Entanglement, fractional magnetization and long range interactions

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Workshop of Young
Researchers in Mathematics (UCM)
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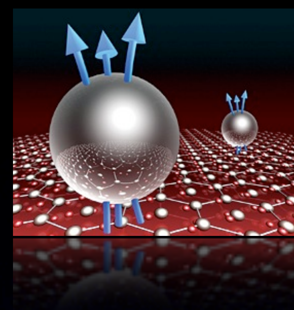
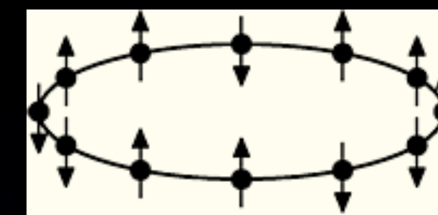
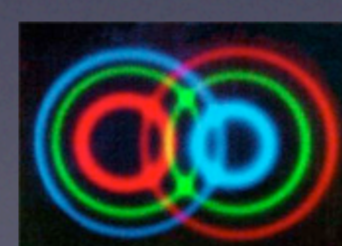


Fractional magnetization

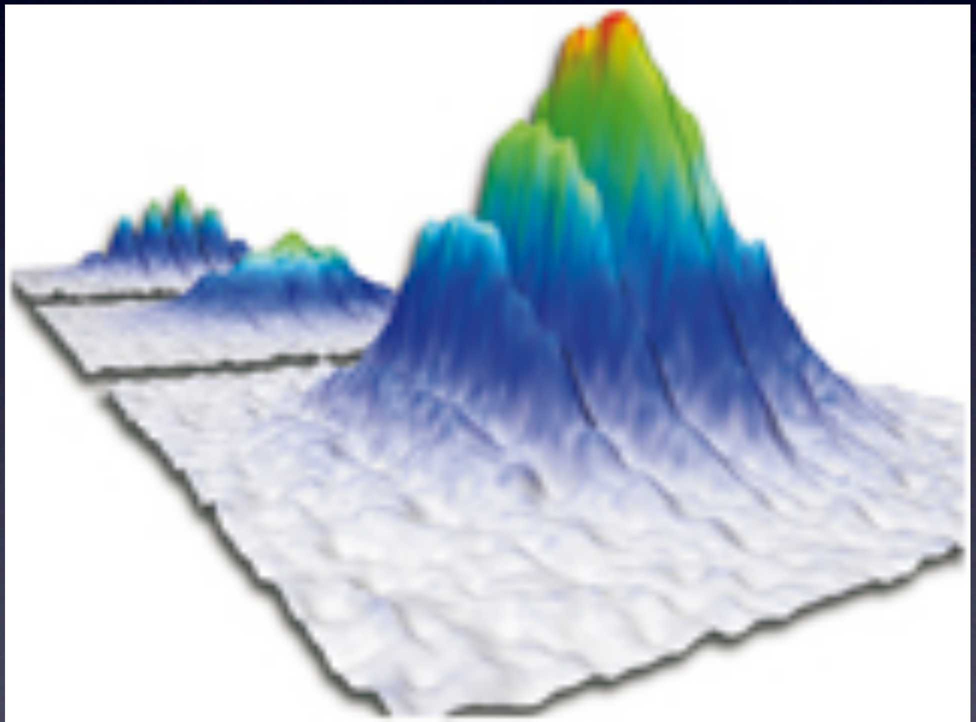


Locality

Entanglement



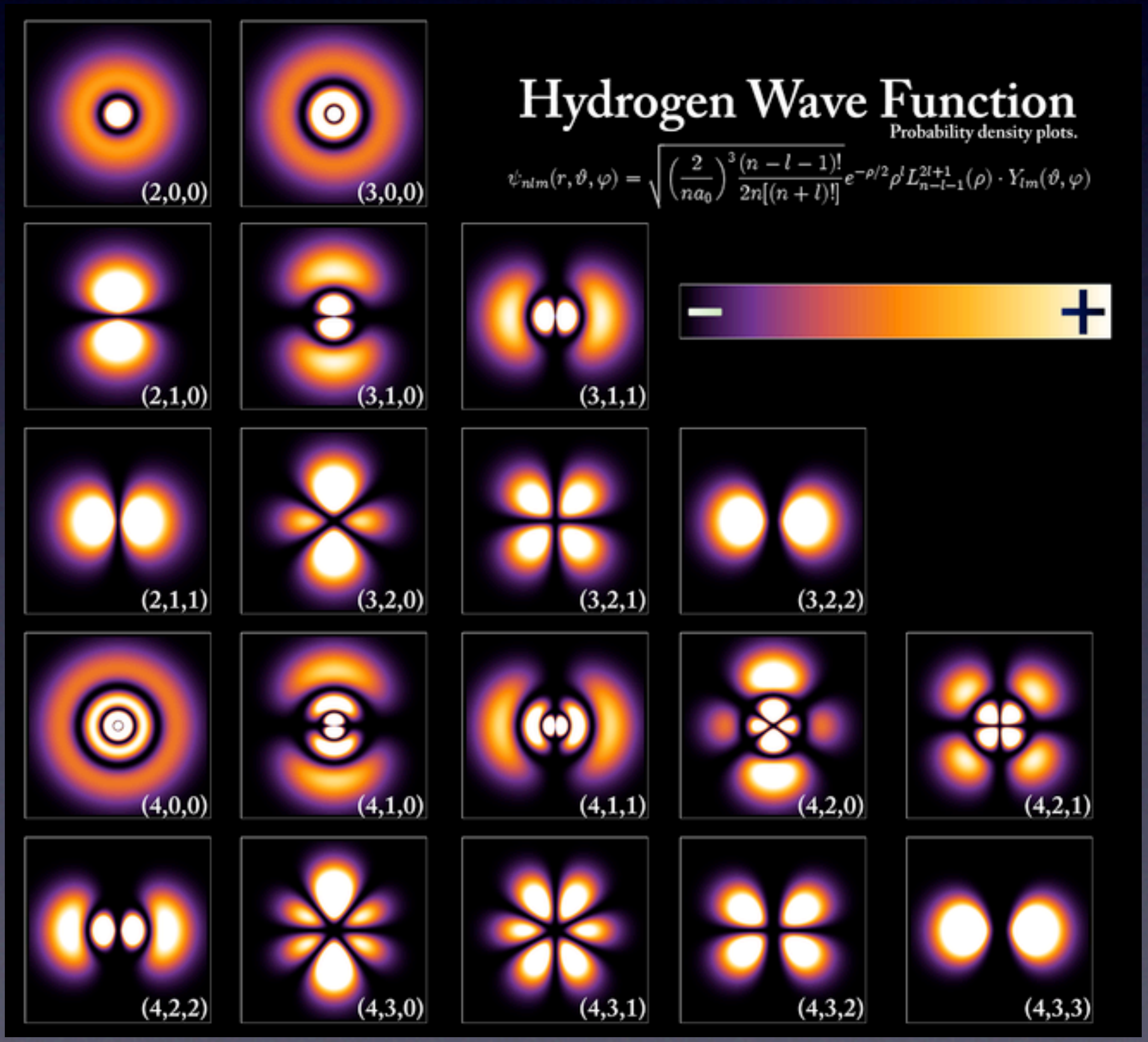
Initial considerations



- What is a quantum many-body system?
- Why are these problems relevant?

Useful background

- What is a quantum state?



