V MODELLING WEEK

Complutense University of Madrid

Faculty of Mathematical Sciences

Estimation of Orientation Distribution of Fibers

Authors:

Davidović A.
Gargallo-Peiró A.
Llontop García M.
Muñoz Ortega F.J.
Toledo Carrasco D.
Wang C.

Advisor:

Dr. Jouni Sampo[1]

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[1]Lappeeranta University of Technology, Finland

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1 Introduction and motivation

Paper is composed mainly by wood fibers and mineral fillers, together with other additives that conform the basic structure of the sheet. The distribution of the fibers that compose the paper determines its properties. Then, it is of the major importance in the paper production industry to be able to properly analyse the distribution properties in order to ensure a good quality paper.

It is in this context where the presented work is placed. In this framework, in order to ensure that the produced paper ensures a certain distribution of fibers (or either ensures that the fibers do not configure a certain critical distribution), the related industries require techniques that estimate the orientation distribution of the produced paper sheets.

Such techniques must be able to, given a certain image of a paper sheet where there can be distinguished the inner fibers, determine the orientation distribution of such set of fibers.

The objective of this project is to study and analyse two specific techniques: a Structure Tensor based method, and a Fast Transform Fourier technique. Once programmed and analysed the limitations and strong points of such methods, new ideas will be tried to be introduced in order to improve their performance.

Several imaging techniques can be used in order to analyse the paper sheets. In these project we have used the following main set of image types:

- Simulated fiber distribution that are represent microscope type-images, see Figure 9.
- Real fiber distributions of paper sheets taken with surface imaging technique (different incident lights on the surface of the paper), see Figure 10(d).
- Real fiber distributions of paper sheets taken with low quality imaging with microscopes, see Figures 10(b) and 10(c).
- Real fiber distribution of paper sheets taken with high quality resolution with electronic microscopes 3(a).

Moreover note that Figure 10(c) corresponds to a newspaper sheet of paper, and that the rest of not simulated images correspond to different standard printing paper sheets.

2 Structure tensor technique

2.1 Theoretical basis

The basic idea of the method is to observe a small part of the image, called *window* and obtain average orientation of the fibers in it. For obtaining distribution of the orientation on all image we slide window along image obtaining different values depending on which part of the image we observe at each time. As we will see distributions will highly depend on the size of the window we use and on the step that we take for moving window along the image.
The windows are defined as a function on the image domain that is zero valued outside the subdomain of interest and weights under a certain criteria. In our case, window is a square that is determined with central point (x_0, y_0) and the length of the square side L.
Let us now explain how to compute average orientation of the fibers in the given window. Consider the weighted inner product of two $\mathbb{R}^2$ valued functions f and g,

$$\langle f, g\rangle_w = \int\int_{\mathbb{R}^2} w(x,y)f(x,y)g(x,y)\mathrm{d}x\mathrm{d}y, \tag{2.1}$$

where $w(x,y) \geq 0$ is a weight function. The simple example of the weight function is the *identity weight* given as $w(x,y) = 1$ for points inside of the window and 0 elsewhere. The norm of f is defined by $\|f\| = \sqrt{\langle f, f\rangle_w}$. In our case f is a function of intensity of grey color, i.e. $f = 0$ for white pixels and $f = 255$ for black pixels. Areas with less fibers in the image will be darker. Also if the fibers are mostly oriented in one specific direction, there will be more variation in intensity of grey in normal direction to that one. Hence, we are interested in directional derivative of the function f given by

$$D_{u_\theta} f(x,y) = u_\theta^T \nabla f(x,y)$$

where $\nabla f(x,y) = (f_x, f_y)$ is the gradient of f and $u_\theta = (\cos\theta, \sin\theta)$ is a unit vector that gives direction. The directional derivative gives a measure of the change of the color in a given direction. We are interested in the direction where this change is maximized, and we will compute it for each window. It is given by

$$\mathbf{u} = \arg\max_{\|u_\theta\|=1} \|D_{u_\theta} f(x,y)\|_w^2.$$

The structure tensor of f is 2×2 matrix defined as

$$J = \left\langle \nabla f, \nabla f^T \right\rangle_w = \begin{bmatrix} \langle f_x, f_x\rangle_w & \langle f_x, f_y\rangle_w \\ \langle f_x, f_y\rangle_w & \langle f_y, f_y\rangle_w \end{bmatrix}. \tag{2.2}$$

Now we can write

$$\|D_{u_\theta} f(x,y)\|_w^2 = \left\langle u_\theta^T \nabla f, \nabla f^T u_\theta \right\rangle = u_\theta^T J u_\theta.$$

The Lagrange function of the optimisation problem is

$$\Lambda(u_\theta, \lambda) = u_\theta^T J u_\theta + \lambda \left(u_\theta^T u_\theta - 1 \right).$$

Then, setting the derivative of Λ w.r.t. u_θ equal to 0 we get that solution of the optimisation problem satisfies

$$J\mathbf{u} = \lambda \mathbf{u}. \tag{2.3}$$

Hence, the directional derivative is maximized in the direction of the eigenvector that corresponds to maximal eigenvalue of J, $\lambda_{\max} = \max \|D_{u_\theta} f(x,y)\|_w^2$, and minimized in orthogonal direction given by the second eigenvector and $\lambda_{\min} = \min \|D_{u_\theta} f(x,y)\|_w^2$. The eigenvalues of J are

$$\lambda_{1,2} = \langle f_x, f_x \rangle_w + \langle f_y, f_y \rangle_w \pm \sqrt{(\langle f_x, f_x \rangle_w - \langle f_y, f_y \rangle_w)^2 + 4 \langle f_x, f_y \rangle_w^2}$$

