Vascular network modeling and simulation

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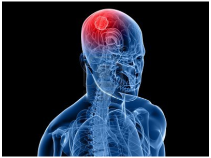
Workshop of young researchers 2011

Contents

Motivation

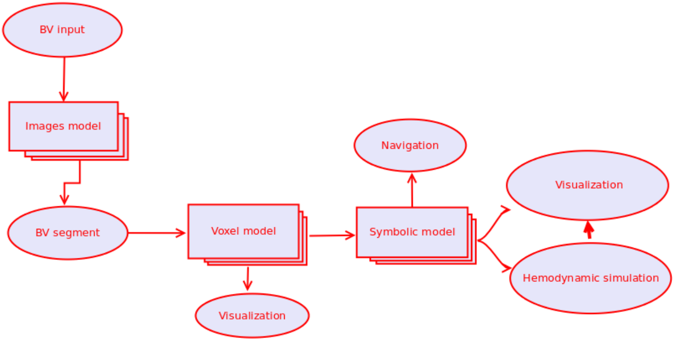
- Design a computer assisted neurodiagnosis system for early diagnosis and risk evaluation of vascular anomalies.



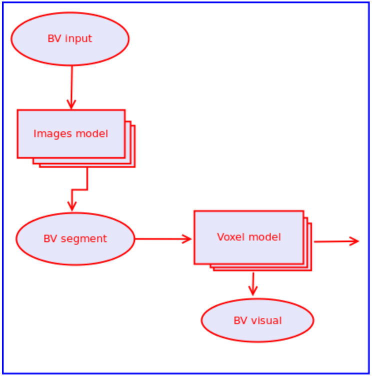
Funcionalities of a computer assisted system

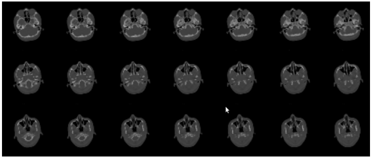
- Visualization of the 3D vascular system: spatial location of the relevant vessels and topological connectivity.
- Automatic detection of the vessel features: stenoses, aneurysms, vessel branchings.
- Blood flow simulation: prediction of vascular accidents.

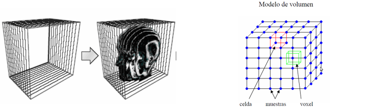
Our proposal of vascular tree model

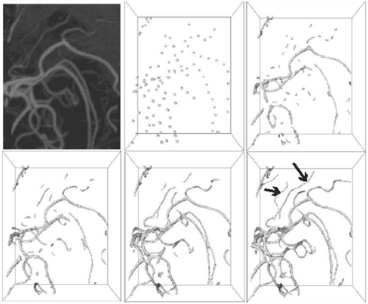


Our proposal of vascular tree model

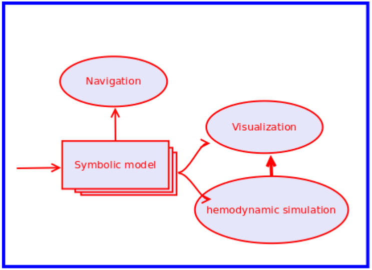






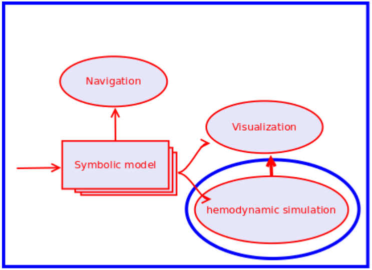


Our proposal of vascular tree model

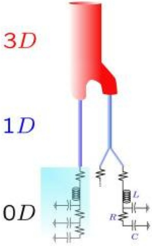


- This module presents:
  - A symbolic model: for the interrogation of the vascular structure.
  - A Genelarized Cilinder method: for the reconstruction and visualization of blood vessels.
  - A 1D mathematical model for simulating the blood flow.

Hemodynamic simulation



- Multiscale approach



Introduction

- Blood flow is modeling using a 1D model for pressure and flow wave propagation in compliant vessels, that is able to capture the main features of pulse wave propagation throughout the cerebral arteries

The couple fluid-structure problem.

