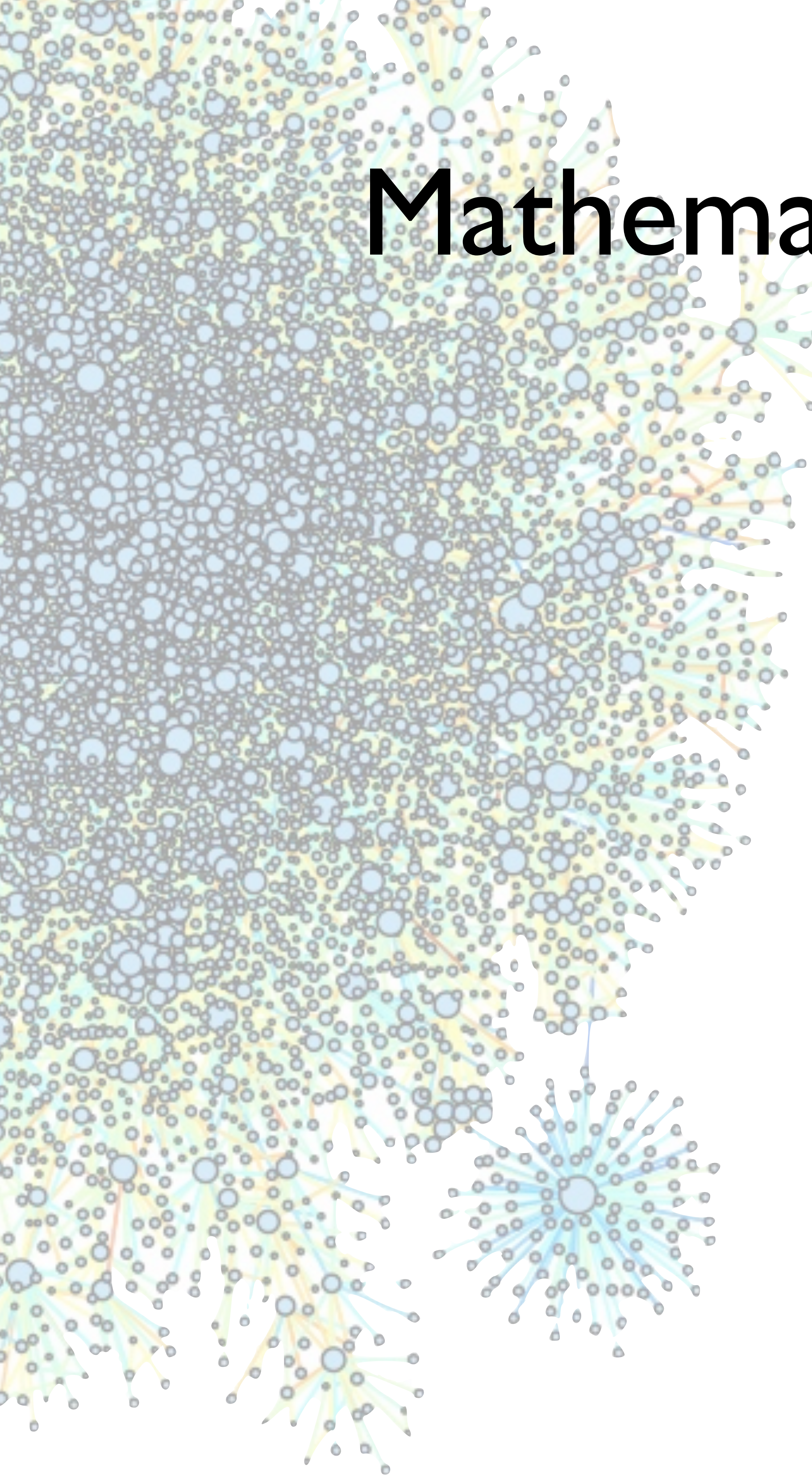


Mathematical Modeling of Human Communication

Giovanna Miritello

Esteban Moro

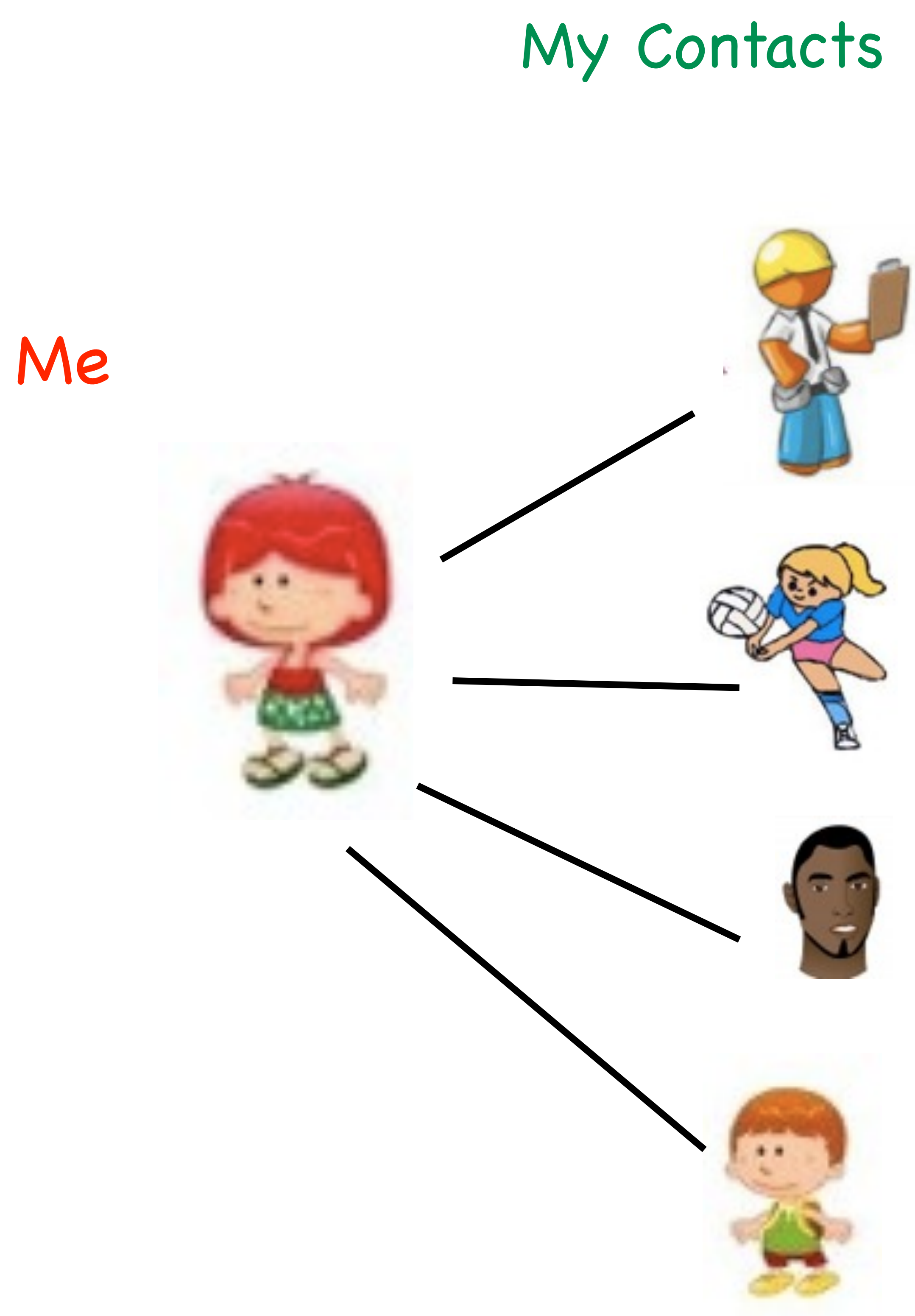
Rubén Lara

Universidad Carlos III de Madrid

Our work and relevance for Telefonica

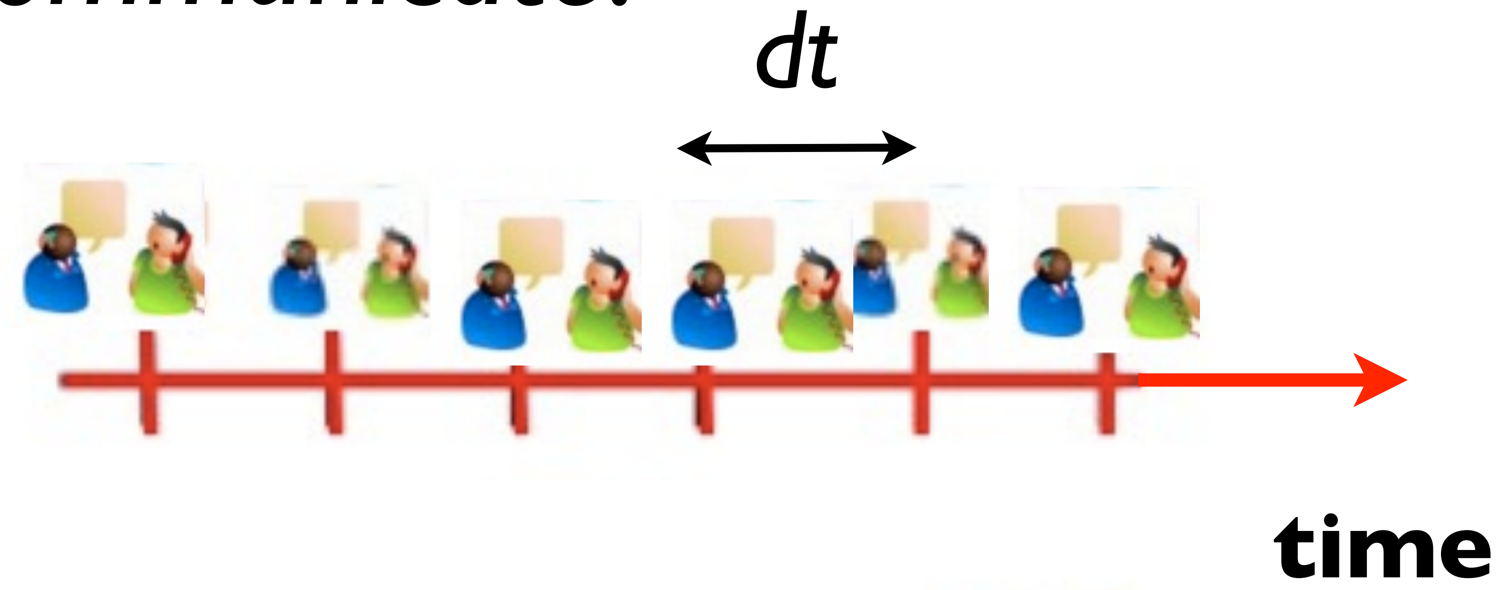
