Estimation of Orientation Distribution of Fibers

V Modelling Week
Complutense University of Madrid

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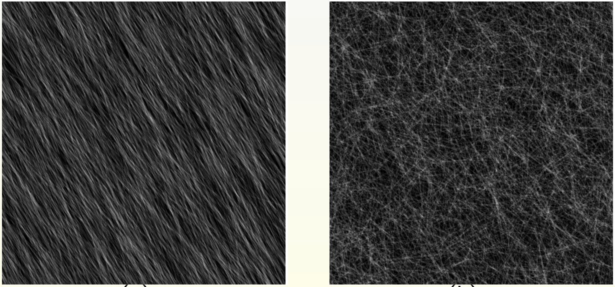
Introduction and motivation

- The distribution of the fibers that compose the paper determines its properties.
- It is of the major importance in the paper production industry to be able to properly analyse the distribution properties.
- Design techniques such that given a certain image of a paper sheet determine the orientation distribution of its fibers.
- Objectives: study and analyse two specific techniques,
  - Structure Tensor based method, and
  - Fast Transform Fourier technique.
- Moreover new ideas will be tried to be introduced in order to improve their performance.

Analysed paper sheets

Several imaging techniques can be used in order to analyse the paper sheets. In these project we have used the following main set of image types:



(a) (b)

Figure: Simulated paper sheet fiber images.

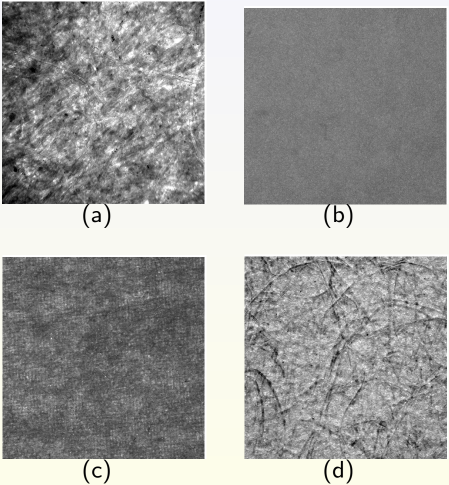


Figure: Real paper sheets: (a) high-resolution microscope, (b) newspaper with low resolution microscope, (c) standard paper with low resolution microscope, and (d) surface imaging of a sheet.

Image Processing for FFT

Processing of the images is useful for FFT method:

- Smoothing Operations: to reduce the amount of intensity variation between one pixel and the next ⟶ applied to reduce noise.
  - Mean Filter[2]: replacing each pixel value with the mean value of its neighbours.
  - Median Filter: the selected value comes from the existing brightness value.
- Rolling Ball algorithm for subtract background: ⟶ to reduce non-uniform defects
- Thresholding: segmenting the foreground from the background
  - Otsu's Method

[2] Mean filter has also used in the structure tensor technique

Structure tensor technique

- Basic idea: observe a small part of the image, called *window* and obtain average orientation of the fibers in it.
- Slide window along image obtaining then different values.
- Distributions dependent on the size of the window and on the step.
- The windows are a function that is zero valued outside the subdomain and weights under a certain criteria.
- In our case, window is a square that is determined with central point (x_0, y_0) and the length of the square side L.

Theoretical basis

Weighted inner product

$$\langle f, g\rangle_w = \int\int_{\mathbb{R}^2} w(x, y) f(x, y) g(x, y) \mathrm{d}x\mathrm{d}y, \qquad (2.1)$$

f is a function of intensity of grey color, i.e. $f = 0$ for white pixels and $f = 255$ for black pixels. Areas with less fibers in the image will be darker.

Directional derivative of the function f:

$$D_{u_\theta} f(x, y) = u_\theta^T \nabla f(x, y).$$

Objective: extract the direction where this change is maximized for each window:

$$\mathbf{u} = \arg \max_{\|u_\theta\|=1} \|D_{u_\theta} f(x, y)\|_w^2.$$

The structure tensor of f is 2×2 matrix defined as

$$J = \left\langle \nabla f, \nabla f^T \right\rangle_w = \begin{bmatrix} \langle f_x, f_x \rangle_w & \langle f_x, f_y \rangle_w \\ \langle f_x, f_y \rangle_w & \langle f_y, f_y \rangle_w \end{bmatrix}. \qquad (2.2)$$

Considering the Lagrange function of the restricted minimization problem we get

$$J\mathbf{u} = \lambda \mathbf{u}. \qquad (2.3)$$

Substituting the eigenvalues expressions in (2.3) and keeping in mind $u_\theta = (\cos\theta, \sin\theta)$ we obtain that the peak of the distribution is given as

$$\theta = \frac{1}{2} \arctan \left(\frac{2 \langle f_x, f_y \rangle_w}{\langle f_y, f_y \rangle_w - \langle f_x, f_x \rangle_w} \right). \qquad (2.4)$$