- Main fluid assumptions.
  - Newtonian.
  - Incompressible.
  - Pulsatile flow.
- Wall vessel assumptions.
  - Small thickness and plain stresses.
  - Cylindrical reference geometry and radial displacements.
  - Small deformation gradients.
  - Incompressibility.
- Main variables:
  - Pressure.
  - Velocity.
  - Vessel wall displacement.

Assumptions

- Axial symmetry.
- Radial displacements.
- Constant pressure.
- No body forces.
- Dominance of axial velocity.

$$u_z(t,r,z) = \overline{u}\frac{\varsigma + 2}{\varsigma}\left[1 - \left(\frac{r}{R}\right)^{\varsigma}\right]$$

where ς describes the velocity profile.

Equations

- Equations:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0$$
$$\frac{\partial Q}{\partial t} + \frac{\partial\left(\frac{\alpha Q}{A}\right)}{\partial x} + \frac{A}{\rho}\frac{\partial P}{\partial x} = -K_r \frac{Q}{A}$$
$$P(A) = \beta \frac{A^{\frac{1}{2}} - A_0^{\frac{1}{2}}}{A_0}$$

- Parameters.
  - $\beta = \pi^{\frac{1}{2}} hE$.
  - E is the Young modulus.
  - h is the vessel wall thickness.
  - K_R is a resistance parameter related to the viscosity of fluid
  - α is indicates the profile velocity law

1D model with algebraic pressure law

- System of equations in conservative form:

$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial z} = B(U)$$

- Boundary conditions:
  - At inlet, a single pulse (half sine).
  - At outlet, non reflecting boundary conditions / absorbing boundary conditions

Numerical scheme

- The system is solved by a splitting approach.
- The hyperbolic homogeneous system:

$$\text{PDE's}: U_t + F(U)_x = 0; \;\; 0 \leq x \leq L$$
$$\text{IC}: U(x, t^n) = U^n$$

is solved using the two-step Richtmyer finite difference method:

$$U_{j+\frac{1}{2}}^{n+\frac{1}{2}} = \frac{1}{2}(U_{j+1}^n + U_j^n) - \frac{\lambda}{2}\left(f\left(U_{j+1}^n\right) - f\left(U_j^n\right)\right)$$
$$U_j^{n+1} = U_j^n - \lambda\left(f\left(U_{j+\frac{1}{2}}^{n+\frac{1}{2}}\right) - f\left(U_{j-\frac{1}{2}}^{n+\frac{1}{2}}\right)\right)$$

- The ODE problem:

$$\text{ODE's}: \frac{dU}{dt} = S(U); \;\; 0 \leq x \leq L$$
$$\text{IC}: \overline{U}^{n+1}$$

where $\overline{U}^{n+1}$ is the solution of the above system, is solved using a Euler method.

Numerical boundary conditions: compatibility conditions

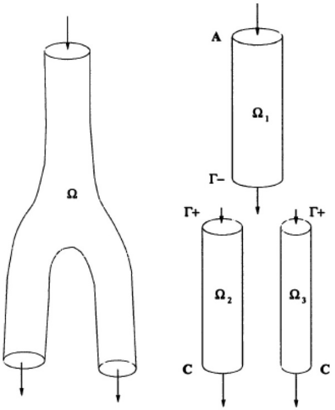
- Two extra relations provided by the differential equations 'projected' along the direction of the outgoing characteristic.

$$l_1^T\left(\frac{\partial U}{\partial t}+\frac{\partial F(U)}{\partial z}-B(U)\right)=0, z=0, t>0$$
$$l_2^T\left(\frac{\partial U}{\partial t}+\frac{\partial F(U)}{\partial z}-B(U)\right)=0, z=L, t>0$$

- Those extra relations together with the boundary conditions complement the discrete numerical scheme.

More complex geometries: branching and general networks

- Domain decomposition method.