Substituting these values in (2.3) and keeping in mind $u_\theta = (\cos\theta, \sin\theta)$ we obtain that the peak of the distribution is given as

$$\theta = \frac{1}{2} \arctan \left(\frac{2 \langle f_x, f_y \rangle_w}{\langle f_y, f_y \rangle_w - \langle f_x, f_x \rangle_w} \right). \tag{2.4}$$

There are some improvements that can be done using some additional information that can be extracted directly from the structure tensor. The first one is *coherency*, given by

$$C = \frac{\lambda_{\max} - \lambda_{\min}}{\lambda_{\max} + \lambda_{\min}} = \frac{\sqrt{\left(\langle f_y, f_y \rangle_w - \langle f_x, f_x \rangle_w \right)^2 + 4 \langle f_x, f_y \rangle_w^2}}{\langle f_y, f_y \rangle_w + \langle f_x, f_x \rangle_w} \in [0,1]. \tag{2.5}$$

If in the window there is one dominant orientation, the coherency will be near 1. On the other hand if in the window coherency is close to 0 it means that fibers are almost uniformly distributed in each direction. In this case window does not give good information, and we can disregard information obtained from this window. The second improvement can be done using *energy*, given by

$$E = \mathrm{Trace}(J). \tag{2.6}$$

The energy will be higher in the windows where we have more fibers. Hence we can disregard windows with the low energies.

Algorithm 2.1 shows schematically the computational procedure that has to be brought up in order to compute the structure tensor for the determination of the orientation distribution. We have implemented Algorithm 2.1 using Matlab. More information about the structure tensor method for image analysis can be found on References [1, 2, 3].

Algorithm 2.1 Structure tensor technique algorithm

Ensure: Grey map f.

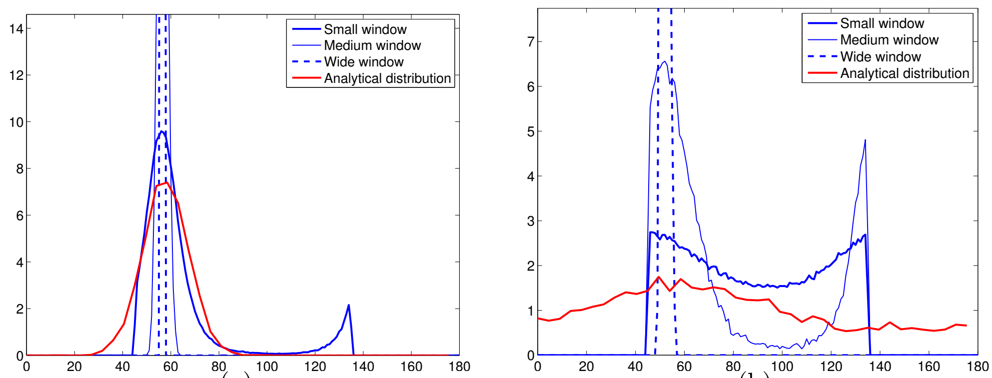
function $Get\theta(f)$
    $f \leftarrow$ filter f
    $p_1, \ldots, p_{np} \leftarrow$ set discretization of np points
    $L \leftarrow$ set window length
    $\theta, E, C \leftarrow$ allocate angle, energy and coherency vectors of size np
    for $k = 1 : np$ **do**
        $p_k \leftarrow$ fix point k of the discretization
        $W_k \leftarrow$ set window domain of edge length L and weights
        $\mathbf{J}_k \leftarrow$ compute structure tensor matrix (see Equation (2.2))
        $\theta(k), E(k), C(k) \leftarrow$ compute parameters (see (2.4), (2.5), (2.6))
    end for
end function

2.2 Results

In this section we present the results corresponding to the structure tensor technique for the paper sheets shown in Figures 9 and 10, corresponding to distribution of fibers of simulated and real papers.

For each analysed fiber distribution we present an image with the results obtained using the structure tensor with different window sizes and discretizations. We specifically have selected to present three types of windows. A wide window (compared to the width of the image domain) in a coarse distribution of points, a midsize window in a refined distribution, and a really narrow window in a dense distribution of points. For all windows, the weighting function is considered as the identity inside the window (and taking zero value outside of it).

Figure 1(a) shows the orientation distribution of the fibers corresponding to Figure 9(a). The x-axis, $[0, 180]$ degrees, corresponds to the angle orientations of the fibers. The y-axis corresponds to the "number" of fibers with the corresponding orientation. The analysed image, Figure 9(a), corresponds to a simulated paper sheet, created using a low randomized pattern of the orientation of the fibers. The fiber distribution in red corresponds to the exact analytical orientation in the paper. The blue distributions correspond to the results of the structure tensor method for different width of the used windows.



(a) (b)

Figure 1: Orientation distribution of fibers of simulated paper sheets presented in Figures 9(a) and 9(b).

Since Figure 1(a) presents the results of a simulated fiber distribution, we are able to plot the exact orientation distribution (in red). Comparing the exact distribution with the one obtained with the three attempts of the method we can extract the relation between the resulting distribution and the window size.

