

# Normal bundle of general Grassmannian $G(k,n)$

**Marta Abril Bucero**

Universidad Complutense de Madrid

We will give a resolution for the conormal bundle of the general Grassmannians  $G(k,n)$ . We will first recall some concepts about Grassmannians, like Plücker embedding, Plücker coordinates, vector bundles, universal bundles,... The main tools we will use are several exact sequences, among them the universal exact sequence, the Eagon- Northcott complex and the Euler sequence in order to build a commutative diagram. Finally the snake lemma applied to the commutative diagram will produce the resolution sought.

# Aspects of higher–order differential equations

**Pablo Álvarez-Caudevilla**  
University Carlos III of Madrid

We will present recent advances in the analysis of certain degenerate fourth–order parabolic equations, the so–called *Thin Film Equations*. In the process we will show some of the motivations and discuss some open problems in the area of higher–order equations. This is a joint work with Prof. Victor. A. Galaktionov.

# An introduction to topological groups

Lydia Außenhofer  
Universität Passau

In this talk an introduction to the theory of topological groups will be given. We start with the definition of a topological group and several relevant examples.

Since topological vector spaces form an important class of abelian topological groups, some analogies and differences to abelian groups in general will be pointed out. Then a short introduction to the duality theory of abelian topological group will be given and several open problems will be mentioned.

## References

- [1] Arhangel'skii, A., Tkachenko, M., *Topological groups and related structures*, 1. Atlantis Press, Paris, 2008. ISBN: 978-90-78677-06-2
- [2] Dikranjan, D.N., Prodanov, I.R., Stoyanov, L.N., *Topological groups. Characters, dualities and minimal group topologies*, Monographs and Textbooks in Pure and Applied Mathematics, 130. Marcel Dekker, New York, 1990. ISBN: 0-8247-8047-7
- [3] Markley, N.G., *Topological Groups*, Wiley, 2010. ISBN: 978-0-470-62451-7

# On a class of orthogonal polynomials with respect to a Jacobi operator

**Jorge Alberto Borrego Morell**  
Universidad Carlos III de Madrid

Let  $\mu$  be a finite positive Borel measure and define the operator  $\mathcal{L}[f] = (1 - x^2)f'' + (\beta - \alpha - (\alpha + \beta + 2)x)f'$  with  $\alpha, \beta > -1$ . We study algebraic, analytic and asymptotic properties of the sequence of monic polynomials  $\{Q_n\}$  that satisfy the orthogonality relations

$$\int_{-1}^1 \mathcal{L}[Q_n](x)x^k d\mu(x) = 0 \quad \text{for all } 0 \leq k \leq n - 1$$

A fluid dynamics model with newtonian and logarithmic potentials for source points location of a flow of an incompressible fluid with preassigned stagnation points is also considered.

## References

- [1] Orthogonality with respect to a Jacobi differential operator and applications. Submitted

# Entanglement, fractional magnetization and long range interactions

**Andrea Cadarso Rebolledo**

Universidad Complutense de Madrid

Matrix Product States enclose many of the physical properties of quantum spin chains. Their importance arises from the fact that, by means of a single tensor, it is possible to describe relevant states of  $N$  spins, which, in principle, require an exponential number of parameters when written in a basis in the corresponding Hilbert space  $\mathcal{H}^{\otimes N}$ . In this talk, we will briefly introduce this concept and use it to give an analytical proof of the fact that a large fractionalization in the magnetization implies large entanglement in a quantum state.

## References

- [1] D. Pérez-García, F. Verstraete, M.M. Wolf, J.I. Cirac, Matrix Product States Representations, *Quantum Inf. Comput.* 7, 401 (2007)

# Noncommutative $L_p$ spaces and Calderón-Zygmund theory

**José Manuel Conde Alonso**  
Universidad Autónoma de Madrid

Noncommutative  $L_p$  spaces have been a subject of study since the early 50's following the works of Dixmier [1] and Segal [2]. In this talk, we will present a basic introduction to noncommutative integration and measure theory including some important examples. We will also outline one of the possible research directions in the area, commenting on some well known results and open problems we are interested in.

