Signal Propagation in Nonlinear Optical Fibers

Michelle De Decker, José María García Roldán,
James Herterich, Alexander Kospach, Tommaso Ristori

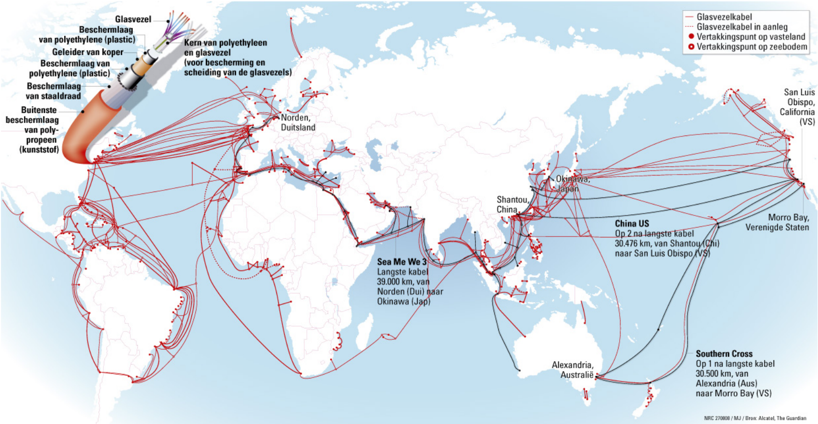
Instructor: Luigi Barletti

Introduction

- Optical fibers made of pure glass (silica) are used as a medium for telecommunication
- information carrying signal travels for hundreds, or even thousands, of kilometers
- several degeneration processes
- signal needs to be periodically reinforced by optical amplifiers over its long journey $\rightarrow$ adds noise
- want to investigate the propagation of this electromagnetic signal

Modelling and simulation of the signal and the degeneration processes is fundamental for the correct interpretation of the output signal and is, therefore, of central interest for the telecommunications industries.

Introduction



Introduction

Degeneration processes that the optical fiber undergoes includes:

- chromatic dispersion
- nonlinear self-phase modulation (Kerr effect)
- dissipation
- nonlinear mixing with noise.

We will look at modelling the electromagnetic signal with the dispersive, nonlinear and dissipative effects with added noise in an optical fiber.
This will provide a tool for the signal degeneration analysis.

Electromagnetic field in the fiber

The electromagnetic field inside the fiber can be assumed to have this form:

$$\vec{E}(x, y, z, T) = F(x, y)\,\mathrm{e}^{i(\beta_0 z - \omega_0 T)}U(z, T)\,\vec{\nu}$$

transverse mode (assumed to be fixed)

longitudinal *carrier wave*

longitudinal *modulation* (information carrying)

polarization vector (assumed to be fixed)

Electromagnetic field in the fiber

The electromagnetic field inside the fiber can be assumed to have this form:

$$\vec{E}(x, y, z, T) = F(x, y)\, \mathrm{e}^{i(\beta_0 z - \omega_0 T)} U(z, T)\, \vec{\nu}$$

transverse mode (assumed to be fixed)

longitudinal *carrier wave*

longitudinal *modulation* (information carrying)

polarization vector (assumed to be fixed)

Electromagnetic field in the fiber

The electromagnetic field inside the fiber can be assumed to have this form:

$$\vec{E}(x, y, z, T) = F(x, y)\, \mathrm{e}^{i(\beta_0 z - \omega_0 T)} U(z, T)\, \vec{\nu}$$

transverse mode (assumed to be fixed)

longitudinal *carrier wave*

longitudinal *modulation* (information carrying)

polarization vector (assumed to be fixed)

Electromagnetic field in the fiber

The electromagnetic field inside the fiber can be assumed to have this form:

$$\vec{E}(x, y, z, T) = F(x, y)\, \mathrm{e}^{i(\beta_0 z - \omega_0 T)} U(z, T)\, \vec{\nu}$$

transverse mode (assumed to be fixed)

longitudinal *carrier wave*

longitudinal *modulation* (information carrying)

polarization vector (assumed to be fixed)

