the model contains just one equation and corresponds with the standard logistic regression model: $\log(\pi_1/\ \pi_2)=\log(\pi_1)$.

General expression for the model:

Main characteristics:
- As many equations as categories Y has.
- For each variable, as many parameters as Y categories minus one are estimated.
- It is required to use a category as a reference.

MODELLING: Model II (multinomial)

MULTINOMIAL LOGISTIC REGRESSION MODEL

- **General Logit Equation**: with respect to a base category J, determines the logit for any pair of categories. Given any two categories 1 and 2:

$$\log\left(\frac{\pi_1}{\pi_2}\right) = \log\left(\frac{\pi_1/\pi_J}{\pi_2/\pi_J}\right) = \log\left(\frac{\pi_1}{\pi_J}\right) - \log\left(\frac{\pi_2}{\pi_J}\right) =$$

$$(\alpha_1 + \beta_1 x) - (\alpha_2 + \beta_2 x) = (\alpha_1 - \alpha_2) + (\beta_1 - \beta_2)x$$

- **Probabilities for the different responses**:

$$\pi 0 = \Pr ob(Y_i = 0) = \frac{1}{1 + \sum_{j=1}^{J-1} e^{\beta'_{kj} X_{ki}}} \text{ for } j = 0$$

$$\pi j = \Pr ob(Y_i = j) = \frac{e^{\beta'_{kj} X_{ki}}}{1 + \sum_{j=1}^{J-1} e^{\beta'_{kj} X_{ki}}} \text{ for } j = 1, 2, ..., (J-1)$$

For a response variable with three categories and two explanatory variables (predictors) we would have:

$$\pi 1 = \Pr ob(Y_i = 1) = \frac{1}{1 + e^{\alpha_2 + \beta_{12} X_{1i} + \beta_{22} X_{2i}} + e^{\alpha_3 + \beta_{13} X_{1i} + \beta_{23} X_{2i}}}$$

$$\pi 2 = \Pr ob(Y_i = 2) = \frac{e^{\alpha_2 + \beta_{12} X_{1i} + \beta_{22} X_{2i}}}{1 + e^{\alpha_2 + \beta_{12} X_{1i} + \beta_{22} X_{2i}} + e^{\alpha_3 + \beta_{13} X_{1i} + \beta_{23} X_{2i}}}$$

$$\pi 3 = \Pr ob(Y_i = 3) = \frac{e^{\alpha_3 + \beta_{13} X_{1i} + \beta_{23} X_{2i}}}{1 + e^{\alpha_2 + \beta_{12} X_{1i} + \beta_{22} X_{2i}} + e^{\alpha_3 + \beta_{13} X_{1i} + \beta_{23} X_{2i}}}$$

Like ordinary regression, logistic regression extends to include qualitative explanatory variables, often called factors. For that purpose, dummy variables are used. A factor with I categories needs I-1 dummy variables.

MODELLING: Model II (multinomial)

MULTINOMIAL LOGISTIC REGRESSION MODEL

Use of Multinomial Logistic Regression Model in the problem

Keeping in mind that Premium is a key variable in our problem and following the hypothesis done regarding relevant explanatory variables, the following multinomial model has been built using SAS:

$$\text{logit}\left(\frac{\pi_j}{\pi_1}\right) = \alpha_j + \beta_j x =$$
$$\alpha_j + \beta_{j1}[\textbf{right} - \textbf{address}] + \beta_{j2} \times [\textbf{NumberOfCampaigns}] + \beta_{j3}[\textbf{SocioEconomic}_{\textbf{Status}}] + \beta_{j4}[\textbf{ProdActive}]$$

SAS Code

```
proc logistic data=&_train;
model target_model2(ref='1')= right_address numberofcampaigns
socieconomic_status prodactive/link=glogit;
output out=pred predprobs=(I);
run;

proc print data=pred;
title 'Predicted probabilities';
run;
```

In the following paragraphs, the SAS outputs for the model built are described:

Goodness of Fit:

Testing Global Null Hypothesis: BETA=0

Test	Chi-Square	DF	Pr > ChiSq
Likelihood Ratio	1525.5060	12	<.0001
Score	1690.2636	12	<.0001
Wald	655.0424	12	<.0001

Global null hypothesis tests the fit of the current model against a null or intercept-only model. The null model has three parameters (one for each logit equation). The test is highly significant, indicating that at least one of the covariates has an effect on response variable (*customer_type*).

MODELLING: Model II (multinomial)

MULTINOMIAL LOGISTIC REGRESSION MODEL

Type III analysis of effects:

Type 3 Analysis of Effects

Effect	DF	Wald Chi-Square	Pr > ChiSq
right_address	3	13.2788	0.0041
NumberofCampaigns	3	334.9377	<.0001
socieconomic_status	3	143.3798	<.0001
ProdActive	3	199.5426	<.0001

This section shows the change in fit resulting from discarding any one of the covariates — *right_address, numberOfCampaigns, socioeconomic_status, prodActive*— while keeping the others in the model. Judging from these tests (p-values obtained) , we see that all the explanatory variables considered have an effect on customer_type (class/response/target variable).