## References

- [1] J. Dixmier, Formes linéaires sur un anneau d'opérateurs, Bull. Soc. Math. France 81 (1953), 9-39.
- [2] I. Segal, A non-commutative extension of abstract integration, Ann. Math. 57 (1953), 401-457.

# Vascular network modeling and simulation

Juan Ramón Duque Rodríguez  
Universidad Complutense de Madrid

The diagnosis and treatment of neurovascular diseases can be greatly improved with the assistance of customized computer applications allowing the reconstruction, visualization, interrogation and simulation of a computational model of the patients vascular structures.

The aim of this work is to present a model of the vascular tree, along with algorithms for its construction, interrogation and hemodynamic simulations. The model is based on the construction of a symbolic model of the vascular tree obtained from vessel images and on the use of the 1D mathematical model for simulating the blood flow in vessels. This last model forms a system of hyperbolic balance laws where the basic equations are obtained from the principles of conservations of mass and momentum.

The model allows an automatic identification and labelling of vessel bifurcation and malformations and is aimed at fulfilling all the requirements of a computer-assisted neurovascular system.

Finally, the effect of partial and total occlusions and other parametric variations of the constitutive vessels of a network on the cerebral flows is also studied numerically and presented along with some numerical simulation of the symbolic model.

## References

- [1] ALFIO QUARTERONI AND LUCA FORMAGGIA. Mathematical modelling and numerical simulation of the cardiovascular system. *Modelling of Living Systems*, Handbook of Numerical Analysis Series, Elsevier, 2002.
- [2] DANIEL LAMPONI. One dimensional and multiscale models for blood flow circulation. *These n. 3006* 2004.

- [3] Anna Puig Puig et al: *An interactive cerebral blood vessel exploration system*. 8th IEEE Conference on Visualization (1997) 443-446.

# A mathematical model of the cell cycle in T lymphocytes. Deciding when to die and when to divide.

Clemente Fernández Arias

Universidad Complutense de Madrid

T cells play a key role in the immune response against pathogenic agents. In the presence of infection signals, T cells undergo dramatic proliferation and differentiate into effector T cells that mediate the removal of the pathogen. They migrate to the site of infection, and once there they actively kill infected host cells and produce protein molecules called cytokines that recruit and activate other cells of the immune system. After the clearance of the pathogen most of the effector cells die but a small number of the responding cells survive as memory T cells.

In spite of the amount of available empirical data about the facts that take place during the immune response, the mechanisms that enable several billions of T cells to display such a complex behavior are currently unknown.

In order to get a satisfactory explanation of the T cell response, two different scales must be simultaneously taken into account. The function of the T lymphocyte system is comprehensible only when considered from a population point of view. Nevertheless, the processes that ultimately lead to the observed population dynamics, i.e. cell division or cell death, take place at a cellular level.

A body of empirical evidence suggests that T cell fate is decided in the early stages of the cell cycle and is determined by the local cytokine environment surrounding the cell. In this talk we will briefly review the biological mechanisms underlying the link between cytokines, cell division and programmed cell death. We will also introduce a mathematical model that integrates these mechanisms and gives a simple answer to the question: How does a T cell know when to divide and when to die?

## References

- [1] Cohen, Irun R. and Segel, Lee A. (Eds.), *Design Principles for the Immune System and Other Distributed Autonomous Systems (Santa Fe Institute Studies in the Sciences of Complexity Proceedings)*. Oxford University Press, USA,. 2001.
- [2] Louzoun, Y. (2007). The evolution of mathematical immunology. *Immunological Reviews*, 216(1), 9-20.
- [3] Molina-París, Carmen; Lythe, Grant (Eds.), *Mathematical Models and Immune Cell Biology*. Springer-Verlag New York. 2011.

# Analytical Sampling and Lagrange-Type Interpolation Series

**Paulo E. Fernández Moncada**  
Universidad Carlos III de Madrid

The classical Kramer sampling theorem provides a method for obtaining orthogonal sampling formulas. This theorem can also be formulated in a more general nonorthogonal setting involving analytic Kramer kernels which are valued in Hilbert spaces. We present here a new problem: To characterize the situations when the nonorthogonal sampling formulas can be expressed as Lagrange-type interpolation series. A necessary and sufficient condition is given in terms of the so-called zero-removing property. We exhibit some examples illustrating the above theory.