Electromagnetic field in the fiber

The electromagnetic field inside the fiber can be assumed to have this form:

$$\vec{E}(x,y,z,T) = F(x,y)\,\mathrm{e}^{i(\beta_0 z - \omega_0 T)}U(z,T)\,\vec{\nu}$$

transverse mode (assumed to be fixed)

longitudinal *carrier wave*

longitudinal *modulation* (information carrying)

polarization vector (assumed to be fixed)

Equation for $U(z, T)$

The modulation $U(z, T)$ obeys a *nonlinear Schrödinger equation (NLSE)*

$$iU_z = -i\beta_1 U_T - \frac{\beta_2}{2} U_{TT} - i\alpha U + \gamma |U|^2 U$$

wave packet drift

chromatic dispersion (CD)

dissipation

self-phase modulation (SPM)

Equation for $U(z, T)$

The modulation $U(z, T)$ obeys a *nonlinear Schrödinger equation (NLSE)*

$$iU_z = -i\beta_1 U_T - \frac{\beta_2}{2} U_{TT} - i\alpha U + \gamma |U|^2 U$$

wave packet drift

chromatic dispersion (CD)

dissipation

self-phase modulation (SPM)

Equation for $U(z, T)$

The modulation $U(z, T)$ obeys a *nonlinear Schrödinger equation (NLSE)*

$$iU_z = -i\beta_1 U_T - \frac{\beta_2}{2} U_{TT} - i\alpha U + \gamma |U|^2 U$$

wave packet drift

chromatic dispersion (CD)

dissipation

self-phase modulation (SPM)

Equation for $U(z, T)$

The modulation $U(z, T)$ obeys a *nonlinear Schrödinger equation (NLSE)*

$$iU_z = -i\beta_1 U_T - \frac{\beta_2}{2} U_{TT} - i\alpha U + \gamma |U|^2 U$$

wave packet drift

chromatic dispersion (CD)

dissipation

self-phase modulation (SPM)

Equation for $U(z, T)$

The modulation $U(z, T)$ obeys a *nonlinear Schrödinger equation (NLSE)*

$$iU_z = -i\beta_1 U_T - \frac{\beta_2}{2} U_{TT} - i\alpha U + \gamma|U|^2 U$$

wave packet drift

chromatic dispersion (CD)

dissipation

self-phase modulation (SPM)

Equation for $u(z,t)$

By the change of variable

$$u(z,t) = U(z, t + \beta_1 z)$$

we get rid of the drift term and obtain the NLSE

$$\boxed{iu_z = -\frac{\beta_2}{2} u_{tt} - i\alpha u + \gamma |u|^2 u}$$

which, from now on, will be our model of signal propagation in a nonlinear optical fiber.

This has to be supplemented with *input conditions* at $z = 0$:

$$\boxed{u(0,t) = u_0(t)}$$

Equation for $u(z, t)$

By the change of variable

$$u(z, t) = U(z, t + \beta_1 z)$$

we get rid of the drift term and obtain the NLSE

$$\boxed{iu_z = -\frac{\beta_2}{2} u_{tt} - i\alpha u + \gamma |u|^2 u}$$

which, from now on, will be our model of signal propagation in a nonlinear optical fiber.

This has to be supplemented with *input conditions* at $z = 0$:

$$\boxed{u(0, t) = u_0(t)}$$

Nondimensionalization

We nondimensionalize the problem by introducting new variables and parameters

$$t = t_0 \hat{t}, \quad z = z_0 \hat{z}, \quad u = u_0 \hat{u}, \quad P_0 = u_0^2,$$

$$N_D = \frac{t_0^2}{\beta_2}, \quad N_{NL} = \frac{1}{\gamma P_0}.$$

where t_0, z_0 and P_0 are reference time, length and power, and the hatted terms are the new dimensionless variables.

Scaled NLSE

It is convienient to set $\epsilon^2 = \frac{N_{NL}}{N_D}$ and then we choose z_0 such that $\epsilon = \frac{N_{NL}}{z_0}$.
In this way we obtain the "semiclassical" scaled form of NLSE:

$$i\epsilon u_z = -\frac{\epsilon^2}{2} u_{tt} + |u|^2 u - \frac{i\alpha}{2} N_{NL} u,$$

where ϵ is a small parameter.

Which t_0 and P_0 values?