- Interface conditions:
  - Conservation of mass.

$$\sum_i Q_i = \sum_o Q_o$$

  - Continuity of the total pressure.

$$\sum_i P_{t,i} = \sum_o P_{t,o}$$

  - + Compatibility conditions

More complex geometries: branching and general networks

- Note: The effect of the bifurcation angles can be introduced imposing:

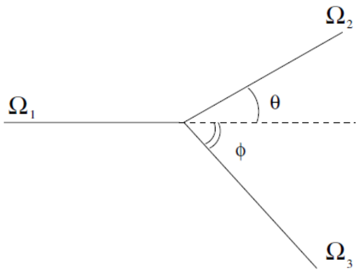
$$p_{t,i} - \text{sign}(u_i) f_i(u_i) = p_{t,o} + \text{sign}(u_o) f_o(u_o, \alpha_o)$$

where

$$f_i(u) = \gamma_i u_i^2$$

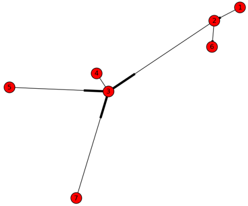
$$f_o(u_o, \alpha_o) = \gamma_o u_o^2 (2(1 - \cos(\alpha_o))^{\frac{1}{2}}$$

are dissipation functions, γ_i positive coefficients and α_j the angles of the branches with respect the incident direction.

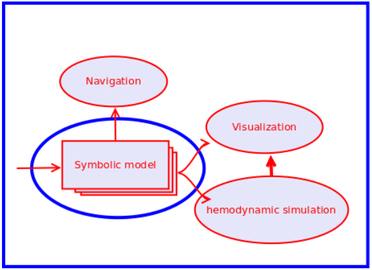


Arbitrary network

- Software tool: Networkx.



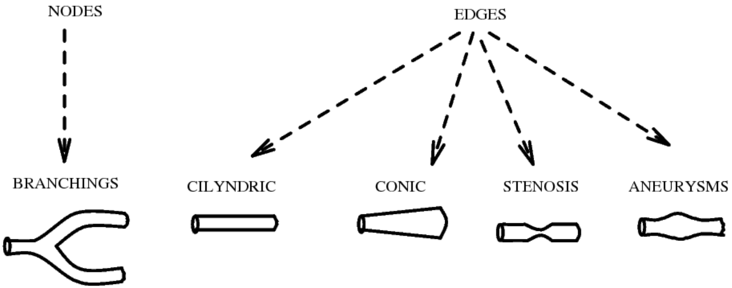
Symbolic model



- Synthesizes the characteristic of the physical vessels and represent the topology and geometry of the vascular system.
- Represented by an oriented graph: Nodes represent bifurcations, graph Edges the vessel sections between branching.

Symbolic model definition

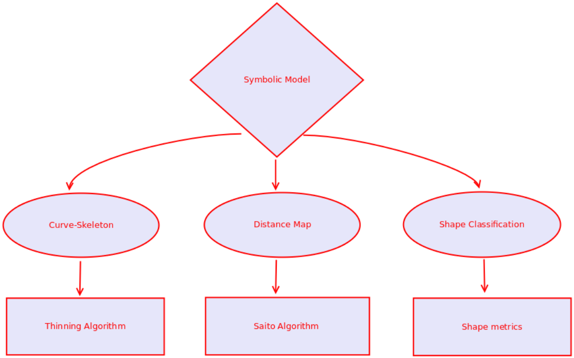
- Each section is composed of a set of path-connected elements of different types.



- Mathematically, it is defined as the tuple: $\{V, E, T, \Sigma\}$, where V is the node set, E is the set of edges, T is the set of anatomical features, and σ a labeling function.

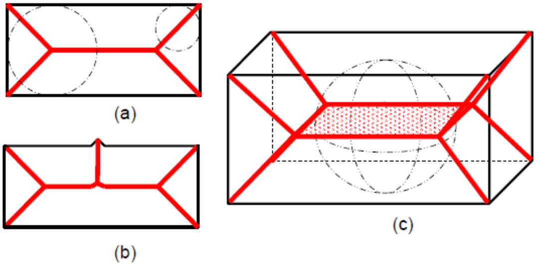
Construction of the symbolic model

- Two different tasks:
  - Computation of a discrete skeleton and distance tranformation.
  - Identification of the nodes and features from these concepts.