- *Characterize interactions between users*
- *Understand how people communicate*
- *Model (and predict) how information spreads between individuals*

How do people communicate?



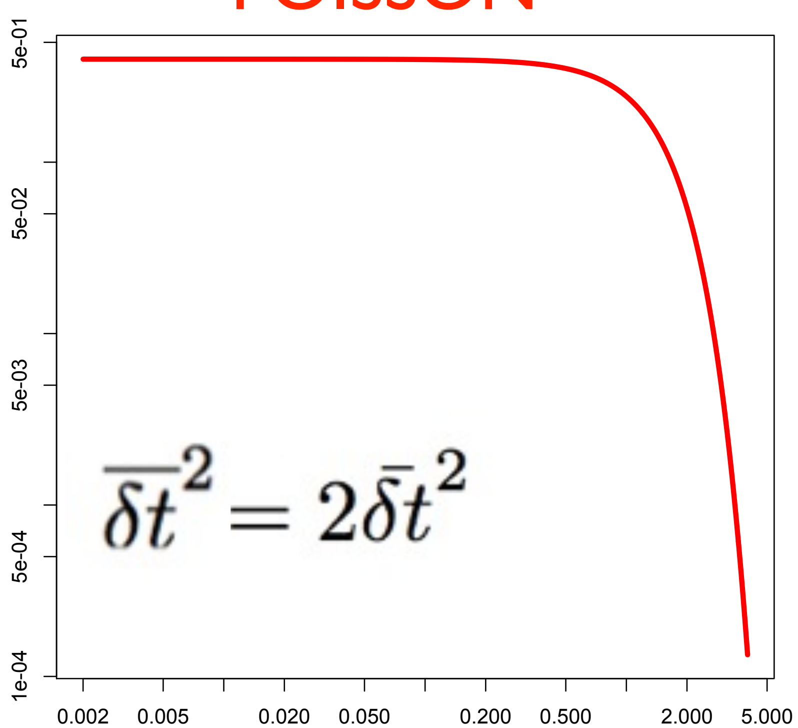
How do people communicate?

Homogeneous



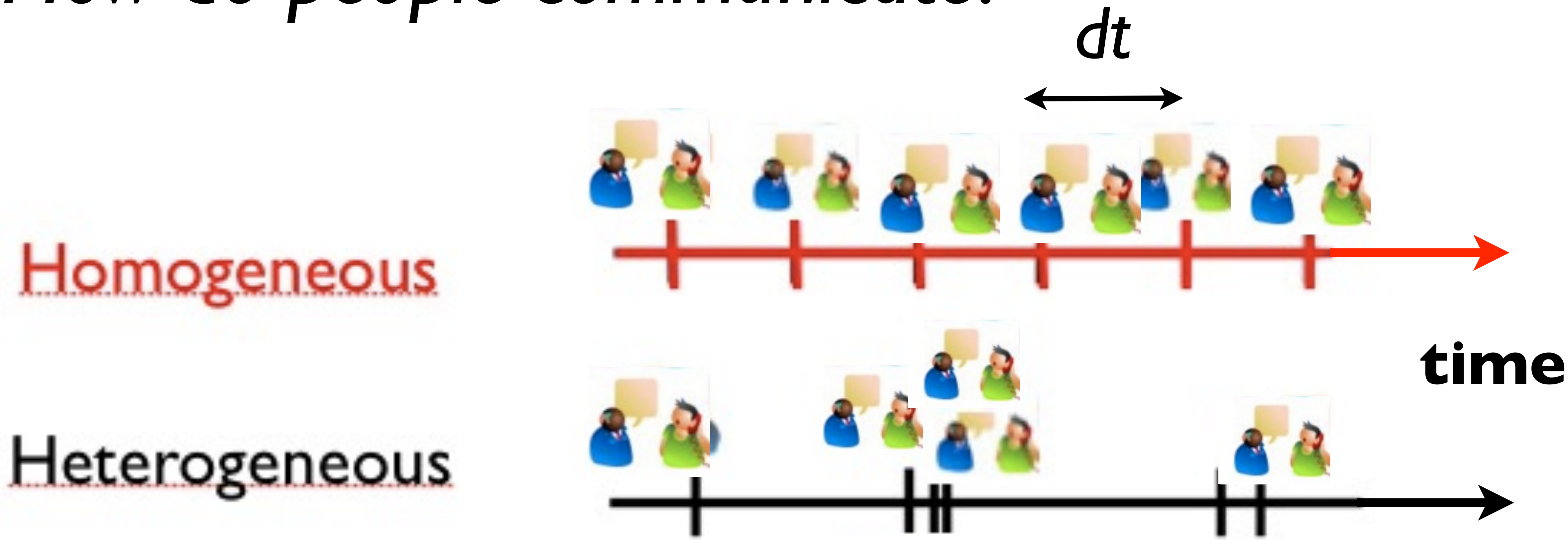
POISSON

P(dt)