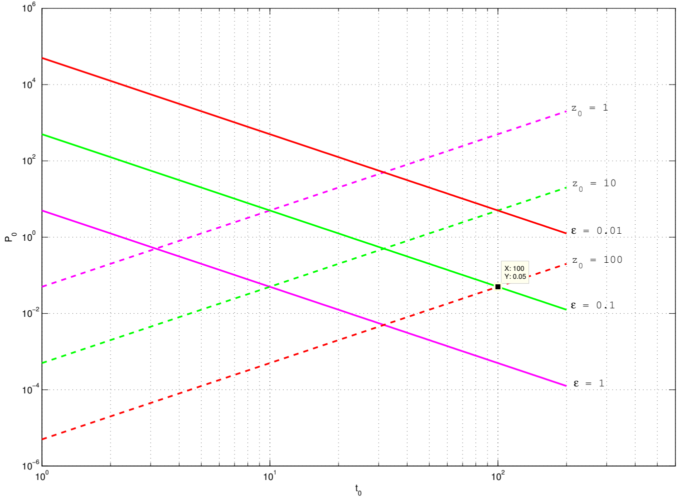
To find a small ϵ value that corresponds to a large z_0 value we plot the two graphs

$$P_0 = \frac{t_0^2}{\gamma\beta z_0^2}, \qquad P_0 = \frac{\beta}{\epsilon^2\gamma t_0^2},$$

for different values of ϵ and z_0.

From the graph we see that a convenient choice is $t_0 \approx 100$ ps and $P_0 \approx 50$ mW.

Which t_0 and P_0 values?



The Madelung transform

We then apply the *Madelung transform*

$$\begin{aligned} u(z,t) &= \sqrt{\rho(z,t)}\,\exp\left(\frac{i}{\epsilon}\phi(z,t)\right), \\ J(z,t) &= \phi_t(z,t), \end{aligned}$$

which brings the NLSE into a fluid-dynamic form:

$$\begin{aligned} \rho_z + (\rho J)_t &= -\hat{\alpha}\rho, \\ J_z + \frac{\partial}{\partial t}\left[\frac{J^2}{2} + \rho - \epsilon^2\left(\frac{\rho_{tt}}{4\rho} - \frac{\rho_t^2}{8\rho^2}\right)\right] &= 0. \end{aligned}$$

These are called *Madelung equations*.

A simplified fluid model

Neglecting the $\mathcal{O}(\epsilon^2)$ terms, we finally obtain a simplified fluid model to work with:

$$\boxed{\begin{cases} \rho_z + (\rho J)_t = -\hat{\alpha}\rho, \\ J_z + \dfrac{\partial}{\partial t}\left(\dfrac{J^2}{2} + \rho\right) = 0. \end{cases}}$$

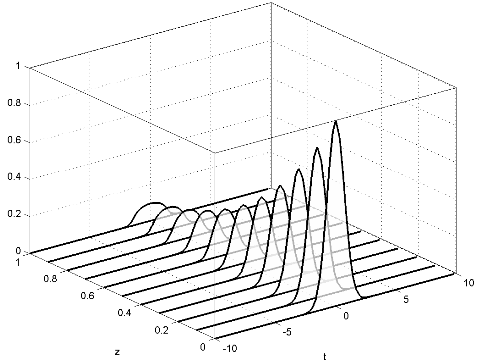
Numerical Experiments

- Implement the simplied fluid model and the full NLSE
- Numerical package COMSOL Multiphysics® (finite element method)
- Will compare both models to demonstrate that the main trend of the solutions are captured by the simplified fluid model
- No natural boundary conditions $\Rightarrow$ choose large domain with trival Neumann conditions
- Initial condition will have a dominating effect on the solution (Gaussian Pulses, Chirped Gaussian Pulses, Super Gaussian Pulses and Hyperbolic Secant Pulses)

Gaussian Pulse

$$u(0,t) = \exp\left(-\frac{t^2}{2\sigma^2}\right),$$

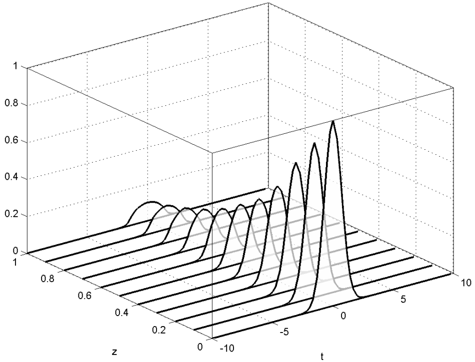
Figure: The fluid model solution.