## References

- [1] P. Fernández Moncada, A.G. García and M. A. Hernández-Medina, *The zero-removing property and Lagrange-type interpolation series*. Numer. Funct. Anal. Optim., 2011. doi:10.1080/01630563.2011.587076

# Motivic Invariants of Hypersurface Singularities

**Manuel González Villa**

Ruprecht Karls Universität Heidelberg

Let  $H \in \mathbb{C}^{n+1}$  be an hypersurface (or a germ of hypersurface) defined by  $f \in \mathbb{C}[x_1, \dots, x_{n+1}]$  (or  $f \in \mathbb{C}\{x_1, \dots, x_{n+1}\}$ ). Understanding the singularities of  $H$  at a given point is a natural and classical problem. Two major milestones in the study of the problem are the fibration theorem by Milnor and the existence of resolution of singularities by Hironaka. In the talk we adopt an alternative approach due to Denef and Loeser and based in the theory of motivic integration introduced by Kontsevich. This approach allows the introduction of new depth algebraic invariants that allow to recover topological information of the singularity. We present some applications of the motivic approach to the quasi-ordinary case.

# Numerical approach to the heat equation with nonlinear flow at the border

**Gustavo Ito**

Universidad Complutense de Madrid

We study an approximation of the problem on heat equation. Specifically, we approximate by finite differences  $u_{xx}$  obtaining a system of ordinary differential equations. After proving a convergence theorem that ensures that the approach taken is "adequate", describe in terms of the parameter  $p$  when the solution of the system explodes in finite time. In this case, we will study the behavior of approximate solution near the time of explosion, comparing that behavior with the original problem.

# Introduction to game theory

**Anna Khmelnitskaya**

Saint-Petersburg State University

The game theory is a theory of rational behavior of agents (in particular, people) with nonidentical interests. It can be defined as the theory of mathematical models of conflict and cooperation between intelligent rational decision makers. While the noncooperative game theory concentrates on the study the strategic behavior of individual players, the cooperative game theory studies cooperative aspects of collective behavior. Our main purpose is to outline the central basic ideas of noncooperative and cooperative game theory. We discuss the Nash equilibrium for games in normal form (strategic-form games) and different solution concepts for cooperative games such as the core, the Shapley value, the nucleolus. Different models of cooperative games with limited cooperation introduces by means of a priori given coalition and communication structures will be also considered.

# Mathematical Modeling of Human Communication

**Giovanna Miritello**

Universidad Carlos III de Madrid

We investigate the temporal patterns of human communication and its influence on the spreading of information in social networks. The analysis of mobile phone calls of 20 million people in one country over a period of one year shows that human communication is bursty and happens in group conversations. These features have opposite effects in information reach: while bursts hinder propagation at large scales, conversations favor local rapid cascades. We show how the observed behavior can be modeled and explained by means of mathematical tools.

## References

- [1] G. Miritello, E. Moro, and R. Lara, Dynamical strength of social ties in information spreading. *Phys. Rev. E* (2011) vol. 83 (4) pp. 045102

# Operator Algebras and the Renormalization Group in Quantum Field Theory

**Gerardo Morsella**

Università di Roma Tor Vergata

The Renormalization Group is a key tool in the analysis of the short distance behaviour of Quantum Field Theory. A version of the Renormalization Group adapted to the operator algebraic approach to Quantum Field Theory has been formulated in the '90s by Buchholz and Verch, and since then it has provided several interesting structural results, such as a general understanding of the scaling limit of superselection charges and an intrinsic notion of charge confinement, or as hints to the appearance of noncommutative geometric invariants in Quantum Field Theory.