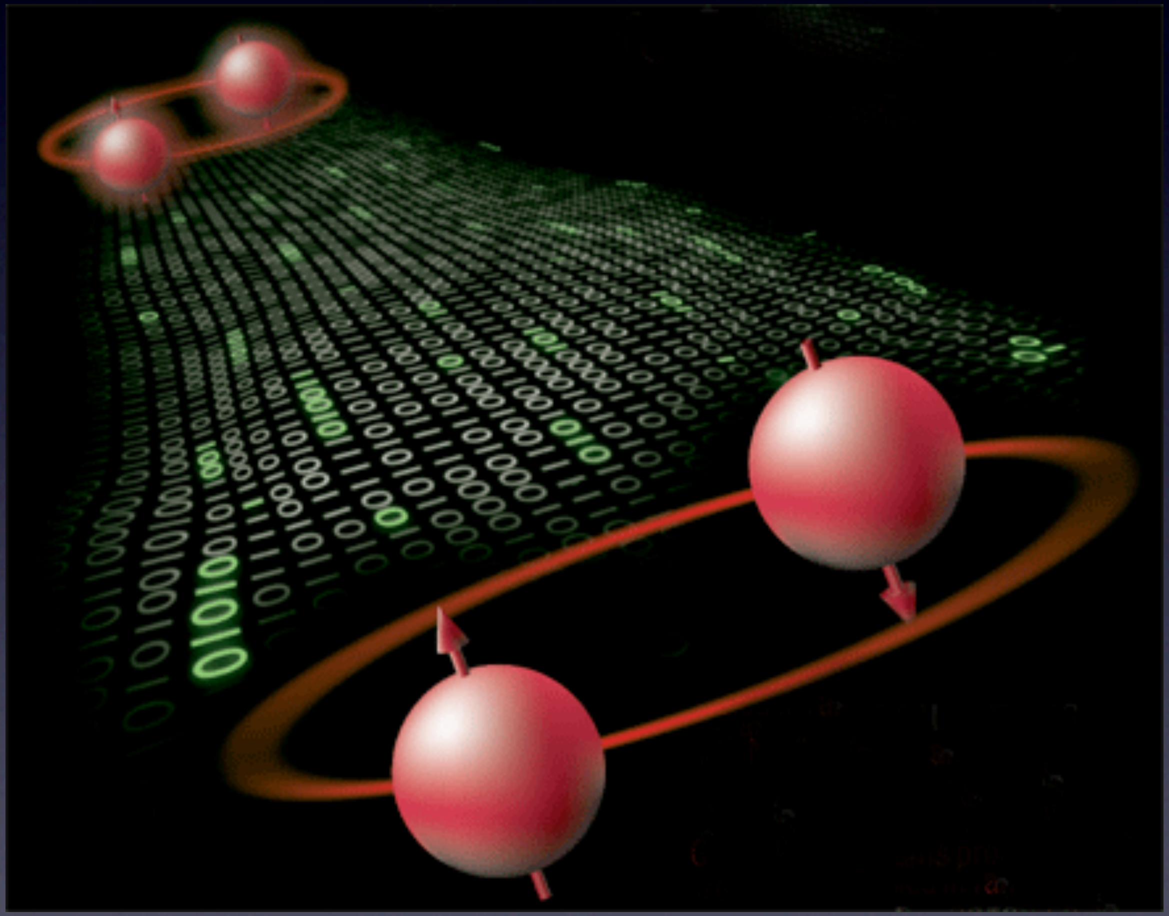
- How can we work with quantum states analytically?

MPS $$|\psi_A\rangle = \sum_{i_1,\ldots,i_L} \mathrm{tr}[A_{i_1} \ldots A_{i_L}]|i_1 \ldots i_L\rangle$$

Injective MPS

- Canonical conditions
- $\Gamma(X) = \sum_{i_1,\ldots,i_L} \mathrm{tr}[X A_{i_1} \ldots A_{i_L}]|i_1 \ldots i_L\rangle$