Gaussian Pulse

$$u(0,t) = \exp\left(-\frac{t^2}{2\sigma^2}\right),$$

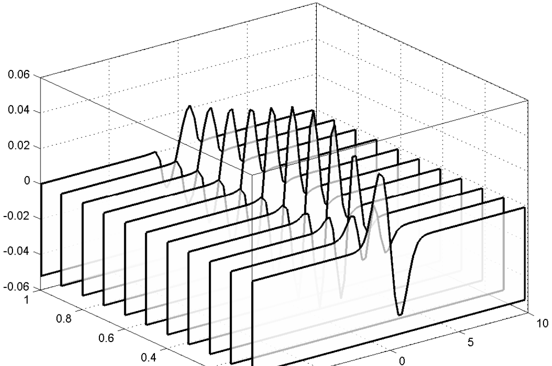
Figure: The full NLSE solution.



Chirped Gaussian Pulses

$$u(0,t) = \exp\left(-\frac{(1+ic)t^2}{2\sigma^2}\right)$$

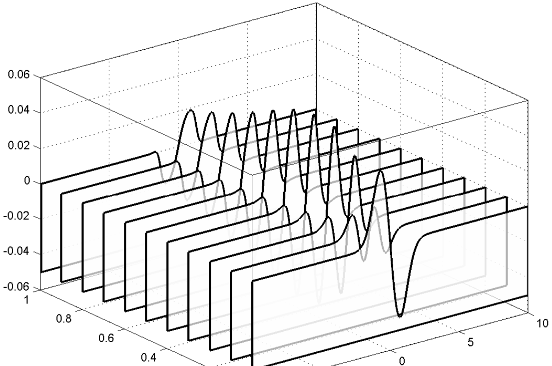
Figure: The fluid model



Chirped Gaussian Pulses

$$u(0,t) = \exp\left(-\frac{(1+ic)t^2}{2\sigma^2}\right)$$

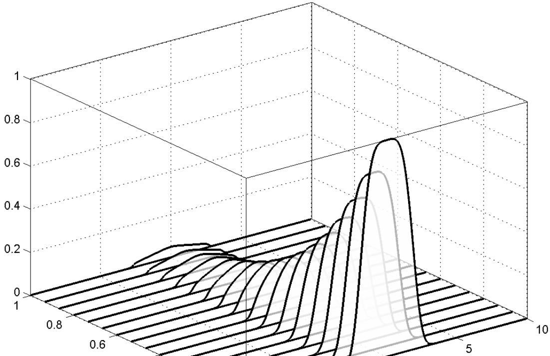
Figure: The full NLSE solution.



Super Gaussian Pulses

$$u(0,t) = \exp\left(-\frac{(1+ic)}{2} \left(\frac{t}{\sigma}\right)^{2m} \right)$$

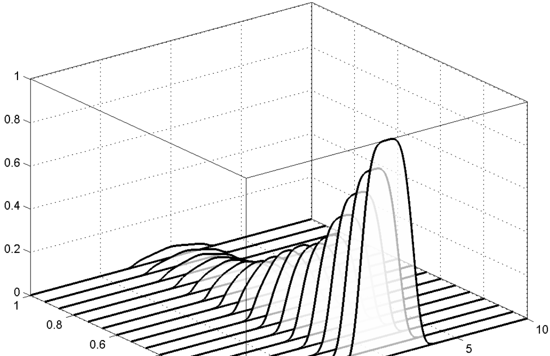
Figure: The fluid model solution.