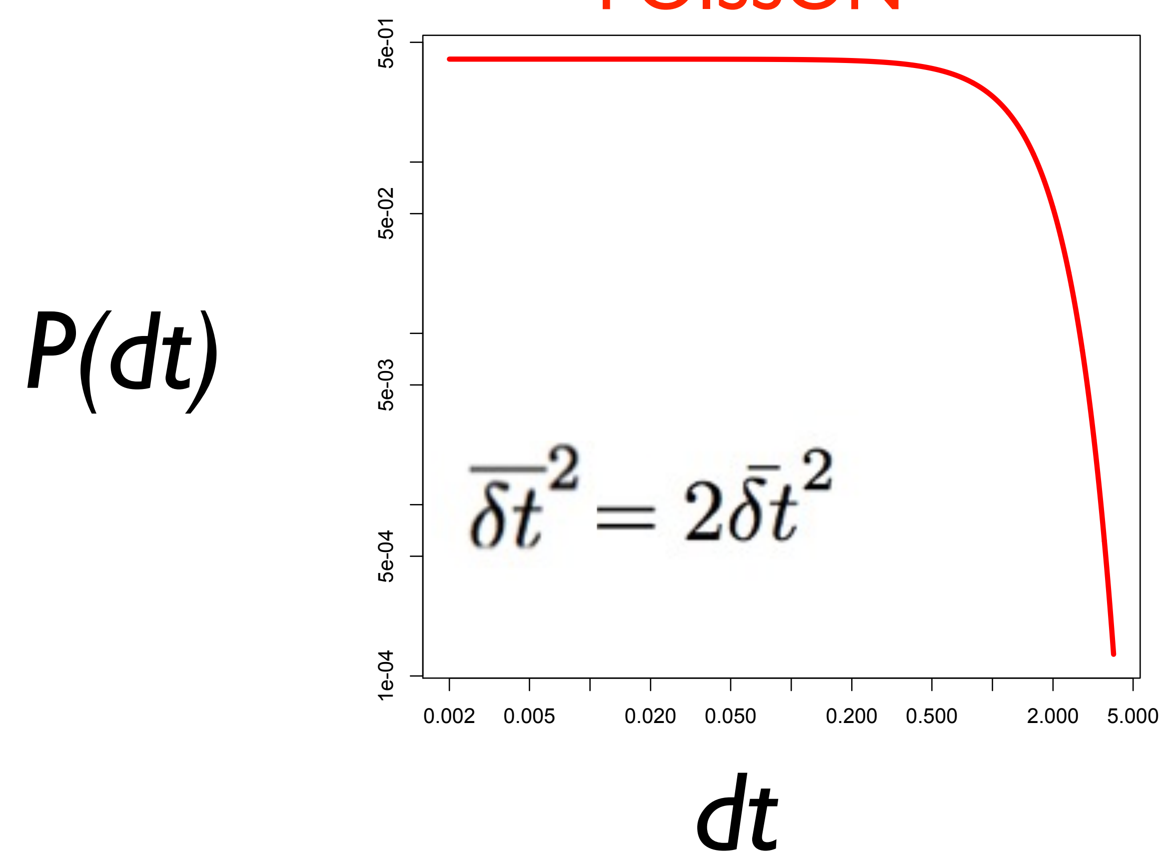


dt

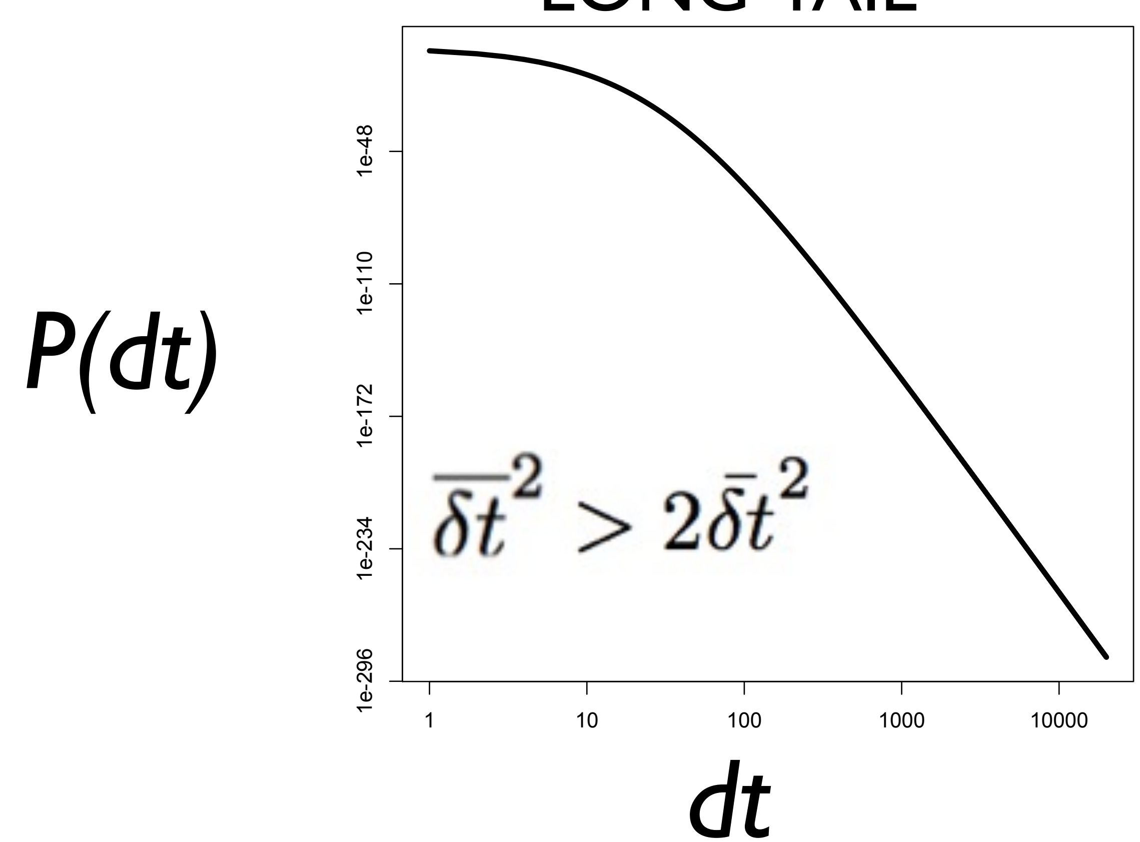
How do people communicate?



POISSON

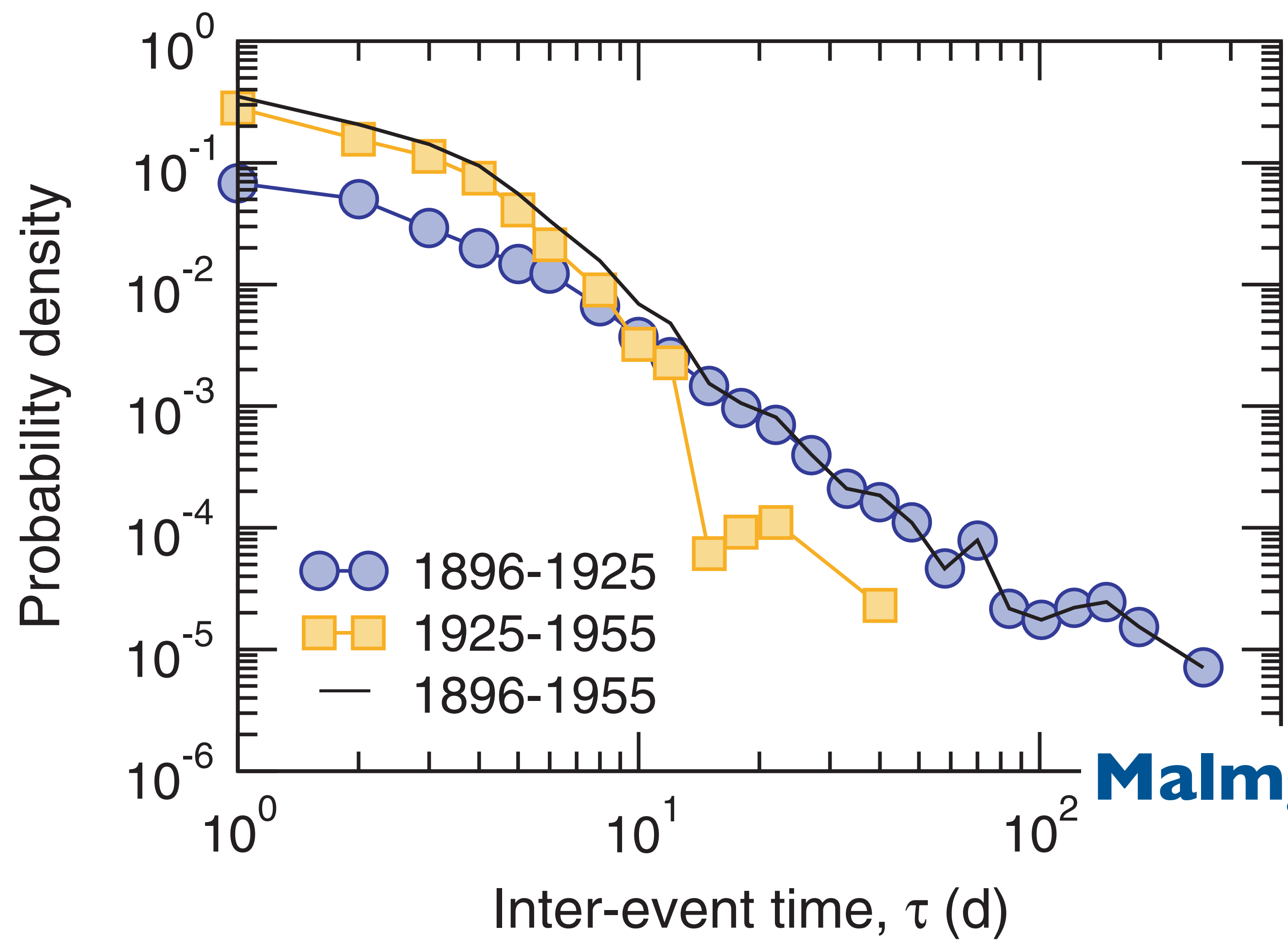


LONG-TAIL

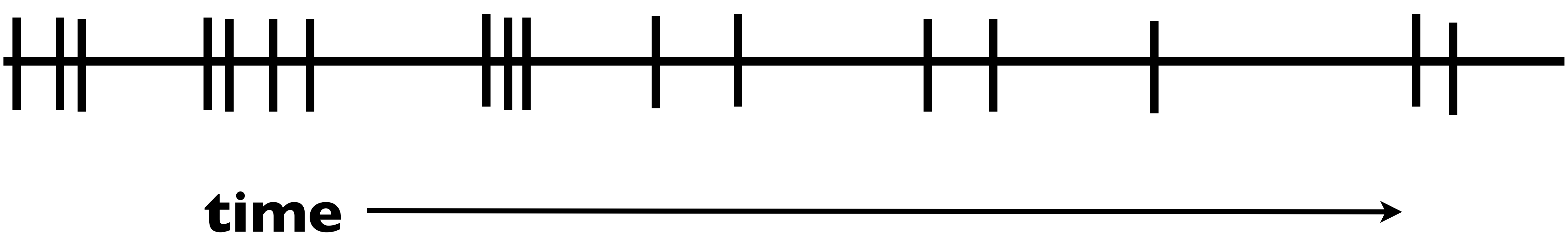


Human communication happens in cascades of events (bursts)