# Analyticity in the Calderón problem with partial data

**Pedro P. Caro**

University of Helsinki

In this talk we present a result related to the stability of an inverse boundary value problem in electrostatics using partial data. The problem we consider consists in recovering the conductivity of a medium using non-invasive measurements, namely, current and voltage measurements. This problem was proposed by Alberto Calderón in 1980 [2]. The Calderón problem is the theoretical base of the so-called electrical impedance tomography. Other possible applications of this imaging method include medical imaging, geophysical prospection and non-destructive testing of mechanical parts. In real applications as in geophysical prospection, it might be expensive or even impossible to get full data. For this reason, it is important to deal with the same problem only using partial data. In this direction, there are two interesting works [1] and [3]. In these papers, the concepts of analyticity and analytic wave front set play an important role. Nevertheless, the use of them in the previous articles is just qualitative, which is very inconvenient in application. In a joint work with Dos Santos Ferreira and Ruiz, we get a stable procedure using the Radon and Segal-Bargmann transforms.

## References

- [1] A. Bukhgeim and G. Uhlmann, *Recovering a potential from partial Cauchy data*, Comm. Partial Differential Equations, **27** (2002), 653–668.
- [2] A. P. Calderón, *On an inverse boundary value problem*, Seminar on Numerical Analysis and its Applications to Continuum Physics, Rio de Janeiro, Sociedade Brasileira de Matematica, (1980), 65–73.

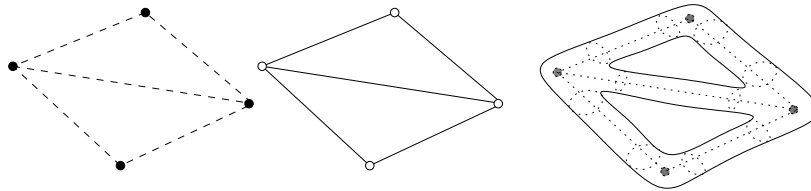
- [3] C. E. Kenig, J. Sjöstrand and G. Uhlmann, *The Calderón problem with partial data*, Ann. of Math., **165** (2007), 567–591.

# Spectral analysis on graph-like spaces

Olaf Post

Humboldt University at Berlin

A graph-like space is a space which reflects the structure of a combinatorial graph as for example a combinatorial graph itself, a topological graph, or the surface of a tubular neighbourhood of a graph embedded in  $\mathbb{R}^3$ .



All these examples carry a natural metric and one can define a Laplace operator on each of these examples.

In the talk, we are going to give an overview of this very active research area, presenting e.g. the concept of quantum graphs, graph-like manifolds and the relation of the corresponding Laplacians. Quantum graphs are often considered to lie inbetween a combinatorial graph and a manifold: they have a rich topological structure, but are still accessible to computations, and therefore serve in many aspects as test objects.

We are going to analyse for example the relation of a Laplacian on a quantum graph with a corresponding discrete Laplacian, as well as approximation results for Laplacians on graph-like manifolds to quantum graph Laplacians. Colin de Verdière used such convergence results e.g. in spectral geometry.

# A way of measuring the complexity of subsets of $\mathbb{R}^3$

**J. J. Sánchez–Gabites**

Universidad de Castilla la Mancha

In this talk we prove that certain arcs and balls in  $\mathbb{R}^3$  cannot be (local) attractors for discrete dynamical systems. For any compact set  $K \subseteq \mathbb{R}^3$  we define a number  $r(K)$  that, roughly speaking, measures the complexity of the embedding of the set in  $\mathbb{R}^3$ . We argue that attractors have finite  $r(K)$  and then exhibit an arc and a ball with infinite  $r(K)$ , so that they cannot be attractors.

# Extreme Interpolation Methods

Alba Segurado López

Universidad Complutense de Madrid

The real interpolation method  $(A_0, A_1)_{\theta, q}$  plays an important role in the applications of Interpolation Theory in Functional Analysis, Harmonic Analysis, Approximation Theory and other areas of Mathematics (see [2], [7], [1]). The definition of  $(A_0, A_1)_{\theta, q}$  requires  $0 < \theta < 1$ ; however, certain problems regarding function spaces have led to the investigation of extreme interpolation spaces, where  $\theta = 0$  or  $\theta = 1$ . See, for instance, [5], [6], [3], [4]. Then it is natural to assume that the couple of Banach spaces is ordered, that is,  $A_0 \hookrightarrow A_1$ . If  $\theta = 0$ , these methods yield spaces that are quite close to  $A_0$ , and when  $\theta = 1$ , these spaces are very close to  $A_1$ . The construction of limiting interpolation spaces and a few of their most important properties will be reviewed in this talk. We will also show how they arise naturally by interpolating over the unit square. The connection between limiting spaces and Extrapolation Theory will be reviewed as well.