When we choose a wide window (size about one order of magnitude smaller than the width of the domain), the presented technique tends to concentrate all the area of the distribution on the main orientation of the fibers of the mesh. That is because for each analysed point of the discretization of the domain a wide range of fibers is analysed. Hence, the most dominant fiber orientation is present in all windows, being too much weighted, and disregarding the other fiber orientations.

However, as the size of the window is decreased, the orientation distribution tends to the analytical behaviour of the given sheet. However, we can realise that noise in the image difficults the precision of the method if we decrease the window size.

Note that when we finally consider narrow windows in a really dense point distribution, we get closer to the analytical distribution, but the distortions of the solution increase. Together with the noise contained in the image, another possible explanation may help to understand the no-sense peaks that appear in the solution distribution.

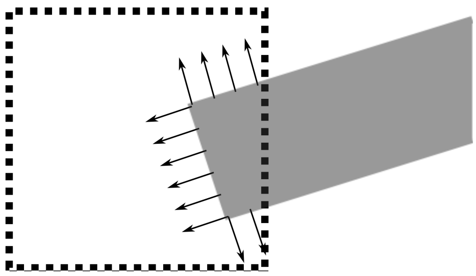


Figure 2: Problematic originated due to the use of narrow windows.

Figure 2 presents an scheme of the added problems that appear due to the use of narrow windows in the analysis. Imagine that a narrow window is opened at the end of one fiber. The arrows that appear in 2 represent the gradients of the function on the boundary of the fiber. If we then consider the orientation in such window using the structure tensor, the presented technique may not distinguish between the edge of the fiber, and its width. Hence, in such window we would extract that there are two main orientations: the orientation of the fiber, and another one about $90°$ translated corresponding to its cross-section. Note that the cross-section gradient is a false orientation angle, and hence, this introduces an error in the solution.

Looking at the results for this first simple image 1(a), we realize that the presented method is going to become a really useful and robust key to determine the main fiber orientation in a paper. However, we have observed an important perturbation when the behaviour of the analytical solution is trying to being approximated by reducing the window size. From this oscillations we realize that the method is really sensitive to noise of the paper image.

Results presented in Figure 3 show the orientation distribution of more complex fiber distributions. All the presented results correspond to real paper sheet distributions, see Figure 10.

Note that in Figure 3(a), the estimation of the orientation distribution is far from being precise. When the structure tensor is created using a wide window, we can clearly see that we are able to catch properly the main orientation of the fibers, but oscillations appear. However, when we decrease the window width, quite a different pattern, with almost no characteristic orientation can be found. Recall that the same commentaries apply to 3(b) and 3(d). Figure 3(c) seems to be a bit stable, although we have no way to totally confirm the given results.

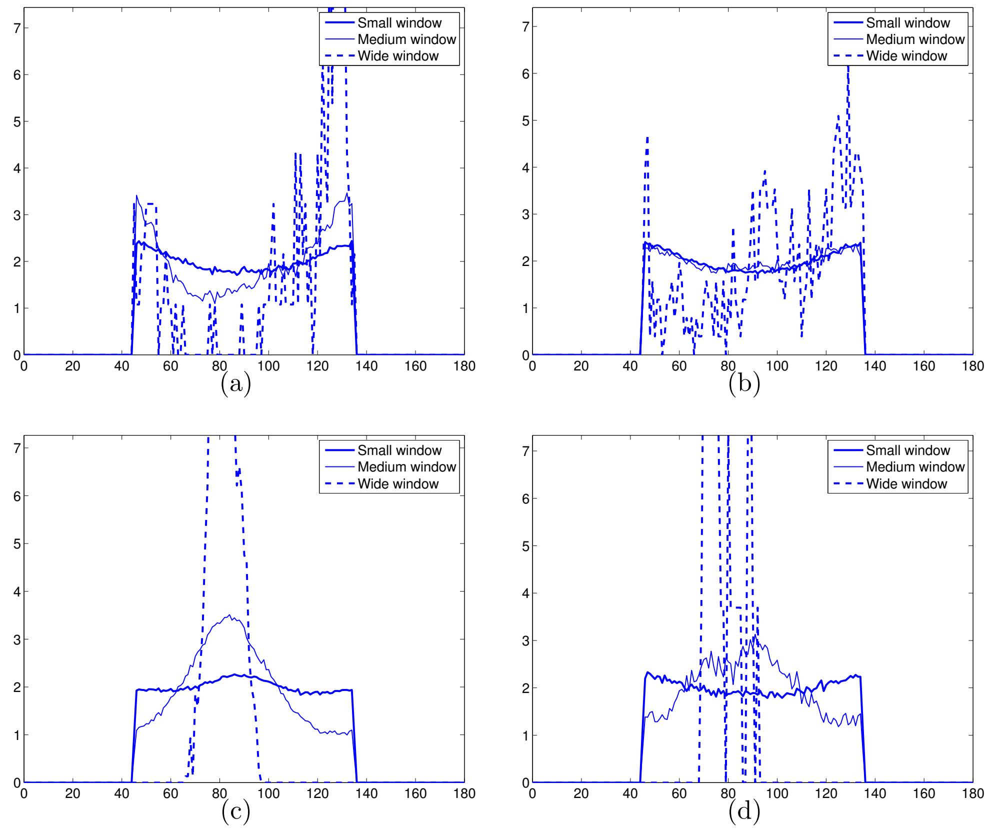


Figure 3: Orientation distribution of fibers of real paper sheets presented in (a) Figure 10(a), (b) Figure 10(b), (c) Figure 10(c), and (d) Figure 10(d).