Albert Einstein's letter correspondence activity

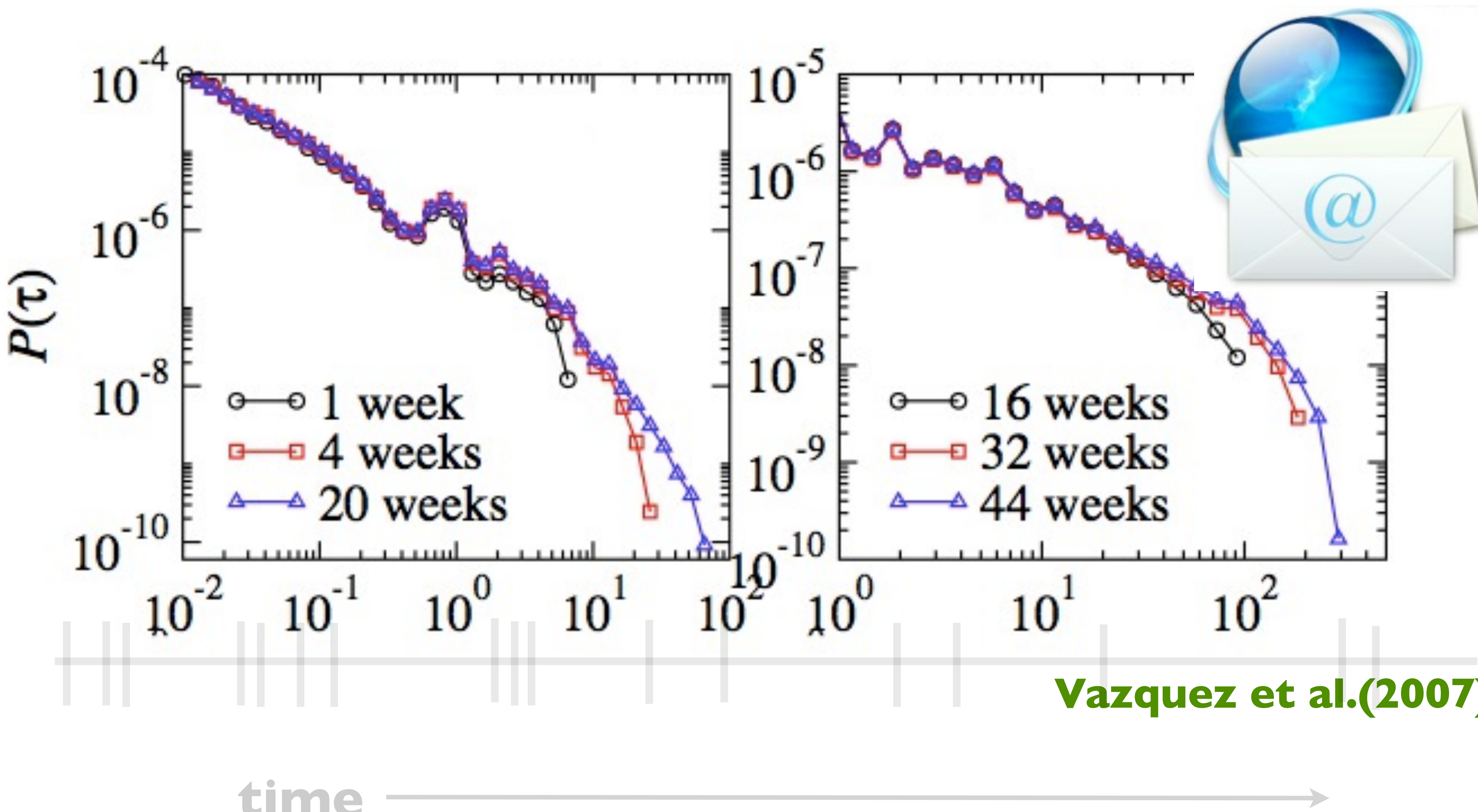


Malmgren et al.(2009)



Human communication happens in cascades of events (bursts)

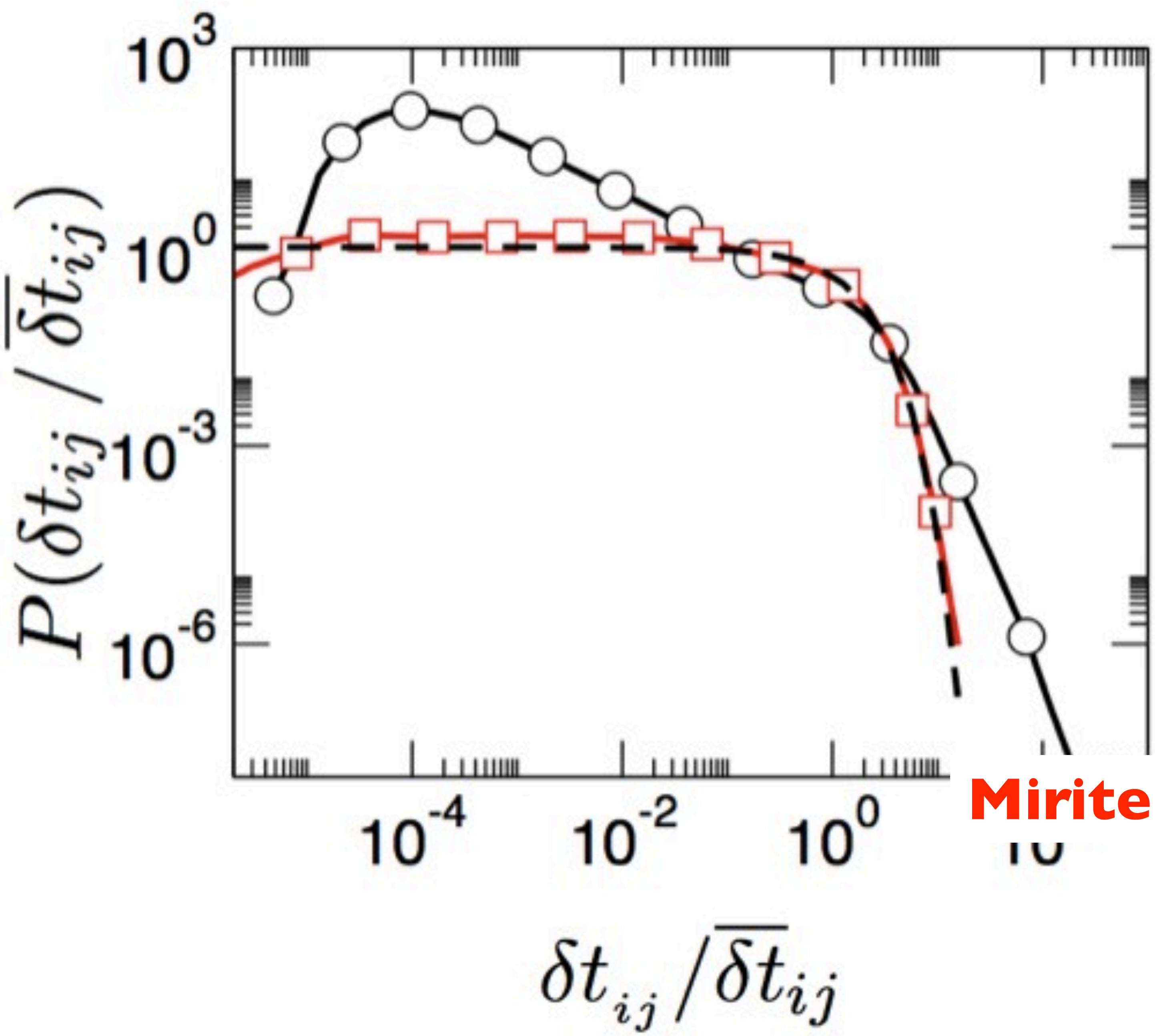
E-mails communication



Vazquez et al.(2007)

Human communication happens in cascades of events (bursts)

Mobile phone communication



Miritello, Moro, Lara (2010)

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What is the impact of real bursty behavior in information spreading?