We also define the *coherency* parameter,

$$C = \frac{\lambda_{\mathrm{max}} - \lambda_{\mathrm{min}}}{\lambda_{\mathrm{max}} + \lambda_{\mathrm{min}}} = \frac{\sqrt{\left(\langle f_y, f_y\rangle_w - \langle f_x, f_x\rangle_w\right)^2 + 4\,\langle f_x, f_y\rangle_w^2}}{\langle f_y, f_y\rangle_w + \langle f_x, f_x\rangle_w} \in [0, 1]. \tag{2.5}$$

The second parameter is the *energy*,

$$E = \mathrm{Trace}(J). \tag{2.6}$$

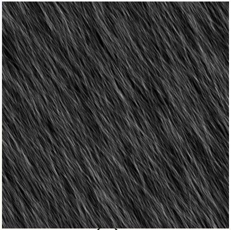
More information about the structure tensor method for image analysis can be found on References [1, 2].

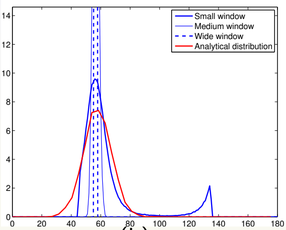
Algorithm 2.1 Structure tensor technique algorithm

Ensure: Grey map f.

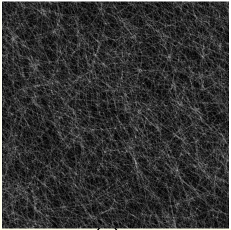
function $Get\theta(f)$
    $f \leftarrow$ filter f
    $p_1, \ldots, p_{np} \leftarrow$ set discretization of np points
    $L \leftarrow$ set window length
    $\theta, E, C \leftarrow$ allocate angle, energy and coherency vectors
    for $k = 1 : np$ **do**
        $p_k \leftarrow$ fix point k of the discretization
        $W_k \leftarrow$ set window of edge length L and weights
        $\mathbf{J}_k \leftarrow$ compute structure tensor matrix (Eq. (2.2))
        $\theta(k), E(k), C(k) \leftarrow$ compute $\theta,\ C,\ E$ (2.4), (2.5), (2.6)
    end for
end function

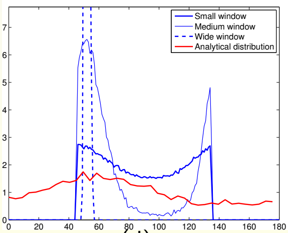
Results


(a)



(b)


(c)

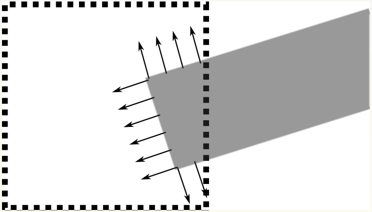


(d)

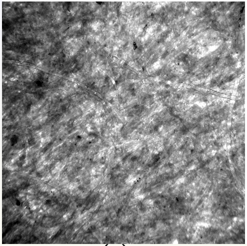
- Good main orientation guess.
- Difficulties on determining the exact orientation distribution.
- Noise peaks.

Problem with narrow windows:

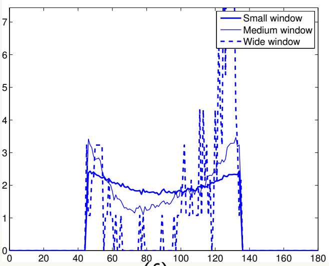
- Ideally, if we consider narrow windows in a really dense point distribution, we get closer to the analytical distribution.
- However peaks appear.



- The arrows symbolize the gradient of the function.
- ST method may not distinguish between the edge of the fiber, and its width.
- Hence, it finds two main orientations: the orientation of the fiber, and another one about 90° translated corresponding to its cross-section.



(e)


Small window
Medium window
Wide window
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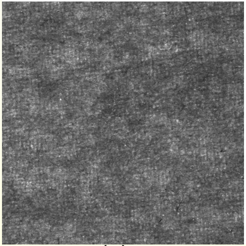
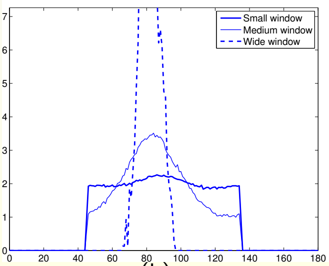
(f)



(g)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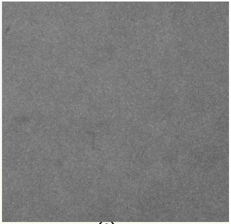
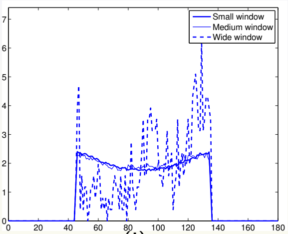
(h)



(i)



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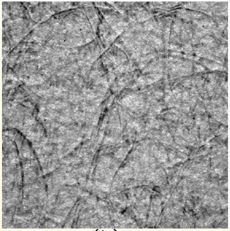
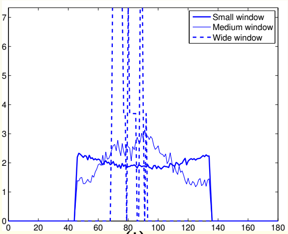


(k)



(l)

- Main orientations more or less correctly found.
- Not possible ensuring always good results.