2.3 New weight strategy: an attempt to improve the method

From the results presented in Section 2.2 we realise that the structure tensor method is limited when we want to use it to determine the exact orientation distribution of the fibers in a paper. However, is worth it remarking that it is a precise method to detect the main orientation of the fibers of the paper sheet.

In an attempt to improve the way that the method considers the orientation distribution of an opened window, we have defined a new weighting function in terms of the energy and the coherency. Fixed a window width, the window function is not any more considered as the identity inside the window and zero outside it. Instead of this simple defined function, a different weight is assigned to each point. The weight assigned corresponds to a function of the energy and coherency on the nearby region of this point. Such region is going to be considered as a small window that contains just the closest pixels to the analysed one.

Recall that the coherency is a parameter in the interval $[0, 1]$, taking zero when there is no a main orientation in a window, and taking one there is just one orientation in the window. Moreover, the energy is a parameter that can take a wide range of values, and that is bigger and bigger in relation to the number of fibers and changes of gradient in function f (gray scale function).

Hence, if we want to somehow weight a point in function of its importance in the determination of the fiber orientation distribution, we require to use both parameters. The proposed weight function is defined as

$$w(x, y) = \frac{C_{sw(x,y)} \cdot E_{sw(x,y)}}{E_{max}}, \tag{2.7}$$

where $C_{sw(x,y)}$ and $E_{sw(x,y)}$ denote the coherency and the energy in the subwindow defined on point (x, y), and E_{max} denotes the maximum energy among all the subwindows that are considered in the image domain. Hence, note that $w(x, y) \in [0, 1]$, taking close to 1 values when the orientation of the subwindow is very clear and there is a high fiber presence, and close to 0 values when either there is no main orientation or there is a low fiber presence.

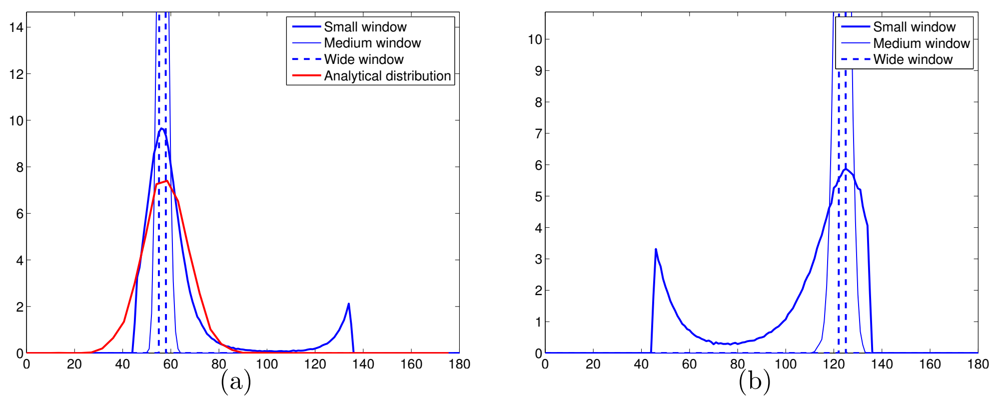


Figure 4: Orientation distribution of fibers using the modified weighted structure tensor of Figures 9(a) and 9(b).

Figure 2.3 presents the results for the simulated paper sheets presented in Figure 9 of the new weighting function. If we analyse in detail the results, we can see that there are slightly improvements in the estimated distribution of fibers. The perturbations in the orientation are decreased, while the main orientation peak is slightly increased. However, the improvement is niggling, and despite increase in the computational time is not worth the slight improvement. Moreover, with more randomised images and with images with more noise, the improvement is even harder to be noticed.

3 Fast Fourier Transform Method

The second analysed technique corresponds to the Fourier Transform method. We have to remark that this method tends to present many oscillations in the determination of the solution. In order to help the method in the determination of an stable orientation distribution, a processing of the images has been brought up. Details on the used techniques and examples of the processed images can be found on Appendix A.2.

3.1 Numerical calculation of fiber distribution

This numerical work is based on [4].

We started from the Fourier transformed images, and then we programmed in Matlab to find the distribution of the fiber orientation in the original images. The complete explanation of the process is:

In the process of determining fiber orientation from the Fourier Transformed images, amplitude of Fourier coefficient was added in the radius direction from the origin, what means, the center of the image toward the peripheral, meaning that was determined from 0 to 180 degrees of center angle θ.

But then, we have a problem because this addition was found to be not as easy as we expected: Each position, representing frequency of the Fourier coefficient in the XY coordinates, cannot be easily converted to the corresponding point in the polar coordinates exactly.