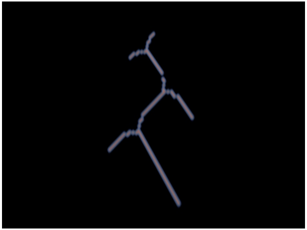
Curve-skeleton definition

- Simplified shape descriptor.
- Discrete approximation of the Medial Axis Transform (MAT).
- MAT is defined as the closure of the set of centers of maximal balls fitted inside the object, not contained in any other ball.



Construction algorithm: thinning

- Iterative process which erodes an object layer by layer until only a “skeleton” of the object remains.



Construction algorithm: thinning

- Based on the concept of simple point, voxel that can be removed without changing the topology of the object.
- The algorithm contains a set of deletion conditions which are applied iteratively to delete 1s (object elements), that is, to change 1s to 0s (background elements).

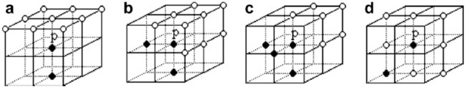


Fig. 2. Four template cores (Classes A, B, C and D) of the fully parallel thinning algorithm. A "●" is an object point. A "○" is a background point. An unmarked point is a "don't care" point that can be either an object point or a background point. For (d), there is an additional restrict that p must be a simple point.

Distance map definition

- A distance map is a representation where the value of any point p within the image is the distance from that point to the closest point on a subset O'.

$$D(p) = \min\{\text{dist}_M(p, q), q \in O'\}$$

27.	22	19	18	19	22	27
22	17	14	13	14	17	22
19	14	11	10	11	14	19
18	13	10	9	10	13	18
19	14	11	10	11	14	19
22	17	14	13	14	17	22
27	22	19	18	19	22	27

Layer 1

22	17	14	13	14	17	22
17	12	9	8	9	12	17
14	9	6	5	6	9	14
13	8	5	4	5	8	13
14	9	6	5	6	9	14
17	12	9	8	9	12	17
22	17	14	13	14	17	22

Layer 2

19	14	11	10	11	14	19
14	9	6	5	6	9	14
11	6	3	2	3	6	11
10	5	2	1	2	5	10
11	6	3	2	3	6	11
14	9	6	5	6	9	14
19	14	11	10	11	14	19

Layer 3

18	13	10	9	10	13	18
13	8	5	4	5	8	13
10	5	2	1	2	5	10
9	4	1	0	1	4	9
10	5	2	1	2	5	10
13	8	5	4	5	8	13
18	13	10	9	10	13	18

Layer 4

27	22	19	18	19	22	27
22	17	14	13	14	17	22
19	14	11	10	11	14	19
18	13	10	9	10	13	18
19	14	11	10	11	14	19
22	17	14	13	14	17	22
27	22	19	18	19	22	27

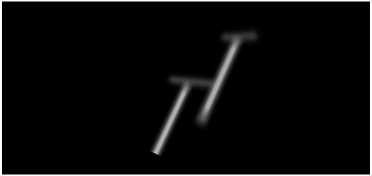
Layer 7

22	17	14	13	14	17	22
17	12	9	8	9	12	17
14	9	6	5	6	9	14
13	8	5	4	5	8	13
14	9	6	5	6	9	14
17	12	9	8	9	12	17
22	17	14	13	14	17	22

Layer 6

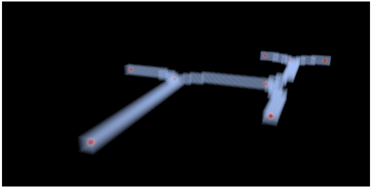
19	14	11	10	11	14	19
14	9	6	5	6	9	14
11	6	3	2	3	6	11
10	5	2	1	2	5	10
11	6	3	2	3	6	11
14	9	6	5	6	9	14
19	14	11	10	11	14	19

Layer 5

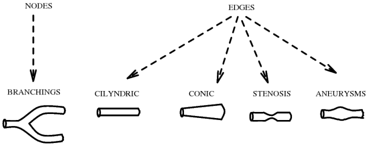


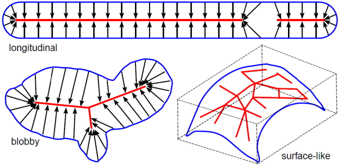
Features identification

- Branchings detection.