Super Gaussian Pulses

$$u(0,t) = \exp\left(-\frac{(1+ic)}{2}\left(\frac{t}{\sigma}\right)^{2m}\right)$$

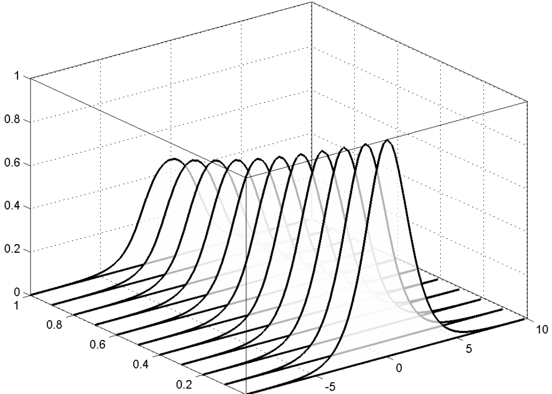
Figure: The full NLSE solution.



Hyperbolic Secant Pulses

$$u(0,t) = \operatorname{sech}\left(\frac{t}{\sigma}\right) \exp\left(-\frac{ict^2}{2\sigma^2}\right)$$

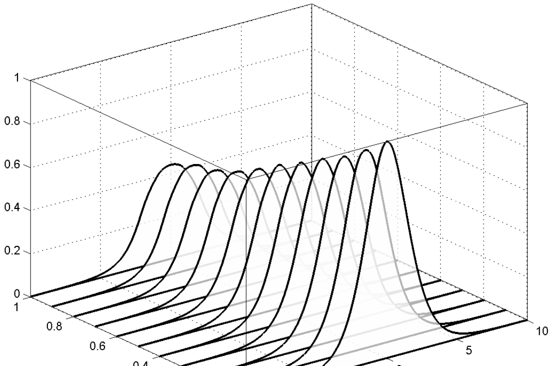
The fluid model solution.



Hyperbolic Secant Pulses

$$u(0,t) = \operatorname{sech}\left(\frac{t}{\sigma}\right) \exp\left(-\frac{ict^2}{2\sigma^2}\right)$$

Figure: The full NLSE solution.



Input noise

The signal travels for hundreds, even thousands, of kilometers and, because of dissipation, needs to be reinforced by optical amplifiers placed along the fiber.

Each amplifier introduces a certain amount of *noise*.

Then, if we consider an amplifier-amplifier or amplifier-receiver span of the fiber, we have to prescribe *stochastic* input data

$$\boxed{u(0,t) = u_0(t) + \lambda g(t)}$$

where u_0 is the deterministic signal and g is some stochastic process.

Assumptions

$$u_0(t) = \bar{u}_0(t)\exp\left(\frac{i}{\epsilon}\phi_0\right) \qquad g(t) = \bar{g}(t)\exp\left(\frac{i}{\epsilon}\phi_0\right)$$

$\bar{g}$ is a white stochastic process with a Chi squared distribution at a fixed time.

Perturbation Solution

$$\rho_z + (\rho J)_t = -\hat{\alpha}\rho,$$
$$J_z + \frac{\partial}{\partial t}\left(\frac{J^2}{2} + \rho\right) = 0.$$

$$\rho(z,t) = \sum_{j=0}^{\infty} \lambda^j \rho_j(z,t) \approx \rho_0(z,t) + \lambda\rho_1(z,t),$$
$$J(z,t) = \sum_{j=0}^{\infty} \lambda^j J_j(z,t) \approx J_0(z,t) + \lambda J_1(z,t),$$

Perturbation Solution

$$
\begin{aligned}
\rho_0(0,t) &= \bar{u}_0^2, \\
\rho_1(0,t) &= 2\bar{u}_0\bar{g}, \\
J_0(0,t) &= \frac{\partial \phi_0}{\partial t}, \\
J_1(0,t) &= 0,
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \rho_0}{\partial z} + \frac{\partial}{\partial t}(\rho_0 J_0) &= -\tilde{\alpha}\rho_0, \\
\frac{\partial J_0}{\partial z} + \frac{\partial}{\partial t}\left(\frac{J_0^2}{2} + \rho_0\right) &= 0, \\
\frac{\partial \rho_1}{\partial z} + \frac{\partial}{\partial t}(\rho_1 J_0 + J_1 \rho_0) &= -\tilde{\alpha}\rho_1, \\
\frac{\partial J_1}{\partial z} + \frac{\partial}{\partial t}(J_0 J_1 + \rho_1) &= 0.
\end{aligned}
$$

