Option management with discontinuities in the payoff: distribution of probability of losses

1. Introduction

Active management for derivatives including discontinuities in the payoff (barrier options, binary options…) is an open topic due to the value of its *Greeks* (especially Delta) in an environment of such points of discontinuity.

The proposal intends to draw conclusions about the probability of losses and its size for different management strategies.

2. Detailed explanation of the problem

For a given portfolio on equity assets, a trader must be capable of replicating the position from simpler assets. The objective in this case is not the speculation (betting) but the construction of a dynamic portfolio with value zero at all time.

Assuming that we are able to modify the exact composition of the portfolio almost instantly, such portfolio will be constructed and updated using sensitivities in a small environment of the studied variables: spot, dividends, volatility, etc.

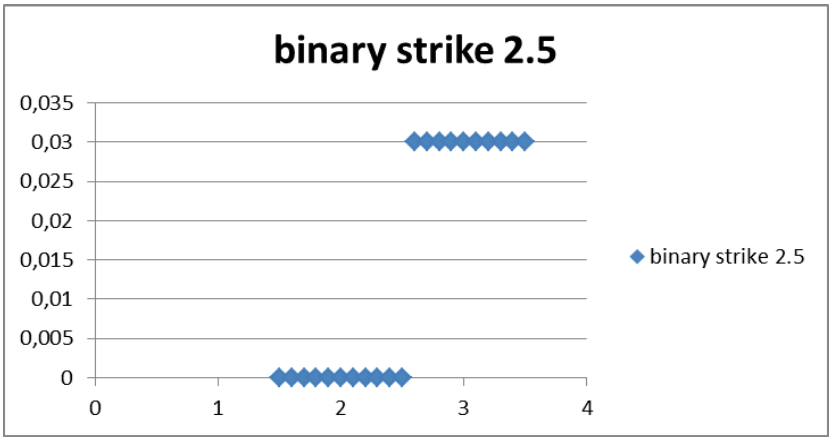
The problem in the case of barriers is that the payoff shows discontinuities and the Delta (the derivative of the present value of the option with respect to the asset price) is infinite in such points and very high near them.

During the active hedging of these positions, and when we start approaching to areas with barriers, the replication portfolio model urges us to buy or sell the asset for small movements in the value of the spot. This dynamic implies what informally has been called "buying expensive and selling cheap" (with the losses that this can cause), and formally can be explained by a great Gamma: the derivative of the Delta.

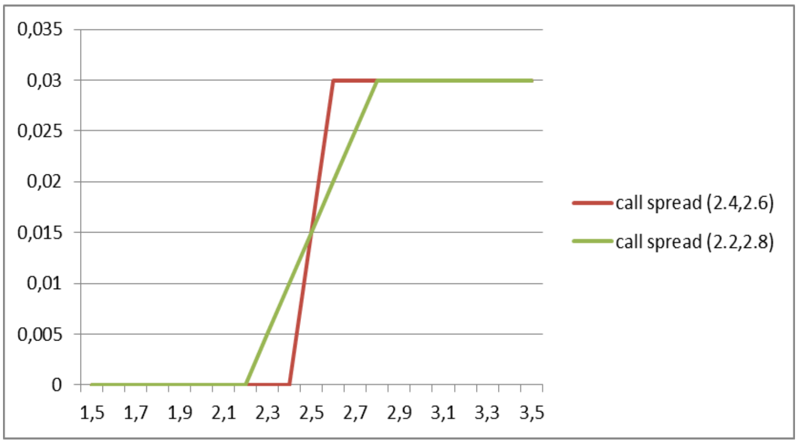
This is the reason why we propose to study the distribution of losses for different strategies of portfolio replication in these cases.

Specifically, the proposal is the use of continuous payoffs to hedge discrete or binary ones. For instance, we will be studying binary options hedging using call spread options with different slopes.

For a given binary option with strike 2.5 and coupon of 3%, this is the payoff al maturity:



We will be using call spreads with strikes fulfilling that the binary option one can be found among them. For example, (k1,k2)=(2.4;2.6) or (k1,k2)=(2.2;2.8). These options will have a payoff at maturity as follows:



3. Goal of the problem

In summary, the aim is to estimate, through simulations, the distribution of P&L (profit and losses) depending on the choice of the hedging strategy and to determine the optimal strategy, for an given risk profile.

4. Recommended skills and knowledge

- Formula and context for pricing binary options (Black-Scholes, interest rates, volatility, dividends, etc.).
- Definition, meaning and use in the hedging context of *Greeks* (Delta, Vega, Gamma, etc.).
- Simulation techniques.

5. Knowledge to be acquired

- Portfolio replication.
- Options management using sensitivities.