## References

- [1] C. Bennett and R. C. Sharpley. *Interpolation of Operators*. Academic Press, London, 1988.
- [2] J. Bergh and J. Löfström. *Interpolation Spaces, An Introduction*. Springer-Verlag, Beijing, 2003.
- [3] F. Cobos, L. M. Fernández-Cabrera, T. Kühn, and T. Ullrich. On an extreme class of real interpolation spaces. *J. Functional Anal.*, (256):2321–2366, 2009.
- [4] F. Cobos, L. M. Fernández-Cabrera, and M. Mastyló. Abstract limit  $J$ -spaces. *J. London Math. Soc.*, (82):501–525, 2010.

- [5] M. E. Gómez and M. Milman. Extrapolation spaces and almost-everywhere convergence of singular integrals. *J. Lond. Math. Soc.*, (34):305–316, 1986.
- [6] M. Milman. *Extrapolation and Optimal Decompositions*. Springer Lecture Notes in Math., vol. 1580, Berlin, 1994.
- [7] H. Triebel. *Interpolation Theory, Function Spaces, Differential Operators*. North Holland, Amsterdam, 1978.

# The fractional Laplacian in the Signorini problem: a semigroup approach to the problem

Pablo Raúl Stinga  
Universidad de La Rioja, Spain

The Signorini problem, that arises in elasticity, was posed by the Italian mathematician Antonio Signorini in 1959. It consists in finding the configuration of a membrane that is in equilibrium and is above some *thin* obstacle. For that reason it is also known as the *thin obstacle problem*.

In this talk we will explain the formulation of the problem. Taking a new point of view, the semigroup language, we will see how it can be derived in a very simple way as a free boundary problem for the fractional Laplacian. In fact, different physical considerations give rise to different fractional operators. Then some mathematical tools, still based on the semigroup language, will be shown.

# Knots with finite integral Menger curvature

Marta Szumanska  
University of Warsaw

Integral Menger Curvature  $\mathcal{M}_p(\gamma)$  of a rectifiable curve  $\gamma$  is defined by the formula below

$$\mathcal{M}_p(\gamma) = \int_{\gamma} \int_{\gamma} \int_{\gamma} R^{-p}(x, y, z) d\mathcal{H}_x^1 d\mathcal{H}_y^1 d\mathcal{H}_z^1,$$

where  $R(x, y, z)$  is the smallest circle passing through points  $x, y$  and  $z$  and  $\mathcal{H}^1$  denotes 1-Hausdorff measure.

It is easy to believe (but not so easy to prove), that for sufficiently large  $p$  curves with finite  $\mathcal{M}_p$  are injective (near self-intersection there are many small circles passing through three different points of the curve, thus the integral blows up). Indeed - for  $p > 3$  closed curves with  $\mathcal{M}_p < \infty$  are homeomorphic to a circle (moreover finiteness of the integral yields higher regularity of the curve - see [1]). Therefore curves with finite energy can be considered as knots. Some properties of such knots will be shown (among others estimation of the average crossing number will be given).

The talk will be self contained. Only basic knowledge of analysis and topology will be assumed.

The talk will be partially based on joint results with P. Strzelecki and H. von der Mosel.

## References

- [1] P. Strzelecki, M. Szumanska, H. von der Mosel, *Regularizing and self-avoidance effects of integral Menger curvature*, Annali della Scuola Norm.Sup. di Pisa 9, no.1 (2010),145-187

# Multiplicity of positive large solutions in a superlinear indefinite problem

**Andrea Tellini**

Complutense University of Madrid

In this talk I will present a recent multiplicity result, obtained in collaboration with Julián López-Gómez and Fabio Zanolin, regarding the positive solutions of the superlinear indefinite problem

$$\begin{cases} -u'' = \lambda u + a(t)u^p & \text{in } (0, 1) \\ u(0) = u(1) = M \end{cases} \quad (1)$$

where  $\lambda < 0$ ,  $p > 1$ ,  $a(t)$  is a symmetric piecewise function that changes sign in  $(0, 1)$  and  $M > 0$  is sufficiently large or even  $M = +\infty$ . The main result says, roughly speaking, that equation (1) has an arbitrarily large number of solutions provided that  $\lambda$  is sufficiently negative.