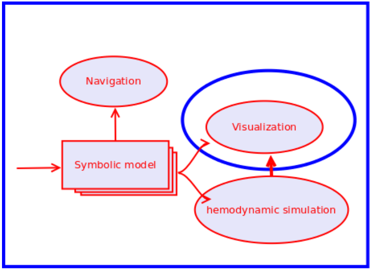


- Region section classification: Tubiness, Surfaceness, Blobbiness descriptors.

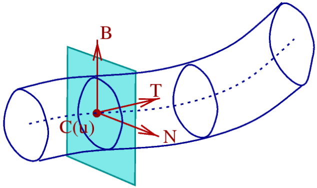


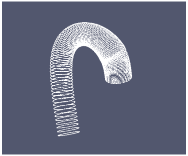


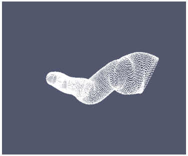
Reconstruction and visualization model: Generalized Cilinder Method



- Generalized cylinders consist of a space curve o axis (spine) and a cross-section function defined on that axis.





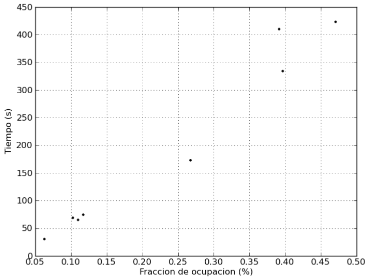


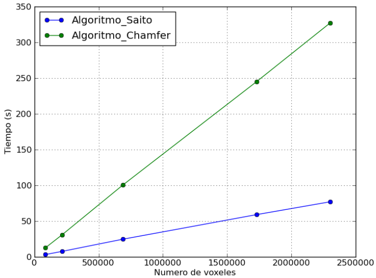
Open source software

- All algorithms have been implemented in python language. Paraview has been used for 3D visualiation purposes.

Symbolic and vascular reconstruction algorithms

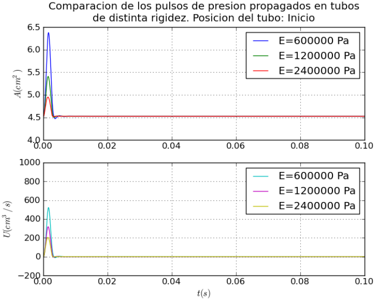
- Linear complexity of the algorithms

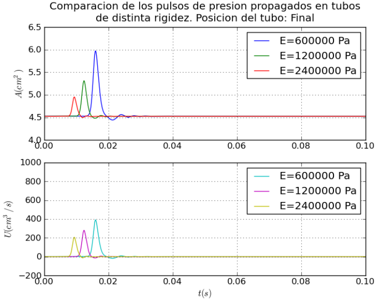




Stiffness

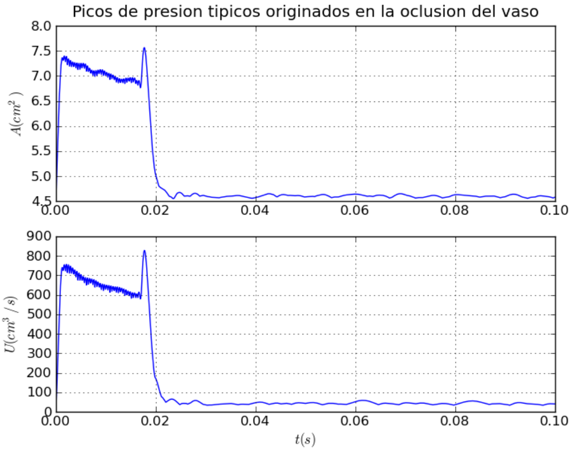
- Stiffness increases the speed of pulse wave propagation and attenuates the wave.





Occlusion

- Occlusion causes overpression and reduction in the outflow.



Summary and Future work

Summary

- A model of the vascular tree has been presented, along with algorithms for its construction and interrogation.
- The model is aimed at fulfilling all the requirements of a computer-assisted neurovascular system and it allows hemodynamic simulations of the blood flow throught it.

Future work

- Accomodate discontinuous variations of material properties in the 1D mathematical model, and extend it to more complex geometries (curvature)
- Study and model the effects of radiotherapy on the vascular network
- Test the visualization model in real images.

Acknowledgements

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