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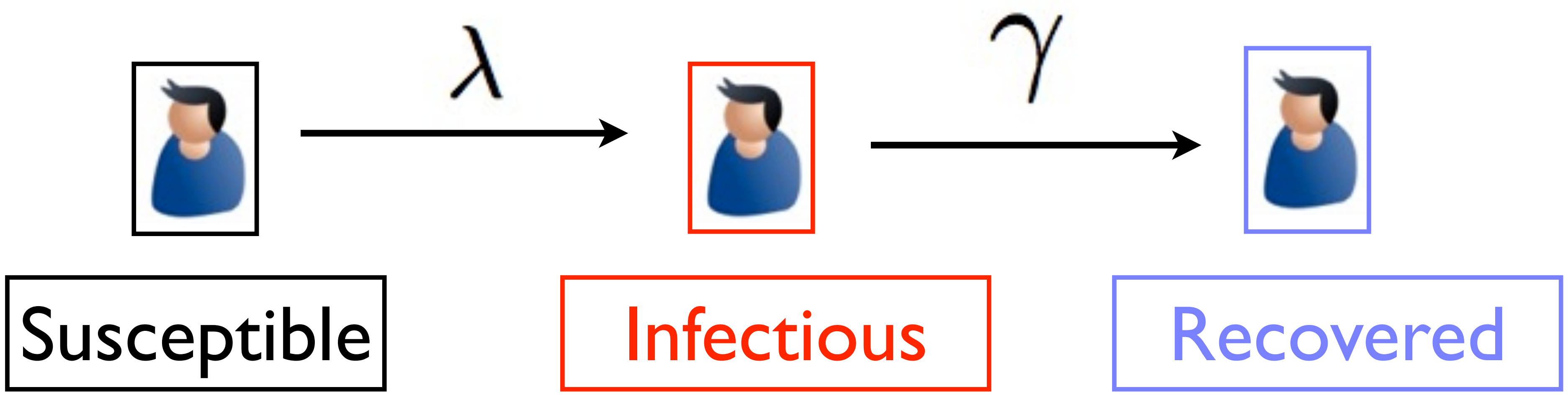
What is the impact of real bursty behavior in information spreading?

Information diffusion model from epidemiology

SIR MODEL

Daley and Kendall , Nature, vol. 204, no. 4963 (1964)

Murray, Math. Biol, vol.17, Springer, NY (2002)



- Fully mixed population
- Each user has the same probability of infection
- Homogeneous time

$$\frac{ds}{dt} = -\lambda i s$$

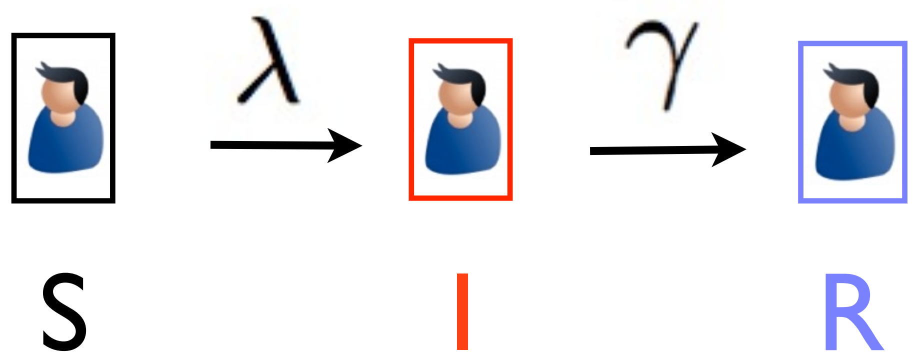
$$\frac{di}{dt} = \lambda i s - \gamma i$$

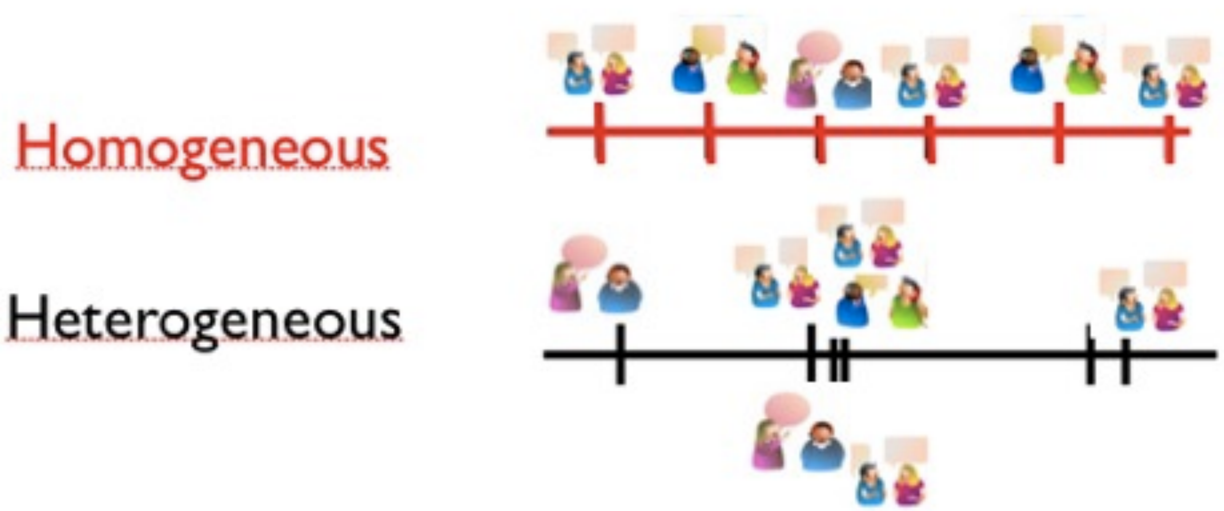
$$\frac{dr}{dt} = \gamma i$$

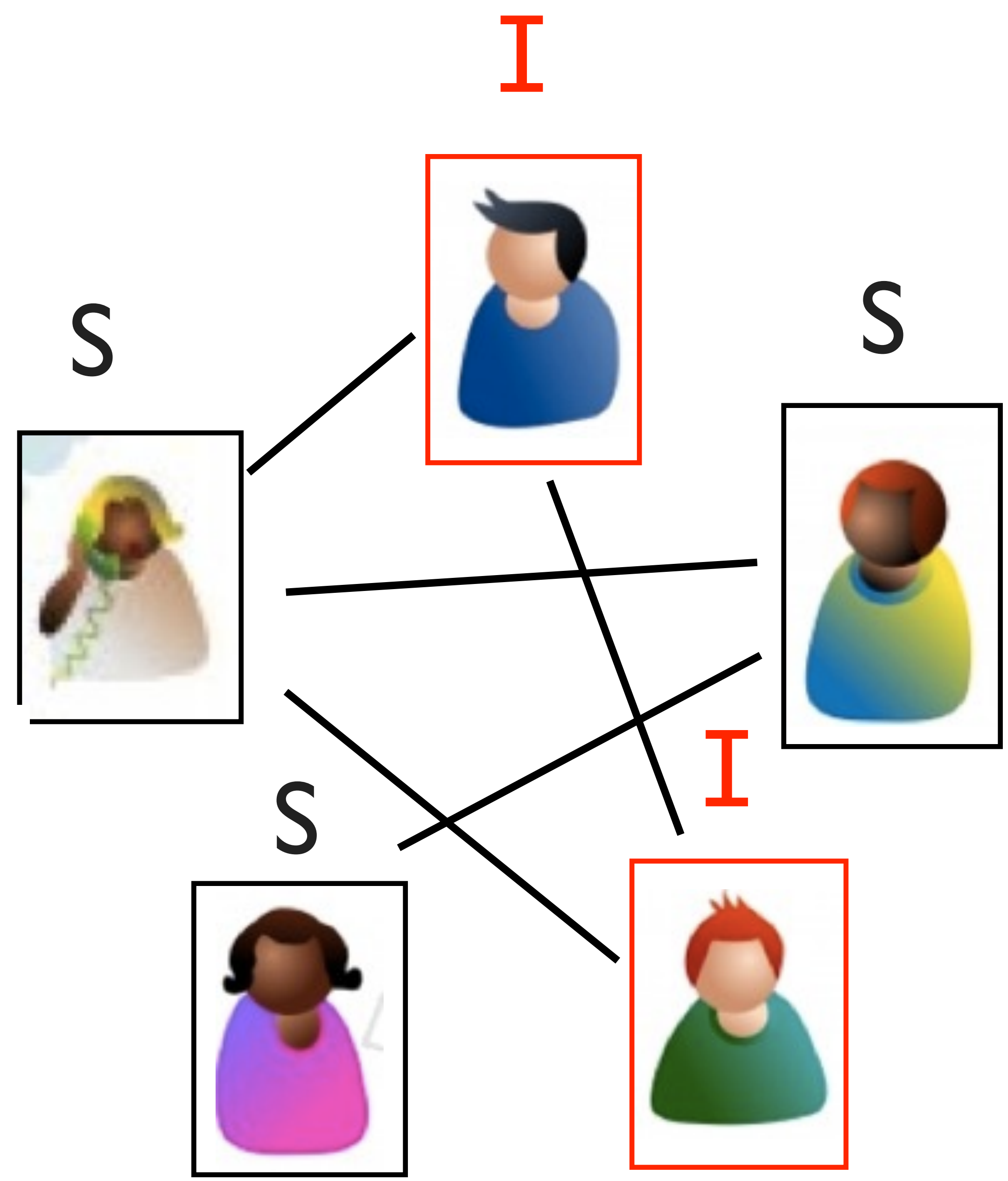
$$s + i + r = 1$$

What is the impact of real bursty behavior in information spreading?