If we drew a radius of a given center angle from the XY coordinate origin, the radius just passed through some positions but mostly only near the positions. Then, interpolation was applied in the following manner:
Suppose amplitude of a Fourier coefficient $A(X, Y)$ at a position (X, Y) in the XY frequency space, what we can see in the following figure:

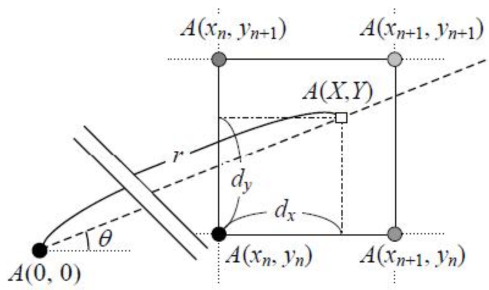


Figure 5: Interpolation [4]

We designed the calculation method that amplitude $A(X, Y)$ at that point with center angle and radius r is given by the equation:

$$\begin{aligned} A(X,Y) &= A(rcos\theta, rsin\theta) \\ &= (1-d_x)(1-d_y)A(x_n, y_n) + d_x(1-d_y)A(x_{n+1}, y_n) \\ &+ (1-d_x)d_yA(x_n, y_{n+1}) + d_xd_yA(x_{n+1}, y_{n+1}) \end{aligned}$$

Then, mean amplitude $\bar{A(\theta)}$ in every direction of center angle θ was calculated according to this equation:

$$\bar{A(\theta)} = \left(\frac{n}{2} - 1\right) \sum_{r=2}^{\frac{n}{2}} A(rcos\theta, rsin\theta) \tag{3.1}$$

And it is defined as the fiber orientation distribution in this work .

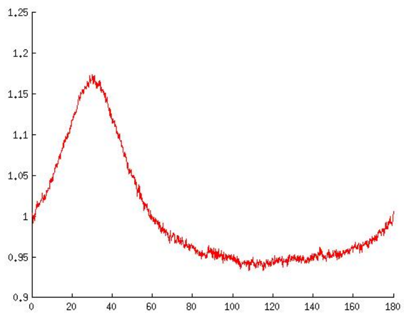
So, we implemented these equations with Matlab, getting some graphics that gave us an idea about those distributions that we were looking for. In next section, we show some of the original images, their Fourier Transformed images and the graphics with the distribution of the fibers.

3.2 Results

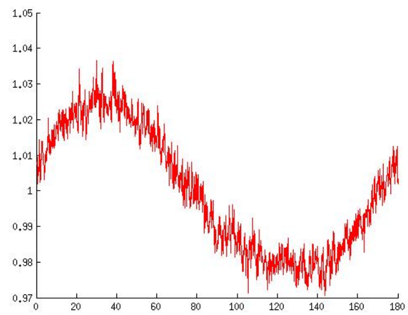
The second method was performed to the five different images of the appendix. As we can see in the graphics, for the first and second ones the results are quite good. Looking at the first graphic there is a maximum at 30 degrees[2] , whereas for the second image the maximum is around 40 degrees

[2]Note that the values from the Fourier transform method differ from the ones of the Structure Tensor metric by 30. This is due to the fact that the angle is measured in the clock wise direction and we are measuring not the orientation direction, but the orthogonal one, that is the direction of the gradient.

and the standard deviation is bigger. However, we can notice a distribution of the fibers of several minor directions besides the main direction. For the following figure we can see a maximum around 0 degrees but the difference between this figure and the two previous ones is that it's difficult to differentiate the main direction clearly. For the fourth image the results with this method is very bad. Now, it's like it doesn't have a main direction which we can determinate accurately, in fact the noise is so big that makes it almost impossible. For the last image we can see two maximums, one for the angles that are close to zero and another for the angles close to 120 degrees. The results are not very good since there is a lot of noise in it and the maximums in both directions aren't very big for the rest of the angles,specially for those bigger than 120 degrees.

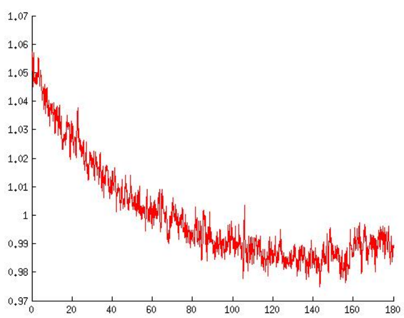


(a)

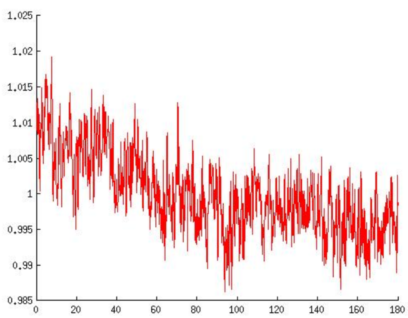


(b)

Figure 6: Results corresponding to (a) Figure 9(a), and (b) Figure 9(b).



(a)



(b)

Figure 7: Results corresponding to (a) Figure 10(b), and (b) Figure 10(c).

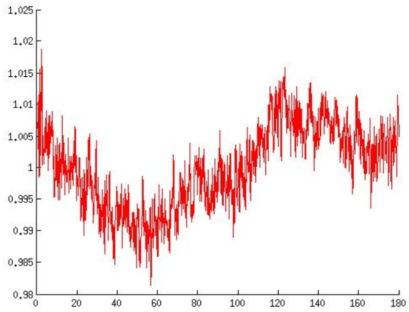


Figure 8: Results corresponding to Figure 10(a)

4 Conclusions

First, we can affirm that the structure tensor method gives a precise determination of the main orientation of the fiber of the paper sheet. However, there is no way to ensure good results when trying to catch the whole fiber distribution. Moreover, the presented modification gives slightly better results, but they can hardly be noticed.