Super Gaussian

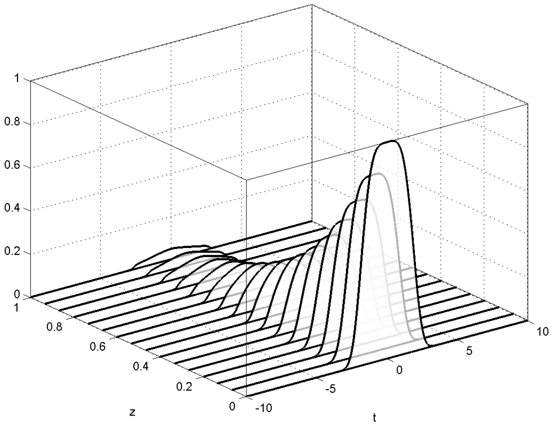
Consider a perturbed super Gaussian. This means that in the initial conditions we have

$$\bar{u}_0 = \exp\left(-\frac{1}{2}\frac{t^{2m}}{\sigma^{2m}}\right) \qquad \phi_t = 0$$

for one realisation with $\bar{g} = 1$, $\sigma = 2$ and $m = 2$.

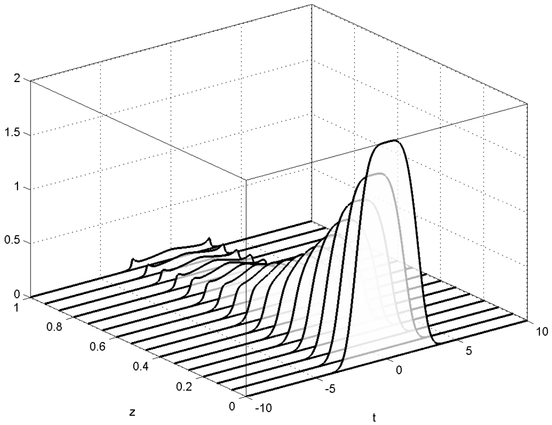
Stochastic Super Gaussian Pulses

Figure: The leading order fluid model solution for ρ.



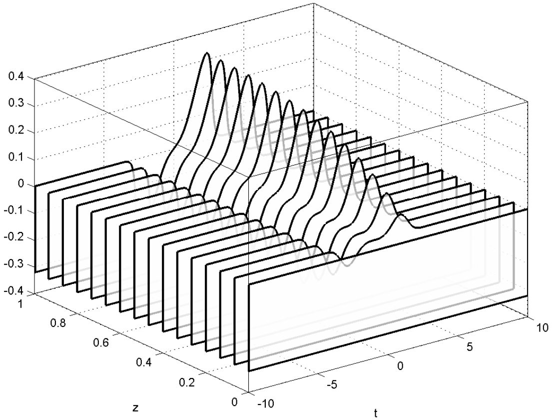
Stochastic Super Gaussian Pulses

Figure: The first order correction fluid model solution for ρ.



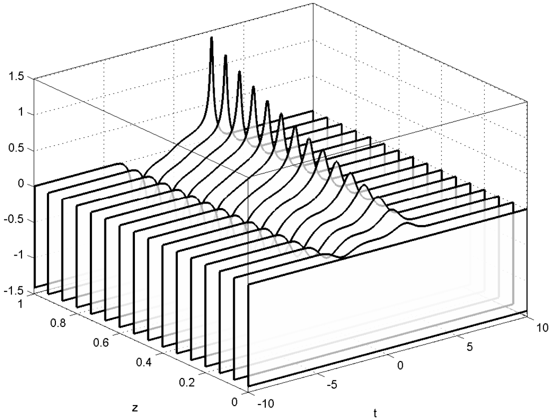
Stochastic Super Gaussian Pulses

Figure: The leading order fluid model solution for J.



Stochastic Super Gaussian Pulses

Figure: The first order correction fluid model solution for J.



Conclusions

- Considered a model of signal propagation in optical fibers based on NLSE
- Applied Madelung transform to obtain a simplified fluid model
- Excellent numerical evidence for agreement between NLSE and simplified fluid model
- Stochastic input produces a perturbation that we treated at first order

Muchisimas gracias por su atenciòn!