Estimated Coefficients by Maximum Likelihood:

Analysis of Maximum Likelihood Estimates

Parameter	target_ model2	DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept	0	1	2.2398	0.3813	34.5135	<.0001
Intercept	2	1	-1.1474	0.3616	10.0668	0.0015
Intercept	3	1	-4.7951	0.7197	44.3867	<.0001
right_address	0	1	-0.8060	0.2641	9.3135	0.0023
right_address	2	1	-0.0760	0.2348	0.1047	0.7463
right_address	3	1	0.1724	0.4622	0.1391	0.7091
NumberofCampaigns	0	1	0.7712	0.0802	92.5445	<.0001
NumberofCampaigns	2	1	0.7264	0.0795	83.5540	<.0001
NumberofCampaigns	3	1	1.0219	0.0812	158.2618	<.0001
socieconomic_status	0	1	-0.8631	0.0849	103.3610	<.0001
socieconomic_status	2	1	-0.0618	0.0781	0.6246	0.4293
socieconomic_status	3	1	-0.3283	0.1956	2.8176	0.0932
ProdActive	0	1	-4.4853	0.3248	190.7363	<.0001
ProdActive	2	1	-0.1548	0.1329	1.3570	0.2441
ProdActive	3	1	-0.0797	0.2605	0.0937	0.7596

According to the logit general equation: $\mathrm{logit}\left(\frac{\pi_j}{\pi_J}\right) = \alpha_j + \beta_j x$ there are three logit equations to predict the log-odds of:

MODELLING: Model II (multinomial)

MULTINOMIAL LOGISTIC REGRESSION MODEL

- Customer does not buy versus customer buys low-premium.

$$\text{logit}\left(\frac{\pi_0}{\pi_1}\right) = \alpha_0 + \beta_0 x = $$
$$2.39 - 0.806 \times [\textbf{right} - \textbf{address}] + 0.771 \times [\textbf{NumberOfCampaigns}] - 0.085 \times [\textbf{SocioEconomic\_Status}] - 4.485 \times [\textbf{ProdActive}]$$

- Customer buys medium-premium versus customer buys low-premium.

$$\text{logit}\left(\frac{\pi_2}{\pi_1}\right) = \alpha_2 + \beta_2 x = $$
$$-1.147 - 0.076 \times [\textbf{right} - \textbf{address}] + 0.726 \times [\textbf{NumberOfCampaigns}] - 0.062 \times [\textbf{SocioEconomic\_Status}] - 0.155 \times [\textbf{ProdActive}]$$

- Customer buys high-premium versus customer buys low-premium.

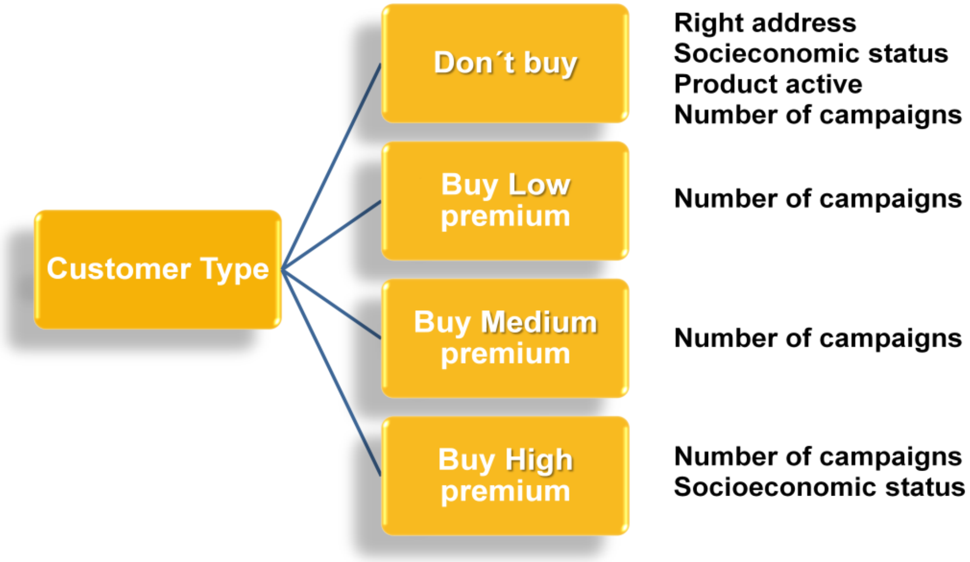
$$\text{logit}\left(\frac{\pi_3}{\pi_1}\right) = \alpha_3 + \beta_3 x = $$
$$-4.795 - 0.172 \times [\textbf{right} - \textbf{address}] + 1.022 \times [\textbf{NumberOfCampaigns}] - 0.328 \times [\textbf{SocioEconomic\_Status}] - 0.079 \times [\textbf{ProdActive}]$$

Note: (customer_type = 1, i.e., customer buys low-premium is taken as reference category).

MODELLING: Model II (multinomial)

MULTINOMIAL LOGISTIC REGRESSION MODEL

From the weights of the coefficients and p-values obtained for each covariate/explanatory variable and response/target level we can conclude that the most relevant explanatory variables according to the response levels are:



MODELLING: Model II (multinomial)

DECISION TREE MODEL

A classification tree for the categorical target variable customer_type (model 2) has been built according to the following characteristics:

Main Tree Parameters/Characteristics

Parameter/Criteria	Value
Splitting criterion (select useful inputs)	Uncertainty measure: Shannon Enthropy
Min number of observations in a leaf (reduce partitions for each input)	6
Min observations required for a split search (reduce partitions for each input)	24
Assessment • **Model Assessment criterion** • **Subtree** (decision/ranking assessment & pruning/ opt.complexity)	Average profit according to the profit/losses matrix. Best assessment value. In many cases, the value of making a true (or false) positive decision differs from the value of making true (or false) negative decision. In such a situation, the concept of accuracy is generalized to profit and the concept of misclassification is generalized to loss.