The main weak point of the structure tensor based technique is the fact that there is no way to ensure that we can get the exact orientation distribution. Above all, we have no way to ensure if the given result may have any sense. This is due to the fact that to get the exact distribution, the user requires to fix a small window size. When doing this, you may be looking to too narrow areas, not ensuring then if quality information is being weighted or not. Hence, Figure 1(b) shows an example of how bad can be a solution with a narrow window when trying to catch the exact behaviour. However, in some cases it may properly work, as happens in 1(a). In the other presented results it is hard to determine the correction of the results, since we do not have the analytical exact orientation distribution.

In contrast, if we are not interested on the exact global fiber distribution, but just in the main one, this may seem a good method. Note that it seems to give some robust well determined main orientation of the fibers.

Second, the Fourier type method seems to usually introduce too much inner oscillations in the given solution. Despite it presents some good results, the presented oscillations reduce the chances on relying on the obtained solution. If a really smooth solution is obtained, then the method is probably in the trust area and the the solution is reliable. However, as happened with the Structure Tensor method, the veracity of the method will depend on the problem.

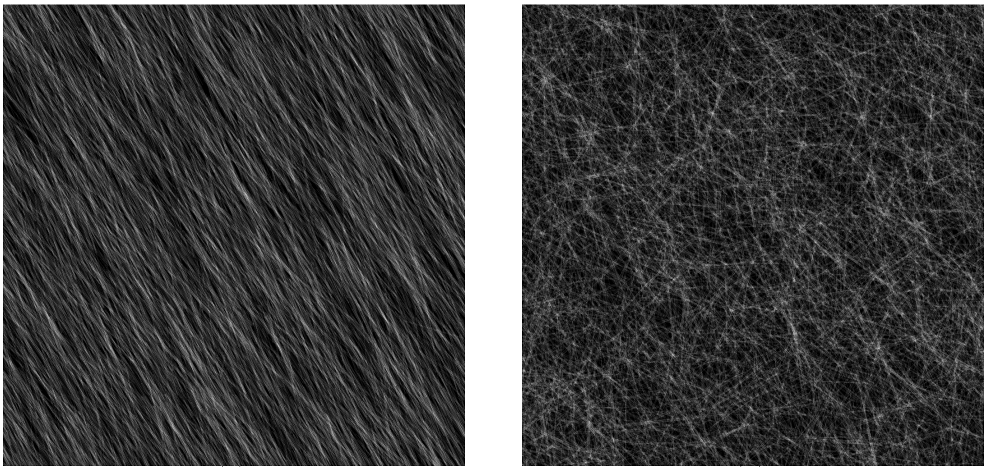
Note that in most of the presented pictures, both methods do give the same result (taking on account the 30 of difference, that are due just to output differences). However, we can also find several differences in the resulting angles, without being able to ensure the most correct able one.

Hence, despite not being able to achieve a global technique that is able to precisely determine the exact orientation distribution of the fibers, a good combination of the two presented techniques may be able to give an approximately good estimation of the orientation. The structure tensor with a wide window can precisely determine the main orientation of the fibers. Then, the Fourier technique can be compared to the result using a narrow window with the ST method and ensure if we are having a good guess of the fiber distribution behaviour.

A Imaged paper sheets and image processing

A.1 Images of fiber distribution of several paper sheets

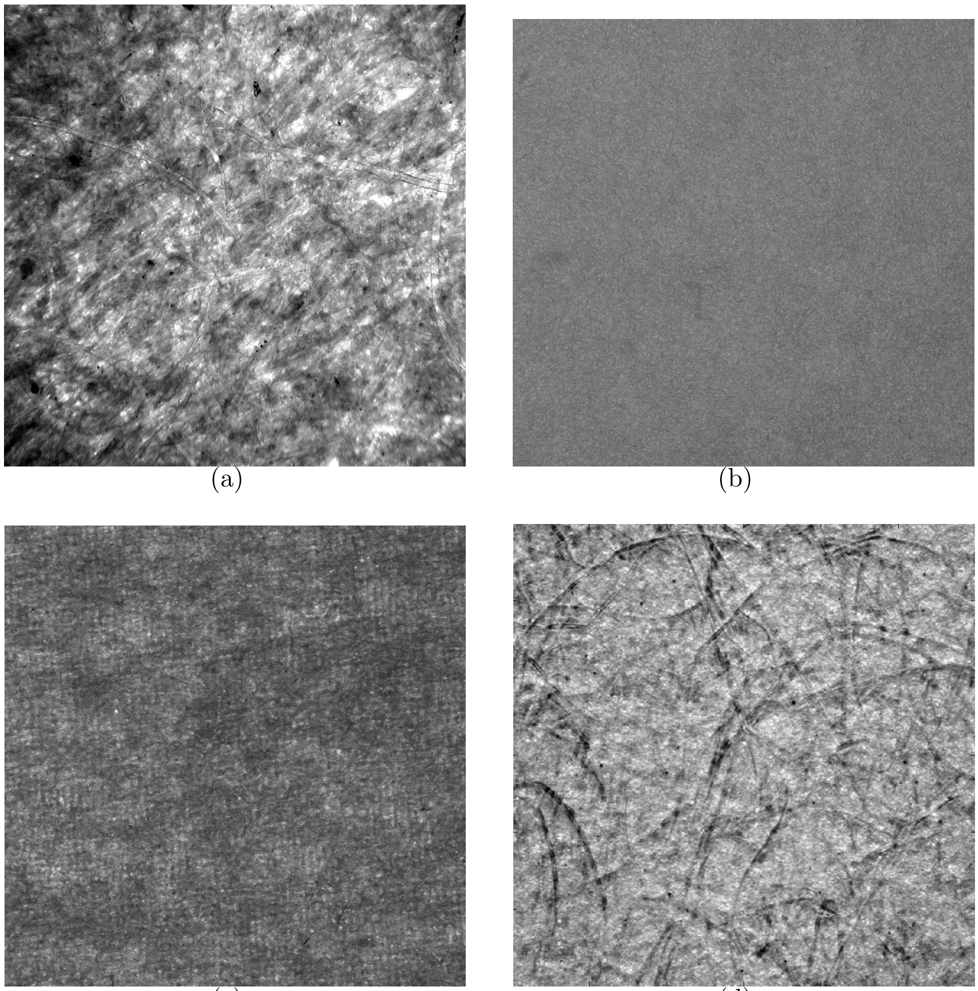
A.1.1 Fiber distributions of simulated paper sheets