SIR MODEL







SIR Model on Real Data from Mobile Phone Calls

Country X

- CDR
- 11 months of data
- 23M people
- 2300M (call) links

07514422955	07872909981	15	000003	C
07802715977	07783276078	15	347	C
07872620521	07872618243	18	000036	C
07740086040	07821842408	24	000069	C
07543053528	07595184583	26	000061	C
07740469727	07766415523	34	20	C
07516626552	07522398677	35	000033	C
07907193548	07856616326	35	000004	C
07907378133	07873905835	41	000021	C
07783276078	07543466075	42	000175	C
07729856086	07769974265	47	000017	C
07894668339	07711598505	48	000009	C
07521620630	07597377490	53	000209	C
07889800622	07776301842	53	145	C
07725326776	07595030724	55	292	C

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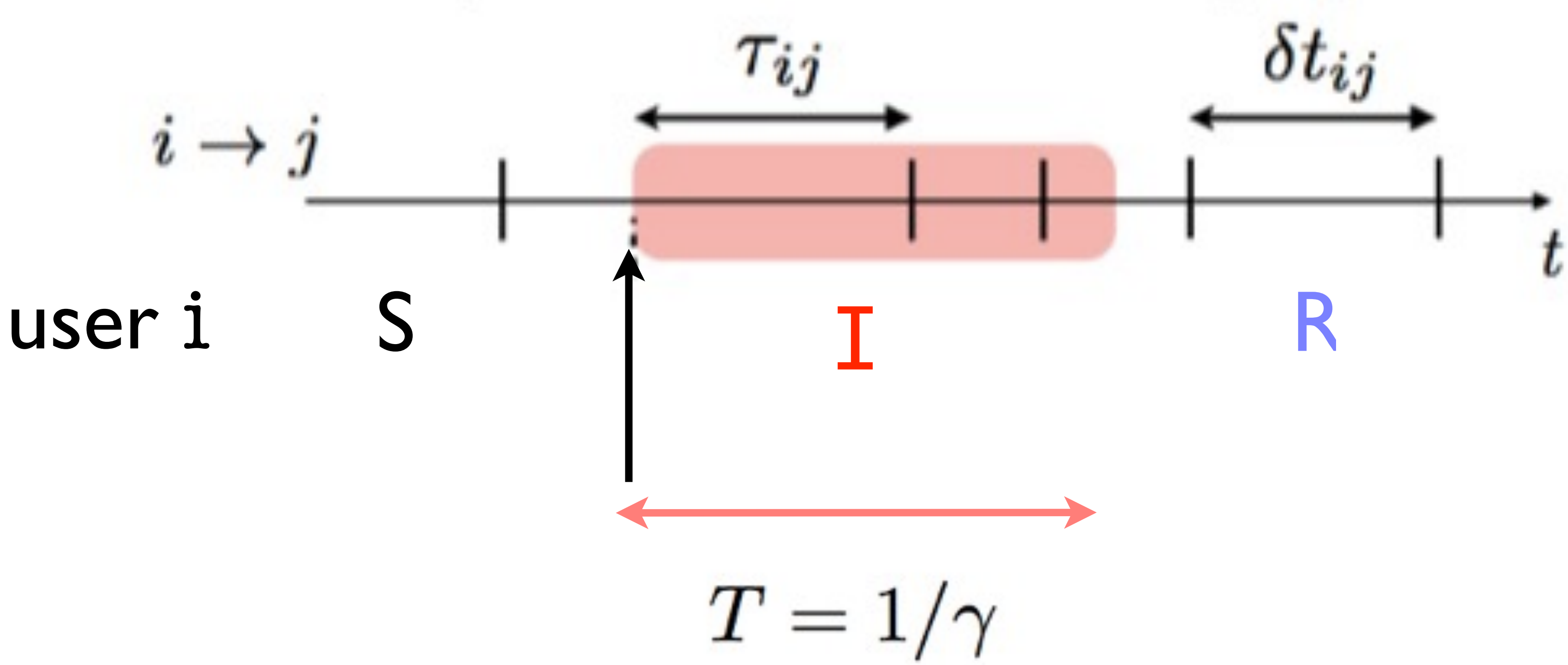
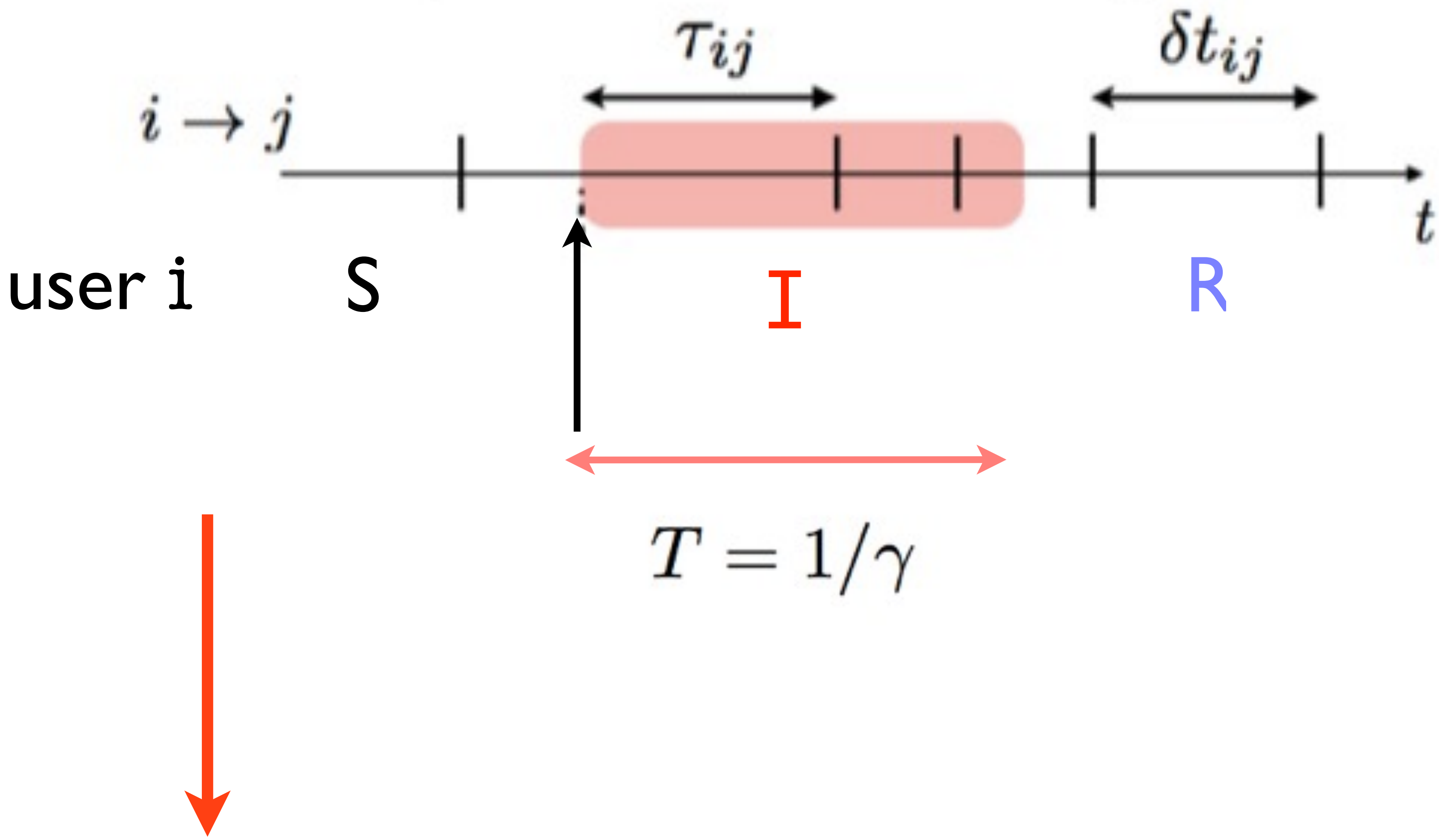
Modeling Human Communication