Profit Matrix associated with Assessment criterion

The following profit matrix has been defined:

		predicted			
		3	**2**	**1**	**0**
r	**3**	3	-1	-1	-3
e	**2**	-1	3	1	-2.8
a	**1**	-1	1	3	-2.5
l	**0**	2	2	2	3

The idea with this matrix is to penalize the cases where the predicted customer type is lower than the observed one (the company would lose incomes in case either a premium is not offered to a potential customer or the premium expected to be offered is lower than the real one).

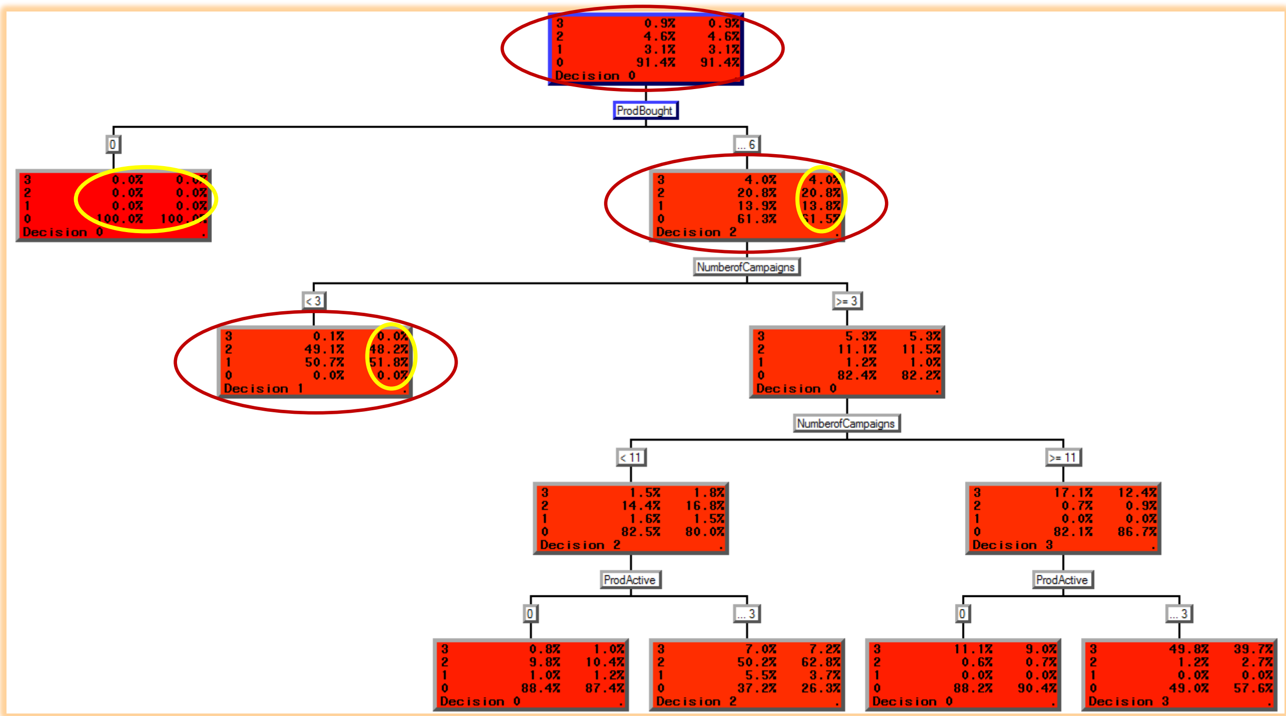
Highlight that whenever the predicted category is 2 and the observed 3 the penalty is higher than the case where moves from 1 to 2 for predicted and observed respectively. That has to do with the fact that the difference between premiums is greater in the first case than in the second one.

MODELLING: Model II (multinomial)

DECISION TREE MODEL

Customer Type Tree

The classification tree obtained according to the parameters/characteristics described above is the following (only four levels of the tree are shown):



The first column of each node shows the customer type category, the second column shows the percentage of cases falling in the corresponding target category in the training set and the third column the percentage for the validation set. Finally, the decision statement provides the best choice according the percentages distribution.

Having a look to the path where the percentages for customer types 1 to 3 (customer buys a premium) **increase** as long as we go deeper in the tree it can be seen that the following decision rule:

$$\boxed{\textbf{[ProdBoughts} > \textbf{0] and [NumberOfCampaigns]} < \textbf{3}}$$

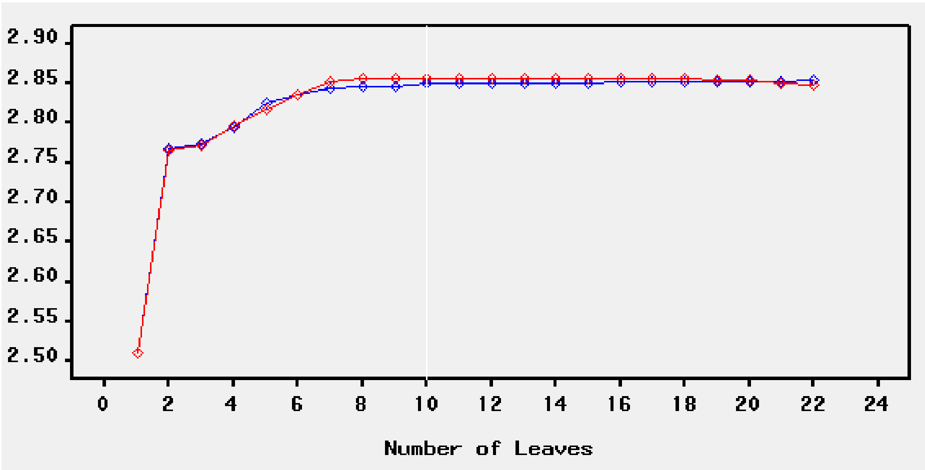
provides the greater probability to capture potential customers. According to this rule, for the validation set, the following premiums are captured (percentages for the corresponding number of cases of the leaf node):

MODELLING: Model II (multinomial)

DECISION TREE MODEL

Validation Set **Product bought > 0** **Number of Campaigns < 3**		**1032 cases** **230 cases** **58 cases**
Premium	% captured	Number of cases
1	**51.8%**	30
2	48.2%	28
3	0%	0
Training Set **Product bought > 0** **Number of Campaigns < 3**		**2388 cases** **529 cases** **136 cases**
Premium	% captured	Number of cases
1	**50.7%**	69
2	49.1%	67
3	0.1%	0

Customer Type Characterization: number of tree leaves vs events captured.