(a) (b)

Figure 9: Simulated paper sheet fiber images.

A.1.2 Fiber distributions of real paper sheets



(a) (b) (c) (d)

Figure 10: Real paper sheet fiber images: (a) high-resolution microscope, (b) newspaper sheet with low resolution microscope, (c) standard printing paper with low resolution microscope, and (d) surface imaging technique of a sheet.

A.2 Image Processing

Image processing is the subject that use computational algorithms to modify and enhance digital images. In this project we are to estimate the orientation of fibers, however in the current images fibers are not easy to recognise. So we perform image preprocess to extract fibers from the noisy background before further FFT analysis.

Firstly we reduce the noise by applying either the smoothing operation or background subtraction. Then we use method of segmentation to identify our image of interests before the next step of Fast Fourier Transform . This includes common image processing procedures of filtering, segmentation (thresholding), and background subtraction.

A.2.1 Smoothing Operations

Smoothing means reducing the amount of intensity variation between one pixel and the next. It is firstly applied to reduce noise. Here both mean filter and median filter are attempted.

- Mean Filter: Mean filtering is a simple and intuitive way to implement method of smoothing images. It works by simply replacing each pixel value with the mean value of its neighbors, including itself. It is linear filter and has the effect of eliminating pixel values which are unrepresentative of their surroundings.
- Median Filter: The median filter is a nonlinear digital filtering technique, often used to remove noise. With a median filter, the value selected comes from the existing brightness value, thus no roundoff error are introduced and our new brightness values are still integers. It has the advantage of preserving edges while removing noise.

A.2.2 Rolling Ball algorithm for subtract background

Above smoothing operation works to a set of images, but does not generate good result for images that have different illumination, for example. Instead we choose subtracting background method, which provides for correction of non-uniform field defects by subtracting a flat field from an acquired image. We use the rolling ball algorithm to reduce non-uniform defects. Imagine a 3D surface with the pixel values of the image being the height, then a ball rolling over the back side of the surface creates the background.

To work out the algorithm we use the free domain program *ImageJ*. That program has develop the subtract background method based on the "rolling ball" algorithm described by Stanley Sternberg [6].

A.2.3 Thresholding

In segmenting the foreground from the background, we use the thresholding instead of edge finding. To identify bright fiber on a dark background, we use

the brightness threshold. We also use similar methods with image of dark fiber on a bright back ground. For the former case, a parameter θ called the brightness threshold is chosen and applied to the image $a[m,n]$ as follows:
If $a[m,n] \geq \theta \qquad a[m,n] = object = 1$
Else $\qquad a[m,n] = background = 0$
For the latter case, the algorithm works the other way around.

- Otsu's Method:

  In thresholding, different method uses different algorithm to decide the threshold θ. Otsu's method [5] calculates the optimum threshold by minimizing the within-class variance of those two classes, which is defined as a weighted sum of the variance of each class:

$$\sigma^2_{\omega(T)} = \omega_B(T)\sigma^2_B(T) + \omega_O(T)\sigma^2_O(T)$$

  where
  $\omega_1(T) = \sum_{i=1}^{T-1} P(i)$
  $\omega_2(T) = \sum_{i=T}^{N-1} P(i)$
  $\sigma^2_B(T)$ = the variance of the pixels in the background (below threshold)
  $\sigma^2_O(T)$ = the variance of the pixels in the foreground (above threshold)