Entanglement



Measure of complexity in a quantum system

- How can we work with quantum states analytically?

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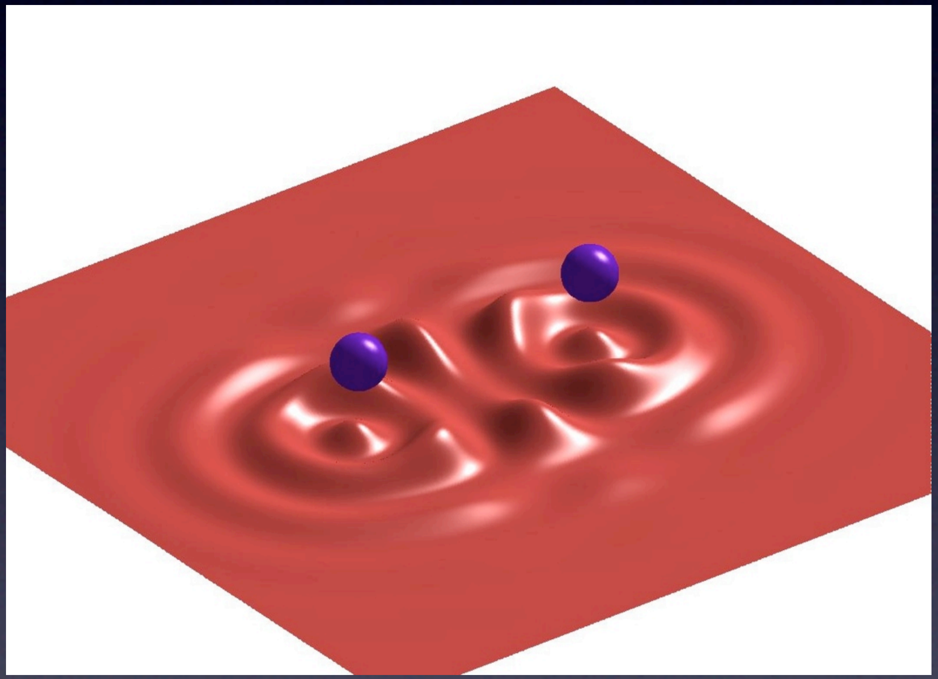
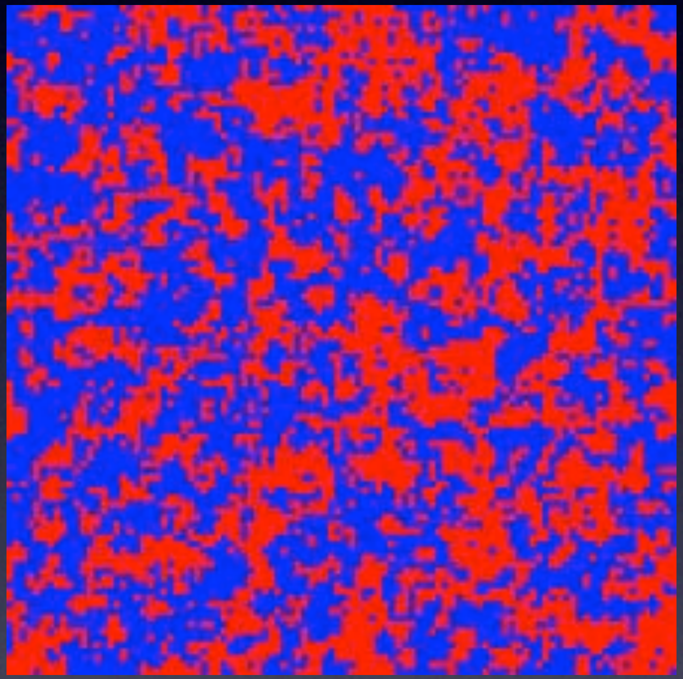
Locality