According to the assessment criterion used, with 10 leaves in the tree, the percentage of events captured is 2.85% (*event* means the cases where customer_type is greater than 0, i.e., the customer buys a premium).

MODELLING: Model II (multinomial)

DECISION TREE MODEL

Model Assessment: Classification Tree Diagnostics - confusion matrix

The information provided for the confusion matrix refers to the training set and is the following:

- Rows represent the **predicted** categories for target variable customer_type.
- Columns represent the **real** categories for target variable customer_type.
- Each cell contains 4 values consisting of:
    - Number of cases matching the corresponding predicted and observed category.
    - Accuracy: % of true positives and negatives with respect to the total.
    - Precision: % of predicted positives by the classifier that are really positive: the higher the precision is, the lower the number of false positives will be. (idem with negative cases).
    - Recall (sensitivity): % of positive/negative cases properly predicted by the classifier. The higher the sensitivity is, the lower the number of wrong positive cases will be.

Frequency Percent Row Pct Col Pct	0	1	2	3	Total
0	1187 49.71 84.66 99.08	16 0.67 1.14 3.75	149 6.24 10.63 23.24	50 2.09 3.57 40.98	1402 58.71
1	0 0.00 0.00 0.00	400 16.75 50.76 93.68	387 16.21 49.11 60.37	1 0.04 0.13 0.82	788 33.00
2	7 0.29 5.07 0.58	11 0.46 7.97 2.58	104 4.36 75.36 16.22	16 0.67 11.59 13.11	138 5.78
3	4 0.17 6.67 0.33	0 0.00 0.00 0.00	1 0.04 1.67 0.16	55 2.30 91.67 45.08	60 2.51
Total	1198 50.17	427 17.88	641 26.84	122 5.11	2388 100.00

With the tree built, the percentage of success/hits obtained is a 78.12%: 49.71% for category 0 (customer does not buy) and the remaining 23.41% for categories corresponding to premiums offered to the customer.

Highlight that the higher % of 0´s properly detected has to do with the fact that the **original proportion of events** (frequencies for the categories of target variable) is kept).

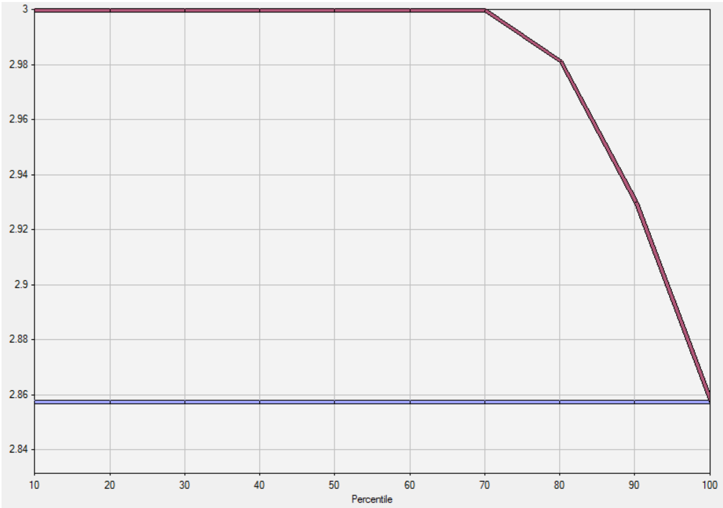
MODELLING: Model II (multinomial)

DECISION TREE MODEL

Model Assessment: Profit Chart (Sensitivity)

For a 70% of the highest scores obtained we get an expected profit of 3. From this 70% till complete the 100% of cases, the average profit decrease sharply till a value of 2.86. This result is in accordance with the values obtained in the confusion matrix.

Blue function represents a random model and red function represents the tree model.



MODELLING: Model II (multinomial)

NEURAL NETWORK

Introduction

A Neural Network has been used as third variant for the multinomial model (model II). The following table provides a summary of some of the main advantages and disadvantages of neural networks as classifiers:

Advantages	Disadvantages
High Accuracy: Neural networks are able to approximate complex non-linear mappings.	Transparency: Neural networks operate as “black boxes”, therefore, lack of transparency
Noise Tolerance: Neural networks are very flexible with respect to incomplete, missing and noisy data	May converge to local minima in the error surface.
Independence from prior assumptions: Neural networks do not make a priori assumptions about the distribution of the data, or the form of interactions between factors.	Totally dependent on the quality and amount of data available.
Flexible: Non-linear model making, flexible for real applications.	Rule extraction is difficult.

The application of Neural Networks in Insurance industry has to do with:

- Profit and growth
- Understanding customer retention patterns (renewal/termination)
- Direct marketing campaigns
- Price setting.

