Modeling species transport through membrane in Haemodialysis

Haemodialysis is an external process of separating urea and creatinine from the human blood (in case of renal failure) by virtue of diffusion due to the concentration gradient across a porous membrane. The principle of dialysis involves diffusion of solutes across a semipermeable membrane. Hemodialysis utilizes counter current flow, where the dialysate is flowing in the opposite direction to blood flow in the extracorporeal circuit. Counter-current flow maintains the concentration gradient across the membrane at a maximum and increases the efficiency of the dialysis. The feed solution (blood) flows on one side of the membrane and the solvent or dialysate stream on the other side (Fig. 1a and 1e). Solute fluxes are dependent on the concentration difference across the semi-permeable membrane. The dialysate is generally a sterilized solution with sodium and chloride levels similar to those of normal plasma to prevent loss

Typically the haemodialysis cartridge consists of a bunch of follow fiber tubes (Fig. 1c) where the blood (to be purified) flows through the core and the dialysate on the outside. The cross-sectional cylindrical wall is porous where the separation essentially takes place (Fig. 1d). The design of a haemodialysis membrane cartridge has very stringent medical specifications. Some of the key requirements are: a minimal hold-up volume (less than 250 ml); the membrane area in the cartridge should be at least 1.2 m²; the membrane material should be bio- and haemo-compatible; and the pore size should be in the range of 2.0-2.9 nm. A typical picture of the haemodialysis cartridge is shown below (Fig. 1b):



Fig. (a): Schematic of the membrane cartridge configuration; (b) actual haemodialyzer cartridge; (c) front section of the cartridge, the small apertures are the inlet to the hollow-fibre membrane tubes; (d) single hollow-fibre membrane; and (e) mechanism of dialysis.

In this modelling problem we aim to quantify the mass transport of the toxins (urea and creatinine) through the membrane, to predict the time required for dialysis. This is one of the key challenges as the duration of the dialysis sessions required is unknown at the beginning of the clinical trial. The key modelling input variables, such as blood urea concentration, rheological constants and blood flow rate, can be easily measured by simple clinical tests. The modelling results will also

provide insight on the effectiveness of the process and allow for its optimization within the physical constraints to maximize the transport rate.

Some of the important answers we would be expecting at the end of the modelling exercise:

- 1. The duration and frequency of the dialysis sessions required is not determined at the beginning of the trial. Generally this is done by using some empirical relationships and observations after the first few trials. So the present modelling effort will provide estimation on the time required for a dialysis session (assuming the blood parameters are known) before the start of the session. Also, since the rate of removal of the toxins is non-linear with time, this can be helpful to understand how much toxins are going to be removed if the patient does not want to continue long enough till the end.
- 2. The design of the membrane module is key. Even though it is largely determined by the medical specifications, its configuration is not customized for individual needs, for certain category of patients because of lack of process knowledge and understanding.
- 3. There is limited knowledge of the contaminant transport mechanisms in the dialyzer unit and therefore prediction of the performance and effectiveness for an individual patient in largely unknown.

Based on the above components of the challenge, a feasible strategy can be -

- Understand the rheology of blood, which is essentially non-Newtonian. For those who are not conversant with fluid flows, it might be necessary to spend some time on understanding the background of the fluid flow equations (Navier-Stokes equation), stress-strain relationships, stress tensors, continuity, inertia, viscosity and compressibility of fluid.
- 2. This is a low Reynolds number laminar flow, so the kind of approximations that might be reasonable to obtain a steady solution for the fluid velocity and pressure profile.
- 3. Solution of the species convective diffusive equation inside the fibers with the transport condition on the porous wall. We need to think different numerical and analytical strategies for solving the steady partial differential equations. Also, approximations / assumptions (for e.g. considering the fluid to be Newtonian) can be tricky for the analytics. Also, a comparison of the analytics with numerics is helpful to confirm our results.
- 4. We may consider the dialysate to be very dilute and can ignore the concentration gradient on the outside of the fibre. Important question: is there a limiting flow rate of the dialysate to hold this assumption true ?
- 5. Estimation of the solute removal rate (as well amount of toxins removed) with time considering an initial contaminant concentration. We may now need to the transient case.
- 6. Determination of some engineering parameters which can judge the efficiency of the process and practically useful during clinical trials.

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Requirement: Experience in working with ordinary and partial differential equations (essential); fluid dynamics, numerical methods and knowledge of working with computational solvers are helpful but not essential.