Which properties arise from local interactions?

Fractionalization of quantum numbers

The Fractional Quantum Hall Effect

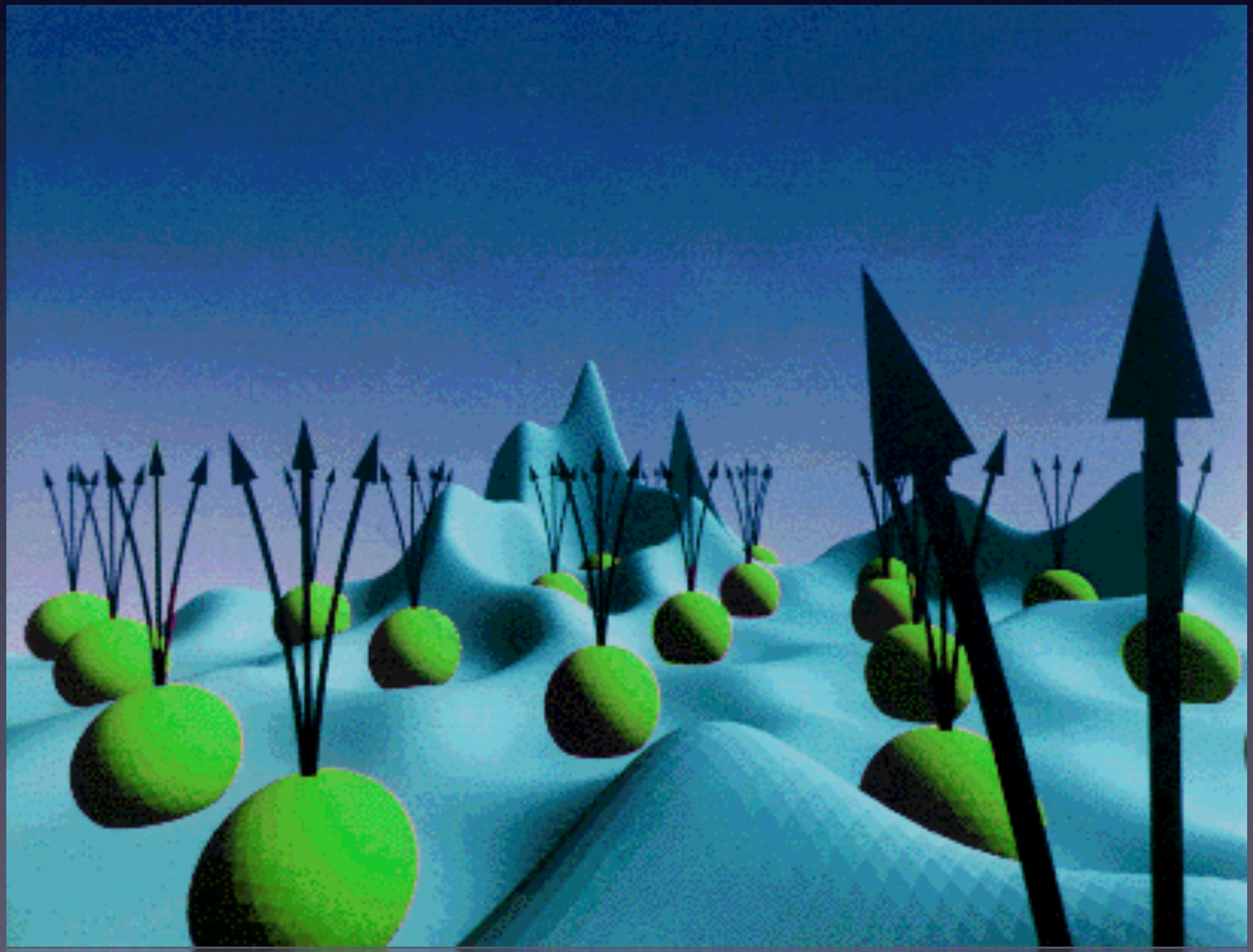
Horst Störmer

Daniel C. Tsui

Robert Laughlin

Fractional Quantum Hall effect

"More is different!"



Large magnetization requires large entanglement

Let $|\psi\rangle$ be a spin J, $U(1)$ invariant MPS with magnetization per particle m verifying $J - m = \frac{q}{p}$ (p and q with no common factors).

Then there exists a multiple of p which we denote by $\tilde{p}$ such that the entropy of the reduced density matrix of any region of size $L = k\tilde{p}$ ($\forall k$) verifies

$$S(\rho_L) \geq \log(p)$$

up to a correction exponentially small in $N - L$.

Theorem 1: Some remarks

- $|\psi_A\rangle = \sum_{i_1,\ldots,i_L} \mathrm{tr}[A_{i_1}\ldots A_{i_L}]|i_1\ldots i_L\rangle$
- If $L\to\infty$, different injective MPS are orthogonal
- If $|\psi\rangle = \sum_{r=1}^{n} \lambda_r|\psi_r\rangle \Rightarrow \rho_L$ is "close" to $\oplus_r|\lambda_r|^2\rho_r$
- When is it true that $|\psi\rangle = \sum_{r=1}^{n} \lambda_r|\psi_r\rangle$?

Theorem 1: Some remarks

- Condition on the number of blocks
  $\Rightarrow$ restriction on $p(J - m) = q$
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  $\Rightarrow$ restriction on number of blocks

Large interaction length implies large entanglement

Given an MPS $|\psi_A\rangle$. If for $\alpha = \frac{1}{6}$ we can upper-bound the α-Renyi entropy by

$$S_\alpha(\rho_A^R) \leq \frac{4}{5}\log\epsilon + \frac{1}{10}(L\log d - \log L) - \log\frac{d}{4}$$

where ρ_A^R is the reduced density matrix of a region of size R, which is large enough.

Then there exists another MPS $|\psi_{\tilde{A}}\rangle$ such that

- $|\psi_{\tilde{A}}\rangle$ is the unique ground state of a gapped frustration free Hamiltonian with interaction length L
- $||\rho_A^L - \rho_{\tilde{A}}^L||_1 \leq \epsilon$

Up to constants the bound on the Renyi entropy is of the form $L + \log\epsilon$.

An interesting bound

$$||\rho_A^L - \rho_{\tilde{A}}^L||_2 \leq 2\mathrm{tr}[\tilde{\Lambda}^{1/2}]\sqrt{L}\delta^{1/4} + (2l+3)\delta$$

$$||\rho_A^L - \rho_{\tilde{A}}^L||_1 \leq 2\sqrt{2}\tilde{D}\sqrt{L}\delta^{1/4} + (2L+3)\delta$$

$$\text{where } \delta = \mathrm{tr}[\Lambda - \tilde{\Lambda}]$$

$$\Lambda = \mathrm{gap}$$

How fast can we reach injectivity?

Every MPS (with exception of a zero-measure set) reaches injectivity in the minimal possible region, that is blocking L sites whenever

$$L \geq \frac{2 \log D}{\log d}$$

Conclusions

- A large fractionalization in the magnetization requires large entanglement in a quantum system.
- The absence of a local model implies a large entanglement in a quantum system.
- MPS allow us to work formally with these physical concepts and deduce consequences in full generality

Thank you!