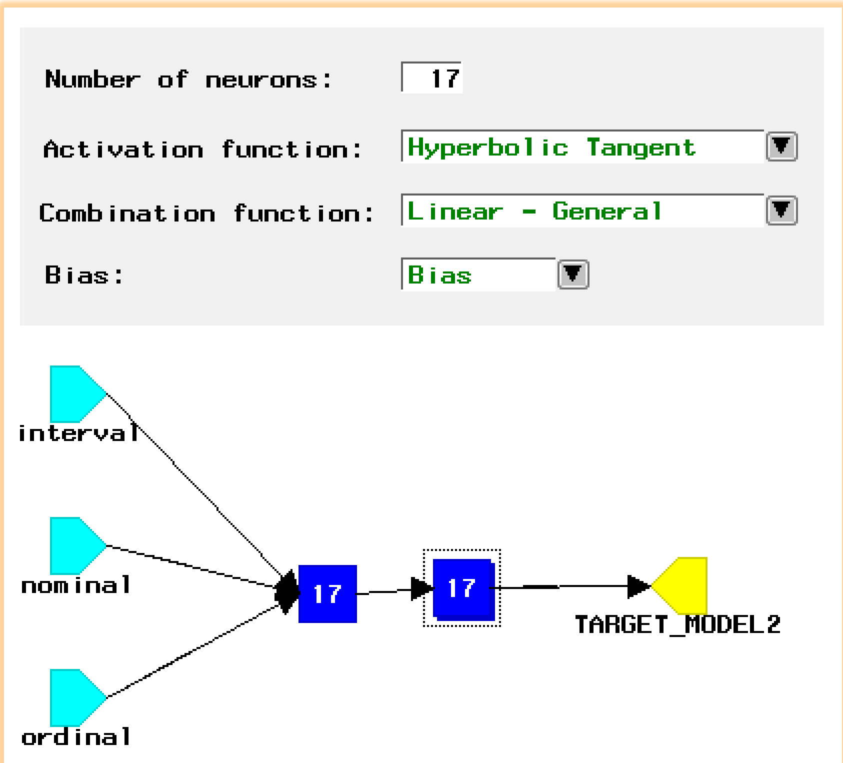
MODELLING: Model II (multinomial)

NEURAL NETWORK

Modelling

The main characteristics of the neural network defined are:

- Multilayer perceptron model
- 2 hidden-layers feed forward neural network with 17 neurons each one:
  - with two hidden layers we make possible to capture non-linear relationships.
  - input layer contains as many neurons as explanatory variables considered (8)
  - according to Lipman rule, the number of neurons for hidden layer are 17.
- Hyperbolic Tangent as activation function
  - due to the fact we have dichotomous variables, we need this activation function.
- Generic Linear combination function
- Selection criterion: profit/loss matrix for back propagation with descend gradient (the one already used for de classification tree).



Explanatory variables (neurons on input layer) have been standardized by average and standard deviation: in this way, hyperbolic tangent function transforms inputs in (-1,1).

MODELLING: Model II (multinomial)

NEURAL NETWORK

Results

Frequency Percent Row Pct Col Pct	0	1	2	3	Total
0	1060 44.39 98.88 88.48	1 0.04 0.09 0.23	6 0.25 0.56 0.94	5 0.21 0.47 4.10	1072 44.89
1	86 3.60 29.35 7.18	85 3.56 29.01 19.91	116 4.86 39.59 18.10	6 0.25 2.05 4.92	293 12.27
2	42 1.76 4.47 3.51	341 14.28 36.32 79.86	514 21.52 54.74 80.19	42 1.76 4.47 34.43	939 39.32
3	10 0.42 11.90 0.83	0 0.00 0.00 0.00	5 0.21 5.95 0.78	69 2.89 82.14 56.56	84 3.52
Total	1198 50.17	427 17.88	641 26.84	122 5.11	2388 100.00

Confusion matrix

Rows in the table represent predicted categories and columns the real/observed categories. Having a look to the diagonal it is observed that a 72.3% of hits (let say "true positives", i.e., % of properly classified cases) are achieved by the model: 44.39% of 0´s contribution, 3.56% of 1´s contribution, 21.52% of 2´s contribution and 2.89% of 3´s contribution.

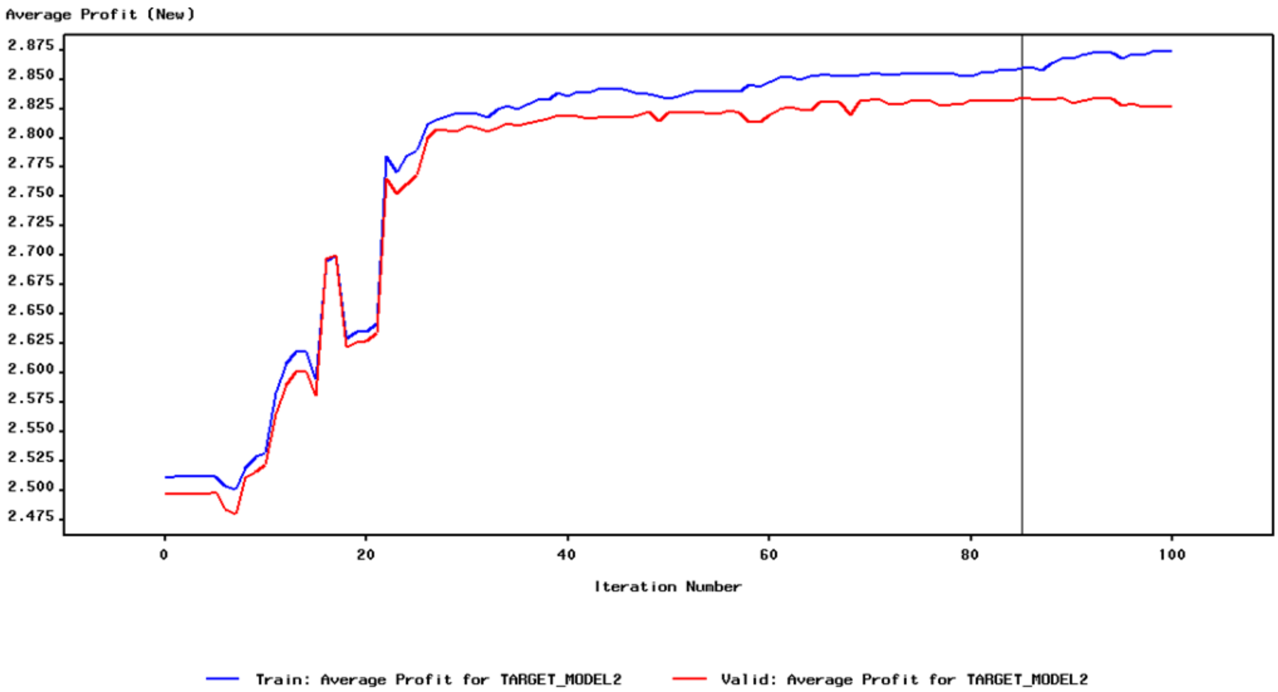
On the other hand, it is observed that:

- Categories 0 and 2 are well predicted: 88.5% (1060/1198) for 0´s and 80.2% (514/641) for 2´s.
- It seems the model confuse 2´s with 1´s. However that´s not really important because both levels of premiums differs only slightly in the price.
- Categories 1 and 3 are worse classified than categories 0 and 2: 19.9% (85/427) for 1´s and 56.55% (69/122) for 3´s: 1´s have the worst prediction power.

MODELLING: Model II (multinomial)

NEURAL NETWORK

With approximately 85 iterations the average profit stops to increase in parallel for training and validation sets and this point is fixed for the optimal generalization. An average profit of 2.9 and 2.8 are obtained for training and validation sets respectively. Considering the profit matrix defined, the values achieved by the model are high and that brings the classification done is good enough.



Average profit by Neural Network iterations number for training and validation datasets

OPTIMIZATION

OPTIMAL MODEL AS COMBINATION OF MODELS

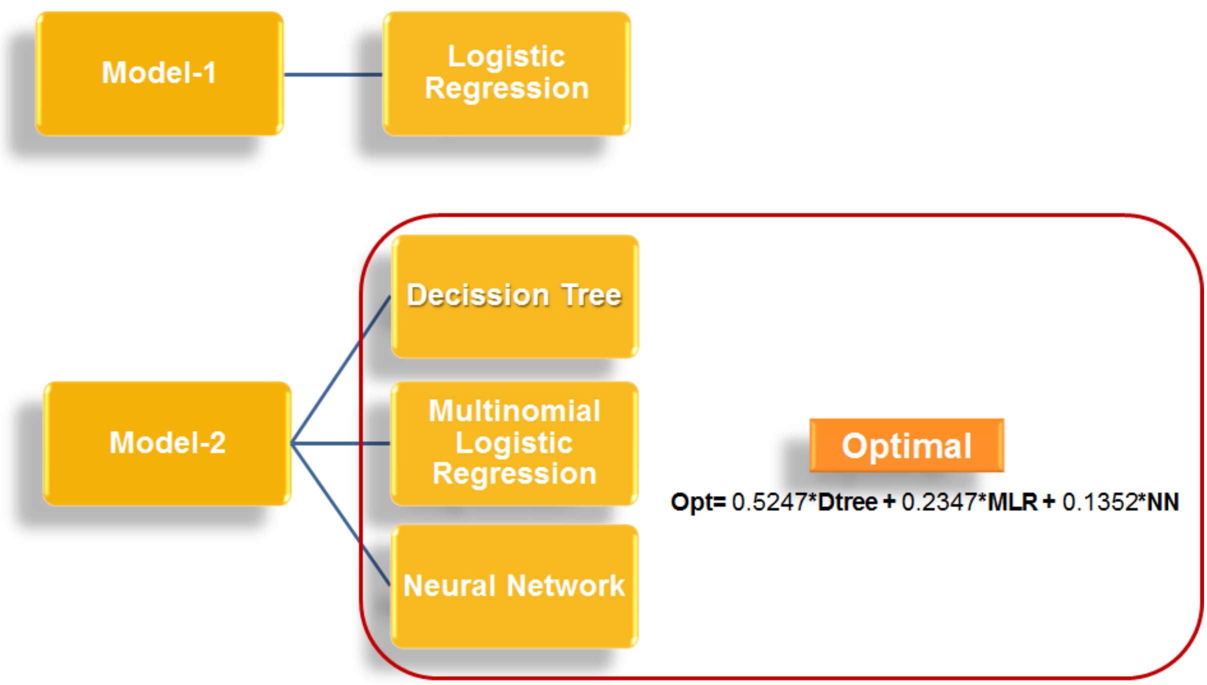
For model II, three variants have been built:

Multinomial model (model II) variants	
M_{TREE} (m1)	**Tree model variant**
M_{MLR} (m2)	**Multinomial logistic regression model variant**
M_{NN} (m3)	**Neural Network model variant**

In order to optimize our multinomial model a convex linear combination of previous models has been used. It can be seen as:

$$M_{opt} = \alpha_1 M_{tree} + \alpha_2 M_{MLR} + \alpha_3 M_{NN}$$

where M_{xxx} refers to the model variant. The following figure summarize the idea of models combination and provide as well the resulting coefficients:



Next paragraphs explain how the optimal model has been built.