Moreover I will present some global bifurcation diagrams obtained by varying the parameter that measures the superlinear effects of  $a(t)$ .

**Acknowledgements** For this work I have been supported by the Spanish Ministry of Science and Innovation through grant BES-2010-039030 associated to the project MTM2009-08259.

# Von Neumann algebras, countable groups and ergodic theory

Stefaan Vaes  
K.U.Leuven

The subject of this talk is at the crossroads of functional analysis, ergodic theory and group theory. Using a construction by Murray and von Neumann (1943), countable groups and their ergodic actions on measure spaces give rise to algebras of operators on a Hilbert space, called von Neumann algebras. My aim is to explain the subtle relation between a group action and its von Neumann algebra. Over the last 10 years, Sorin Popa's deformation/rigidity lead to deep classification results, up to superrigidity theorems where the von Neumann algebra entirely determines the group and its action.

## References

- [Ga10] D. Gaboriau, Orbit equivalence and measured group theory. In *Proceedings of the International Congress of Mathematicians (Hyderabad, India, 2010)*, Vol. III, Hindustan Book Agency, 2010, pp. 1501-1527.
- [Io10] A. Ioana,  $W^*$ -superrigidity for Bernoulli actions of property (T) groups. To appear in *J. Amer. Math. Soc.* [arXiv:1002.4595](#)
- [IPV10] A. Ioana, S. Popa and S. Vaes, A class of superrigid group von Neumann algebras. *Preprint.* [arXiv:1007.1412](#)
- [OP07] N. Ozawa and S. Popa, On a class of  $II_1$  factors with at most one Cartan subalgebra. *Ann. Math.* **172** (2010), 713-749.
- [Po01] S. Popa, On a class of type  $II_1$  factors with Betti numbers invariants. *Ann. of Math.* **163** (2006), 809-899.

- [Po03] S. Popa, Strong rigidity of  $\text{II}_1$  factors arising from malleable actions of  $w$ -rigid groups, I. *Invent. Math.* **165** (2006), 369-408.
- [Po04] S. Popa, Strong rigidity of  $\text{II}_1$  factors arising from malleable actions of  $w$ -rigid groups, II. *Invent. Math.* **165** (2006), 409-452.
- [Po06] S. Popa, Deformation and rigidity for group actions and von Neumann algebras. In *Proceedings of the International Congress of Mathematicians (Madrid, 2006)*, Vol. I, European Mathematical Society Publishing House, 2007, p. 445-477.
- [PV08] S. Popa and S. Vaes, Actions of  $\mathbb{F}_\infty$  whose  $\text{II}_1$  factors and orbit equivalence relations have prescribed fundamental group. *J. Amer. Math. Soc.* **23** (2010), 383-403.
- [PV09] S. Popa and S. Vaes, Group measure space decomposition of  $\text{II}_1$  factors and  $W^*$ -superrigidity. *Invent. Math.* **182** (2010), 371-417.
- [Va10a] S. Vaes, Rigidity for von Neumann algebras and their invariants. In *Proceedings of the International Congress of Mathematicians (Hyderabad, 2010)*, Vol. III, Hindustan Book Agency, 2010, p. 1624-1650.

# Entropy on LCA groups

Simone Virili

Universitat Autònoma de Barcelona

Let  $G$  be a locally compact Abelian (LCA) group,  $\mu$  a fixed Haar measure on  $G$ , and  $\phi : G \rightarrow G$  a continuous endomorphism. The entropy of  $\phi$  can be interpreted as a measure of “expansiveness” of  $\phi$ . In particular, for every compact neighborhood  $U$  of 0 in  $G$ :

- (1) *the topological entropy* of  $\phi$  (see [2]) measures the growth of the “cotrajectories”  $\mu(U \cap \phi^{-1}U \cap \cdots \cap \phi^{-n}U)$  with  $n$  varying in  $\mathbb{N}$ ;
- (2) *the algebraic entropy “à la Peters”* of  $\phi$  (see [10]) measures the growth of the “trajectories”  $\mu(U + \phi^{-1}U + \cdots + \phi^{-n}U)$ , with  $n$  varying in  $\mathbb{N}$ .