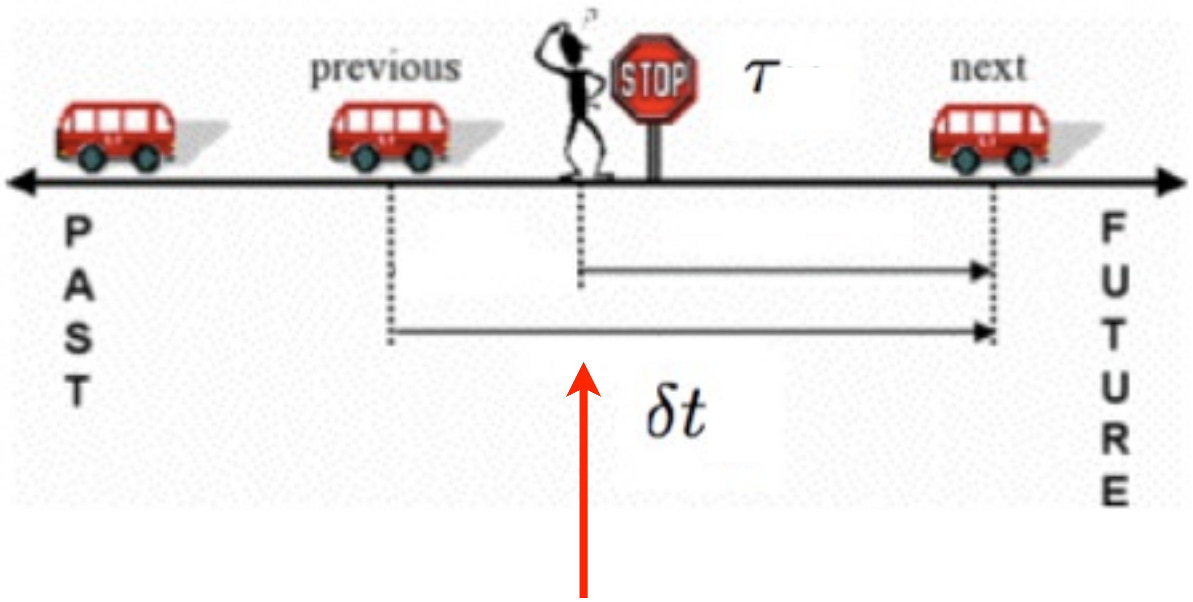


Modeling Human Communication



Bus Waiting Time Paradox: how long do you wait for the bus?

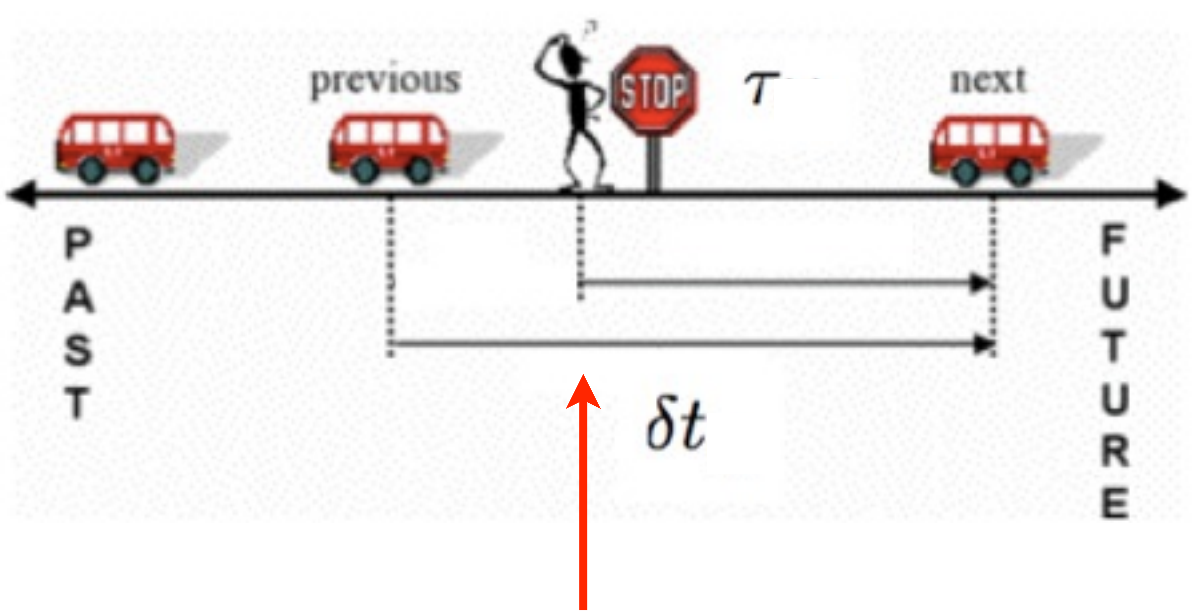
Bus Waiting Time Paradox: how long do you wait for the bus?



- you arrive at random t
- buses arrive according to $Q(\delta \mathrm{t})$
- the mean interval between buses is $\bar{\delta t}$

$$\bar{\tau}\ ?$$

Bus Waiting Time Paradox: how long do you wait for the bus?



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$$\bar{\tau}\ ?$$

Probability to arrive at an interval δt

$$\frac{\delta t Q(\delta t)}{\bar{\delta t}}$$

Bus Waiting Time Paradox: how long do you wait for the bus?

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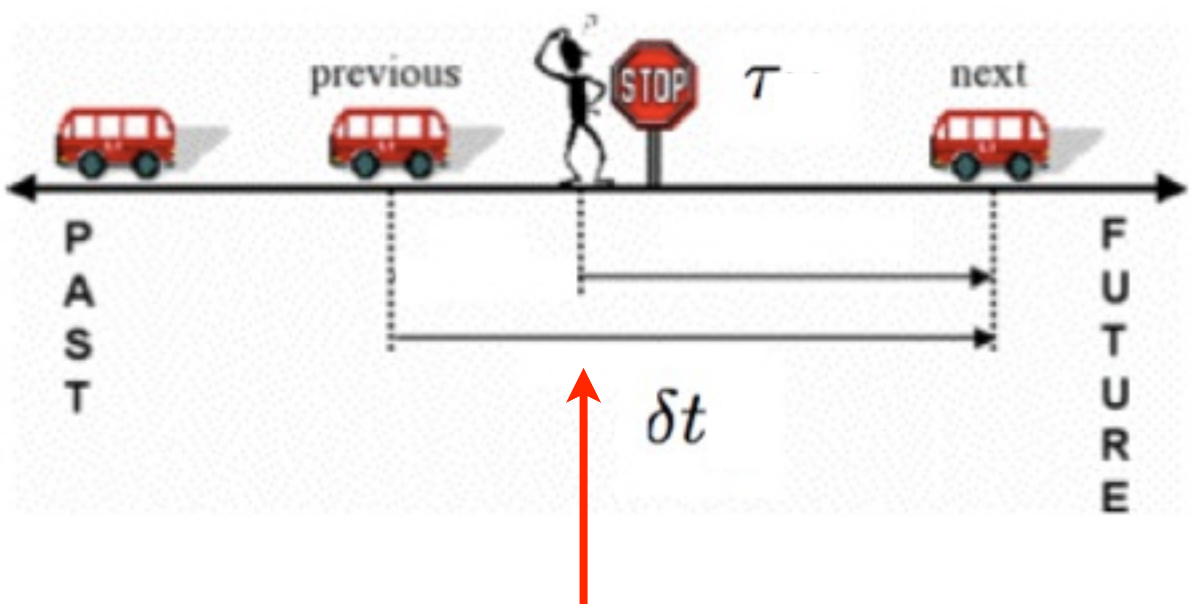


$$\bar{\tau}\ ?$$

given the interval δt

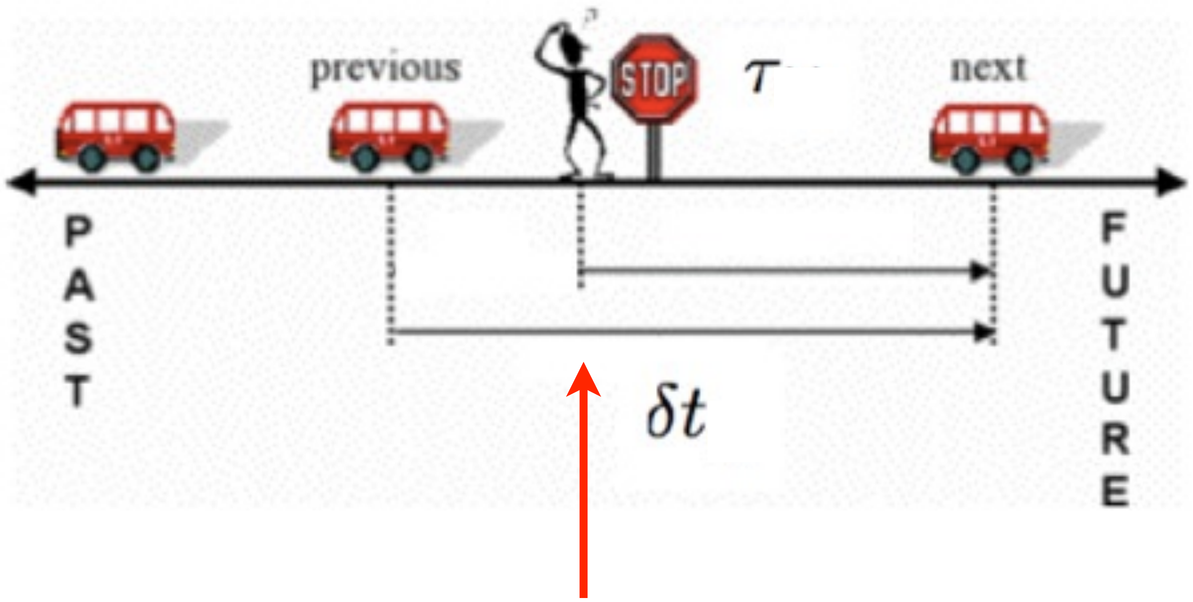
τ is equally distributed

Probability to arrive at an interval δt_{ij}

$$\frac{\delta t Q(\delta t)}{\overline{\delta t}} \quad \frac{1}{\delta t}$$

Bus Waiting Time Paradox: how long do you wait for the bus?