OPTIMIZATION

OPTIMAL MODEL AS COMBINATION OF MODELS

First of all let's define the variable and indexes to be considered in the combination of models:

var	description
i index	**i:0..3 for: customer does not by; low premium; medium premium; high premium**
j index	**j:1..3 for models: tree, multinomial reg.,neural network**
k index	**k:1..dataset size (number of cases)**
$pp_i\_m_{j(k)}$	**Probability of category "i" for customer_type obtained by model j for case k.**
$pp\_m_{j(k)}$	**Maximum probability obtained by model "j" for case k (probability associated to the most likely category according to model "j")**
$tp\_m_{j(k)}$	**Target category predicted by model "j" for case "k"**
$to_{(k)}$	**Target category observed (real) for case "k"**
$op_{(k)}$	**Optimal probability for case "k"**
$ot_{(k)}$	**Optimal target category for case "k"**

The following steps explain the combination process:

- For each case of our training and validation sets, four predicted probabilities are obtained: each one corresponds to the different levels of our class/target variable customer_type (the scoring calculated by SAS provides us this data).

probabilities considered by the models					
M1	**pp0_m1**	**pp1_m1**	**pp2_m1**	**pp3_m1**	**pp_m1= max(pp0_m1, pp1_m1, pp2_m1, pp3_m1)**
M2	**pp0_m2**	**pp1_m2**	**pp2_m2**	**pp3_m2**	**pp_m2= max(pp0_m2, pp1_m2, pp2_m2, pp3_m2)**
M3	**pp0_m3**	**pp1_m3**	**pp2_m3**	**pp3_m3**	**pp_m3= max(pp0_m3, pp1_m3, pp2_m3, pp3_m3)**

- For each case, the maximum of these probabilities (**$pp\_m_{j(k)}$**) provides the most likely classification for the customer_type, i.e., the predicted premium to be offered (in case).

- Considering the three models, we can get for each case of the set, the predicted customer_type category together with the associated probability and get the maximum of these three probabilities and therefore the most likely customer_type category according to the three models.

OPTIMIZATION

OPTIMAL MODEL AS COMBINATION OF MODELS

Higher probability for each case according to models max.probs. (and therefore most likely target category)				
Case 1	pp_m1(1)	pp_m2(1)	pp_m3(1)	pp(1)=max(pp_m1, pp_m2, pp_m3)
...	-	-	-	-...
Case k	pp1_m3(k)	pp2_m3(k)	pp3_m3(k)	pp(k)=max(pp_m1, pp_m2, pp_m3)
...	-	-	-	-

- Once customer_type is set for each case according to previous steps, the probabilities for this category in each model are available.

- Finally a function in charge of minimize the absolute error between observed and predicted customer_type categories for each case considering a linear combination of the three models is defined:

$$\min Z = \sum_k |to_{(k)} - \{ot_{(k)} \times (\alpha_1 pp\_m1_{ot(k)} + \alpha_2 pp\_m2_{ot(k)} + \alpha_3 pp\_m3_{ot(k)})\}|$$

$$and \sum_j \alpha_j = 1$$

In order to solve this model and get the values of α_j a genetic algorithm has been used in MatLab (gob library).

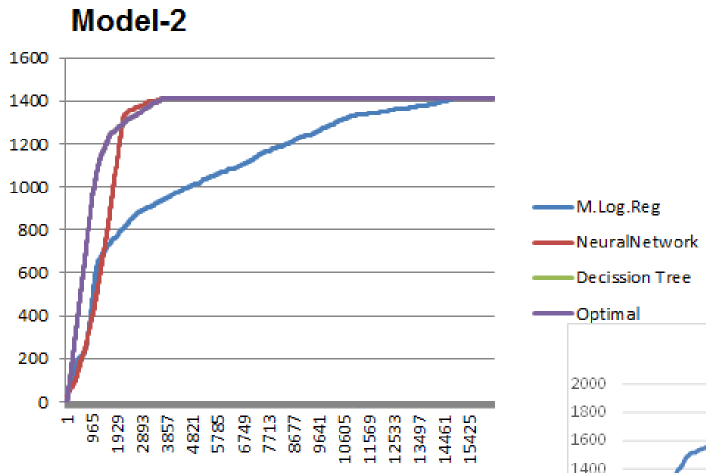
As result, the α vector obtained is: **(0.5247;0.2347;0.1352)**

VALIDATION OF MODELS

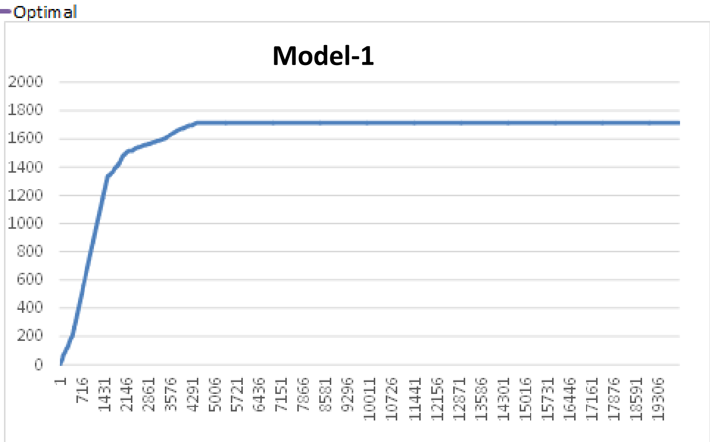
ROC curves for model-I & model-II variants and optimal

According to the shape of the ROC curves (area under curve) obtained for multinomial model (model II) we can see that optimal, Decision Tree and Neural Network models provide better prediction power than multinomial logistic regression model.