The intention of Peters in his papers [9]–[10] is to show that his notion of algebraic entropy is “dual” to the topological entropy.

I’ll describe a new invariant called *algebraic entropy* (defined in [3]–[12]) measuring the growth of the trajectories of the form  $\mu(U + \phi U + \cdots + \phi^n U)$  with  $n$  varying in  $\mathbb{N}$ .

Some of the basic properties of this entropy will be discussed and in particular I’ll give precise formulae to compute the algebraic entropy of the endomorphisms of  $\mathbb{Z}^n$ ,  $\mathbb{R}^n$ , and  $\mathbb{C}^n$  (proved in [12]–[7]). We will see with an easy example that Peters’s entropy is not “dual” to Bowen’s entropy for endomorphisms of  $\mathbb{R}$ , while the algebraic entropy has this property.

If time allows, in the second part of the talk I will try to specialize to morphisms of *discrete* Abelian groups, trying to give an overview of the main results of [3]–[4]–[5]. In this more restricted context, the algebraic entropy is very well understood, in particular we are able to characterize it as the unique function satisfying four very natural axioms (a similar characterization is given in [11] for Bowen’s entropy on compact groups).

## References

- [1] R.L. Adler, A. G. Konheim, and M. H. McAndrew, *Topological entropy*, Trans. Amer. Math. 114 (1965), 309–319.
- [2] R. Bowen, *Entropy for group endomorphisms and homogeneous spaces*, Trans. Amer. Math. Soc. **153** (1971), 401–414.
- [3] D. Dikranjan, A. Giordano Bruno, *Entropy on Abelian groups*, preprint, arXiv:1007.0533.
- [4] D. Dikranjan, A. Giordano Bruno, *The Pinsker subgroup of an algebraic flow*, submitted. <http://arxiv.org/abs/1006.5120>.
- [5] D. Dikranjan, K. Gong and P. Zanardo, *Endomorphisms of Abelian groups with small algebraic entropy*, submitted.
- [6] D. Dikranjan, M. Sanchis and S. Virili, *New and old facts about entropy in uniform spaces and topological groups*, submitted.
- [7] A. Giordano Bruno, S. Virili, *Algebraic Yuzvinski Formula*, preprint.
- [8] D. A. Lind and T. Ward, *Automorphisms of solenoids and  $p$ -adic entropy*, Ergod. Th. & Dynam. Sys. **8** (1988), 411–419.
- [9] J. Peters, *Entropy on discrete Abelian groups*, Adv. Math. **33** (1979), 1–13.
- [10] J. Peters, *Entropy of automorphisms on L.C.A. groups*, Pacific J. Math. **96** (1981), no. 2, 475–488.
- [11] L. N. Stojanov, *Uniqueness of topological entropy for endomorphisms on compact groups*, Boll. Un. Mat. Ital. B (7) **1** (1987), no. 3, 829–847.
- [12] S. Virili, *Entropy for endomorphisms of LCA groups*, to appear.
- [13] S. Yuzvinski, *Calculation of the entropy of a group-endomorphism*, Sibirsk. Mat. Ž. **8** (1967), 230–239.
- [14] S. Yuzvinski, *Metric properties of endomorphisms of compact groups*, Izv. Acad. Nauk SSSR, Ser. Mat. **29** (1965), 1295–1328 (in Russian); Engl. Transl.: Amer. Math. Soc. Transl. (2) **66** (1968), 63–98.

# Boundedness and Topology

**Tom Vroegrijk**

University of Antwerp

Many notions of boundedness exist in topology: relative compactness and pseudocompactness in topological spaces, total and Bourbaki boundedness in uniform spaces, equicontinuity in function spaces ... In recent years the concept of a topological space endowed with a collection of bounded sets, called a bornological universe, has been studied extensively. This has led to numerous interesting results in function space and hyperspace topologies, variational analysis and completeness theory. In this talk we will give a brief survey of the existing theory on bornological universes.