A.2.4 Results of the processing

- Median Filter

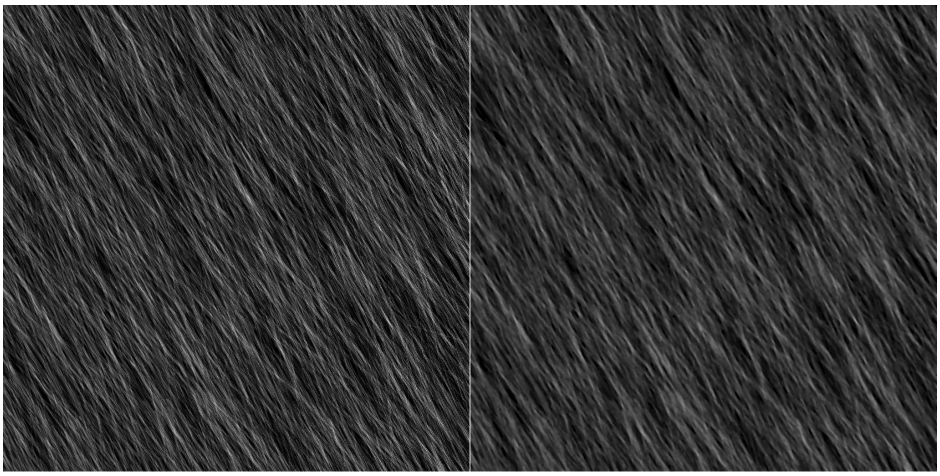


Figure 11: Sheet presented in Figure 9(a): Left, before the Median Filter; Right, after the Median Filter.

- Mean Filter

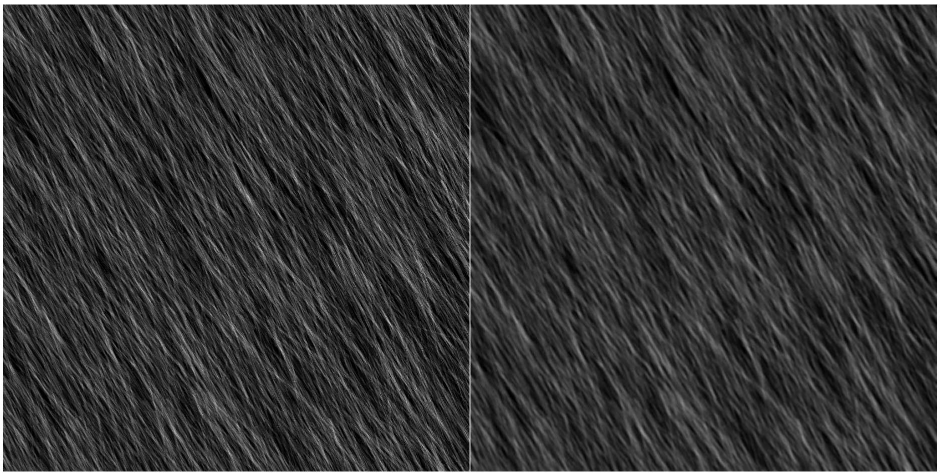


Figure 12: Sheet presented in Figure 9(a): Left, before the Mean Filter; Right, after the Mean Filter.

- Subtract Background

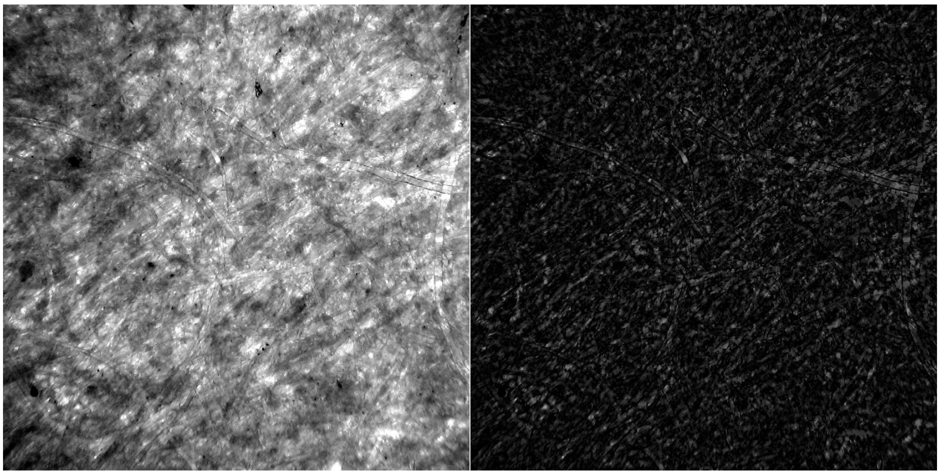


Figure 13: Sheet presented in Figure 10(a): Left, before subtracting background; Right, after subtracting background.

- Otsu threshold (and subtract background algorithm)

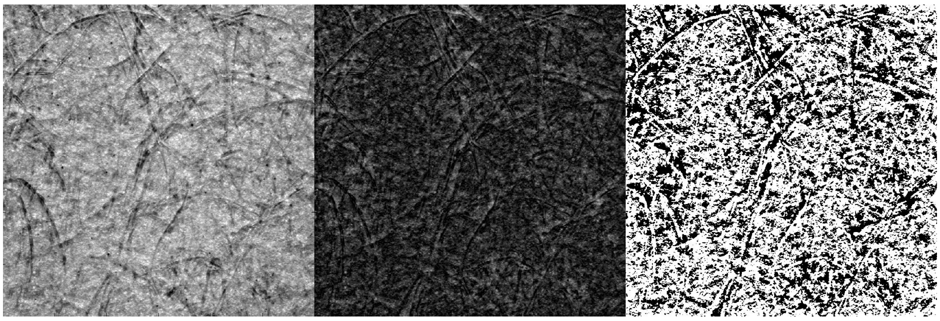


Figure 14: Sheet presented in Figure 10(a): Left: original image; Centre: after subtract background method; Right: after Otsu's threshold.

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