Keep in mind that no decision matrix (profit/loss) is used by this model. Optimal and Decision Tree models overlap due to the fact that the tree is the most accurate and the weight of the tree in the optimal model is the highest.



Note: Decission tree curve hidden by Optimal Curve

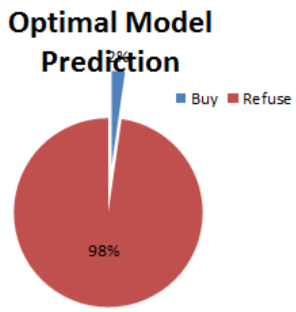


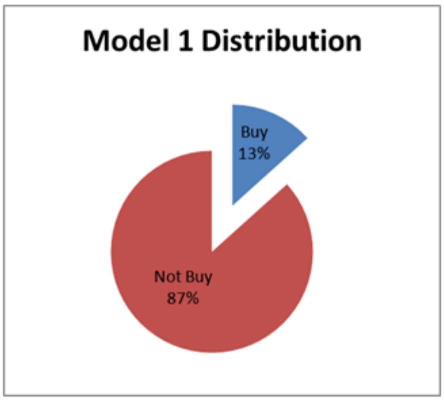
For model I similar results are obtained althought the area under ROC curve seems to be lower than optimal or decision tree in model II. Therfore model II is prefered than model I to predict buyer/not buyer.

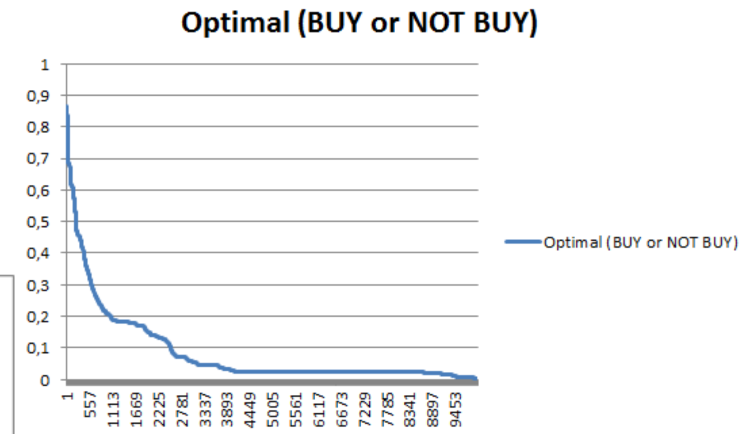
VALIDATION OF MODELS

Merge of Optimum & Model-1 selection: Why?

- The number of buyers catched by model II is lower than the ones catched by model 1 and in both of them lower than the required.
- Way to proceed: Catch buyers with model II, complete with model I and take no buyers from model I offering the premium detected by this model.



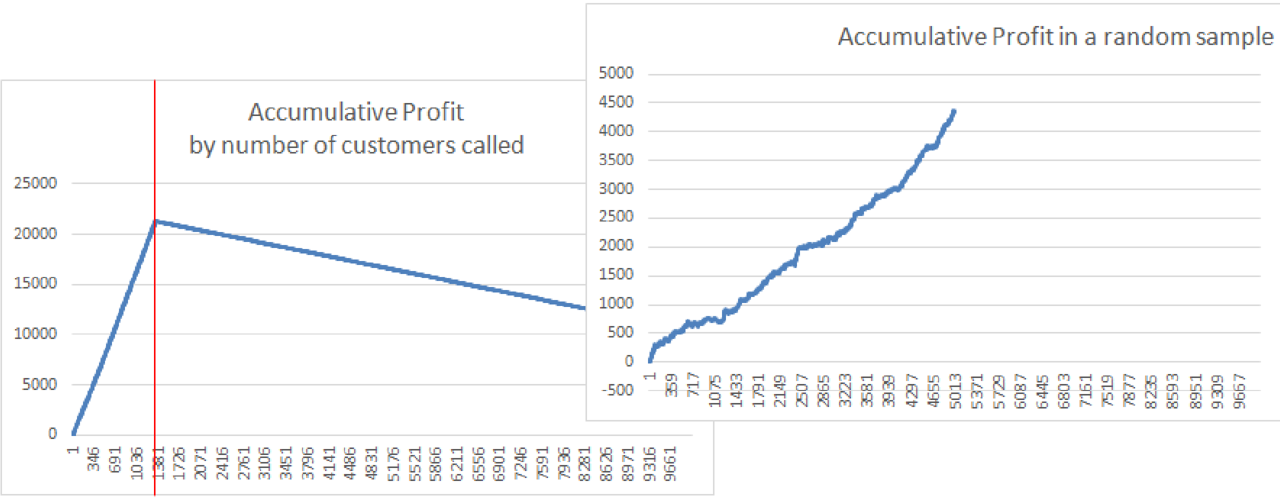




CONCLUSIONS, REMARKS and NEXT STEPS

Having a look to the profit analysis, we can conclude that:

- **First insight**: we have found the optimal number of customers to call should be less than 5000.
- **Second insight**: the benefits with optimal model are much higher than the obtained with the random model: around 20000 euros if we call the optimal number of potential customers in the first case versus around 4500 euros calling to 5000 in the second case (random model).



Remarks

- Followed two research lines. They converged in some aspects. They shed light about the drivers and the forecasting ability
- Saved some difficulties:
    - Missing values
    - Ideate the models
    - Decide the most relevant aspects

Next Steps

- More time to know the insurance company interests about the model
- Analyze the relationship among the explanatory variables in depth, non linearities and so on…
- Analyze robustness of all the models more carefully.
- With more information about costs, to build a better benefit function.