New weight strategy: an attempt to improve the method

- The window function weight is assigned differently to each point.
- The assigned weight corresponds to a function of the energy and coherency on the nearby region of this point.
- Such region is going to be considered as a small window that contains just the closest pixels to the analysed one.

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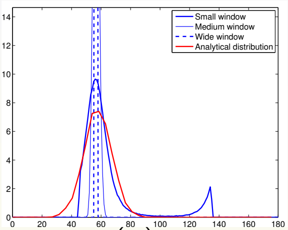
The proposed weight function is defined as

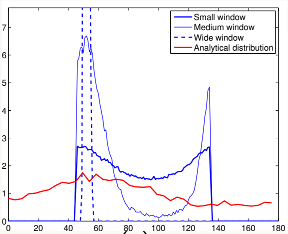
$$w(x, y) = \frac{C_{sw(x,y)} \cdot E_{sw(x,y)}}{E_{max}} \in [0, 1], \tag{2.7}$$

- w close to 1 values when the orientation of the subwindow is very clear and there is a high fiber presence, and
- close to 0 values when either there is no main orientation or there is a low fiber presence.

Results of the modified technique



(m)



(n)

- Small improvements in the estimated distribution of fibers.
- The perturbation peak is decreased, while the main orientation peak is increased.
- However, the improvement is niggling. With images with more noise, the improvement is not even noticed.

Fast Fourier Transform Method

- Start from Fourier Transformed images
- Add the amplitude of Fourier coefficient in the radius direction from the origin
- Problem: conversion XY coordinates $\rightarrow$ polar coordinates

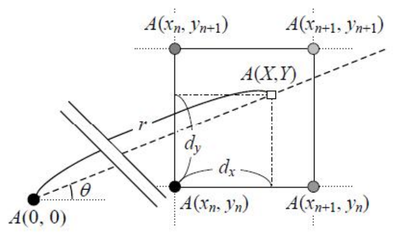


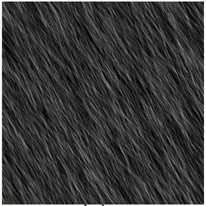
Figure: Interpolation [3]

$$\begin{aligned} A(X,Y) &= A(rcos\theta, rsin\theta) \\ &= (1-d_x)(1-d_y)A(x_n,y_n) + d_x(1-d_y)A(x_{n+1},y_n) \\ &+ (1-d_x)d_yA(x_n,y_{n+1}) + d_xd_yA(x_{n+1},y_{n+1}) \end{aligned}$$

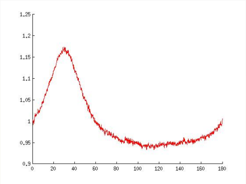
Mean amplitude in each direction

$$A(\bar{\theta}) = \left(\frac{n}{2}-1\right)\sum_{r=2}^{\frac{n}{2}} A(rcos\theta, rsin\theta)$$

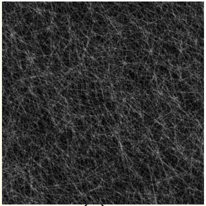
Simulated paper sheets: original images and corresponding distribution



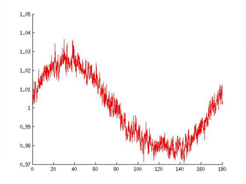
(a)



(b)

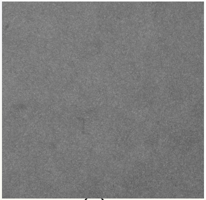


(c)

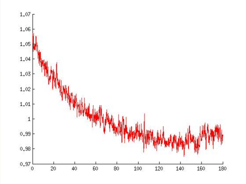


(d)

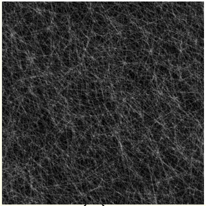
Real paper sheets: original images and corresponding distribution



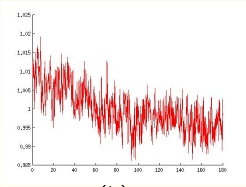
(e)



(f)

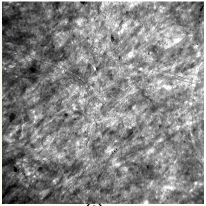


(g)

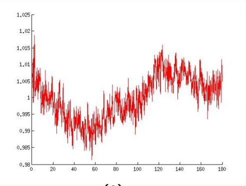


(h)

Real paper sheet: original images and corresponding distribution



(i)



(j)

Conclusions

- Good guess of the main orientation
- ST technique does not give good estimation of exact distribution
- FT has too much oscillations

J. Bigun and G. Granlund, *Optimal Orientation Detection of Linear Symmetry*, Tech. Report LiTH-ISY-I-0828, Computer Vision Laboratory, Linkoping University, Sweden 1986; Thesis Report, Linkoping studies in science and technology No. 85, 1986.

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