Signal propagation in nonlinear optical fibers

Problem raised by CNIT, Pisa

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Exposition of the problem:

The proposed problem concerns the propagation of an electromagnetic signal in an optical fiber, with dispersive, nonlinear and dissipative effects. In general, such propagation can be described by a one-dimensional Schrödinger equation with cubic nonlinearity, where the roles of space and time variables are inverted with respect to the usual Schrödinger equation of quantum mechanics. The concomitant effects of chromatic dispersion (different wavelenghts travel with different speed) and of nonlinear refraction (the refraction index depends of the field intensity) lead to a signal distorsion. The signal, moreover, which travels for hundreds, or even thousands, of kilometers, needs to be periodically reinforced by optical amplifiers. In this way, undesired noise is introduced into the fiber. Usually, such noise is modelled as an input gaussian white process. Then, signal and noise nonlinearly mix up during the propagation, and this contributes to the degeneration of the output.
Modelling and simulation of all such phenomena is fundamental for the correct interpretation of the output signal and is, therefore, of central interest for the telecommunications industries.

The proposed model, rather than using directly the Schrödinger equation, exploits the so-called Madelung transform which leads to an Euler-like system of equations for a compressible fluid. Such system has some advantages with respect to the original formulation; in particular, a "semiclassical" approximation (which is somehow related to the geometrical optics approximation) can be performed, leading to a considerable simplification of the model.

Scheme of the work to be done:

1) Introduction of the basic ingredients of the model: the nonlinear Schrödinger equation, the Madelung transform and the resulting fluid-like description. Discussion about input and boundary data which, as a first step, will be supposed to be deterministic.

2) Introduction of a suitable scaling and discussion about possible approximations, in particular the "semiclassical" one.

3) Numerical solution of the deterministic and semiclassical propagation problem.

4) Discussion about possible refinements of the model, such as insertion of the stochastic input and/or dropping of the semiclassical approximation. Then, if possible, numerical implementation of the refined model.