- you arrive at random t
- buses arrive according to $Q(\delta t)$
- the mean interval between buses is $\bar{\delta t}$



$$\bar{\tau}\,?$$

given the interval δt_{ij}

τ_{ij} is equally distributed

Probability to arrive at an interval δt_{ij}

Finally we average over all the $\delta t \geq \tau$

$$P(\tau) = \int_{\tau}^{\infty} d\delta t \, \frac{\delta t Q(\delta t)}{\bar{\delta t}} \, \frac{1}{\delta t}$$

Bus Waiting Time Paradox: how long do you wait for the bus?

$$P(\tau) = \int_\tau^\infty d\delta t \; \frac{\delta t Q(\delta t)}{\overline{\delta t}} \; \frac{1}{\delta t} \; = \frac{1}{\overline{\delta t}} \int_\tau^\infty d\delta t Q(\delta t)$$

$$\bar{\tau} \, ?$$

$$\bar{\tau} = \int_0^\infty d\tau \; \tau \; P(\tau) \; = \frac{\overline{\delta t}}{2} \left(1 + \frac{\sigma_{\delta t}^2}{\overline{\delta t}^2} \right)$$

$$\bar{\tau} = \frac{1}{2} \frac{\overline{\delta t^2}}{\overline{\delta t}}$$

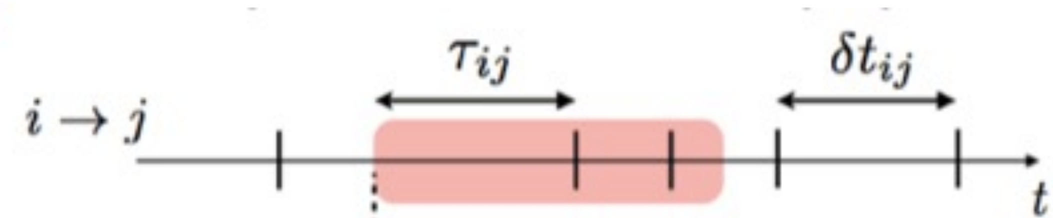
Bus Waiting Time Paradox: how long do you wait for the bus?

$$P(\tau) = \int_\tau^\infty d\delta t \, \frac{\delta t Q(\delta t)}{\overline{\delta t}} \, \frac{1}{\delta t} = \frac{1}{\overline{\delta t}} \int_\tau^\infty d\delta t Q(\delta t)$$

$$\bar{\tau} \, ?$$

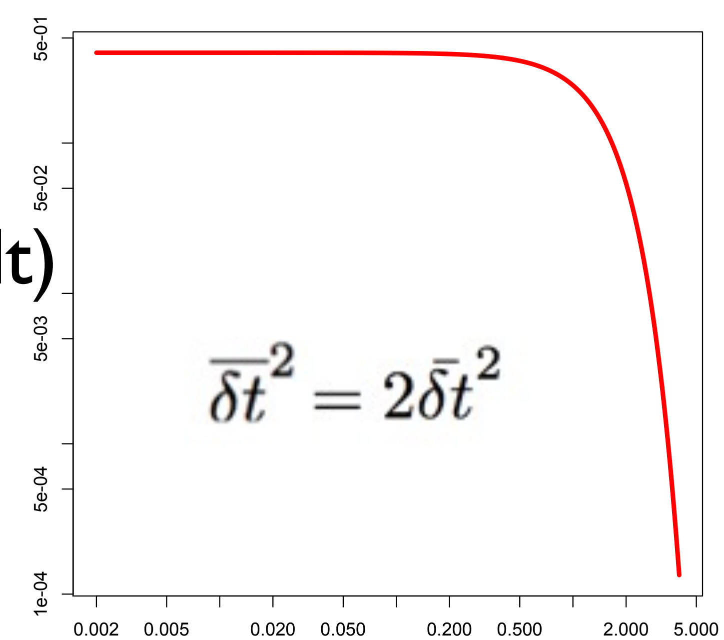
$$\bar{\tau} = \int_0^\infty d\tau \, \tau \, P(\tau) = \frac{\overline{\delta t}}{2} \left(1 + \frac{\sigma^2_{\delta t}}{\overline{\delta t}^2}\right)$$

$$\bar{\tau} = \frac{1}{2} \frac{\overline{\delta t^2}}{\overline{\delta t}}$$



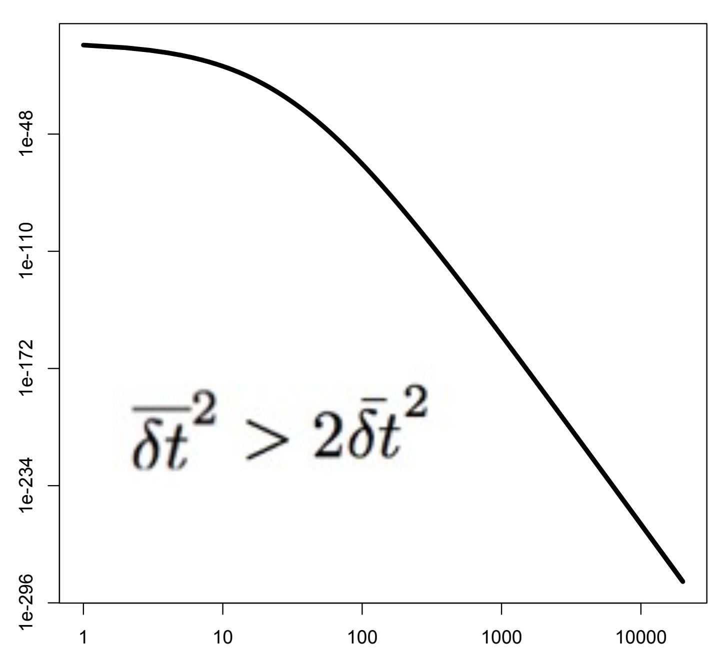
POISSON

Q(dt)



dt

LONG-TAIL



dt

The long-tail of

$Q(\delta t)$

makes the waiting time longer than the Poisson case

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Implications in information spreading

SIR Model on Real Data from Mobile Phone Calls

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07889800622	07776301842	53	145	C
07725326776	07595030724	55	292	C

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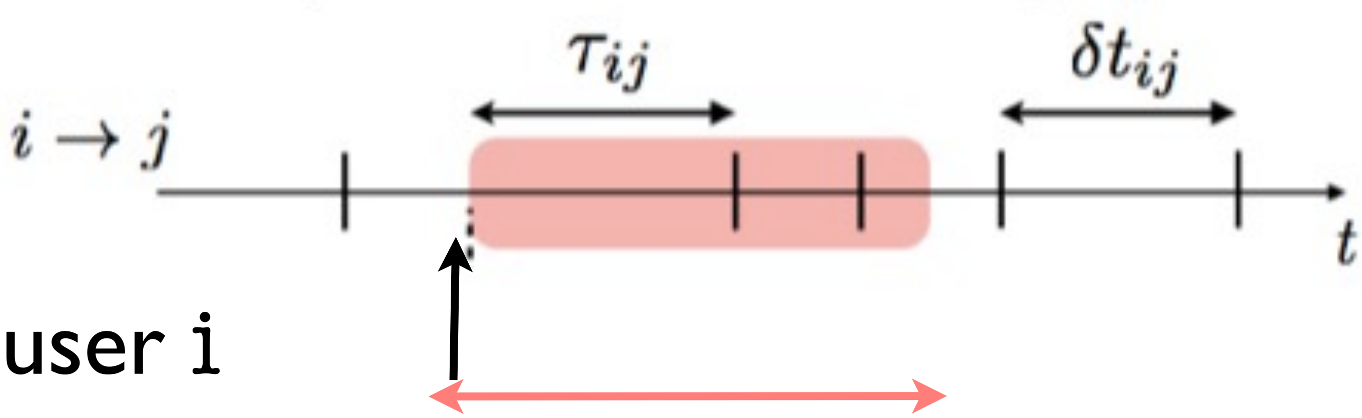
Implications in information spreading

probability that the information
will be transmitted from i to j

Case $\lambda = 1$

M.E.Newman, Phys. Rev. E 66, 16128 (2002)

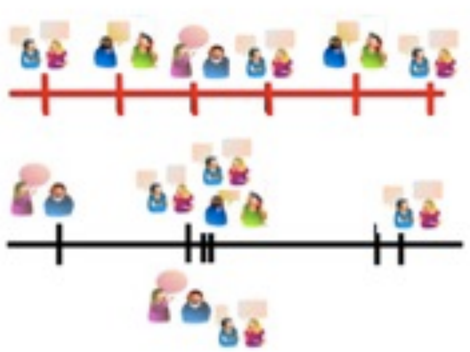


user i

$T = 1/\gamma$

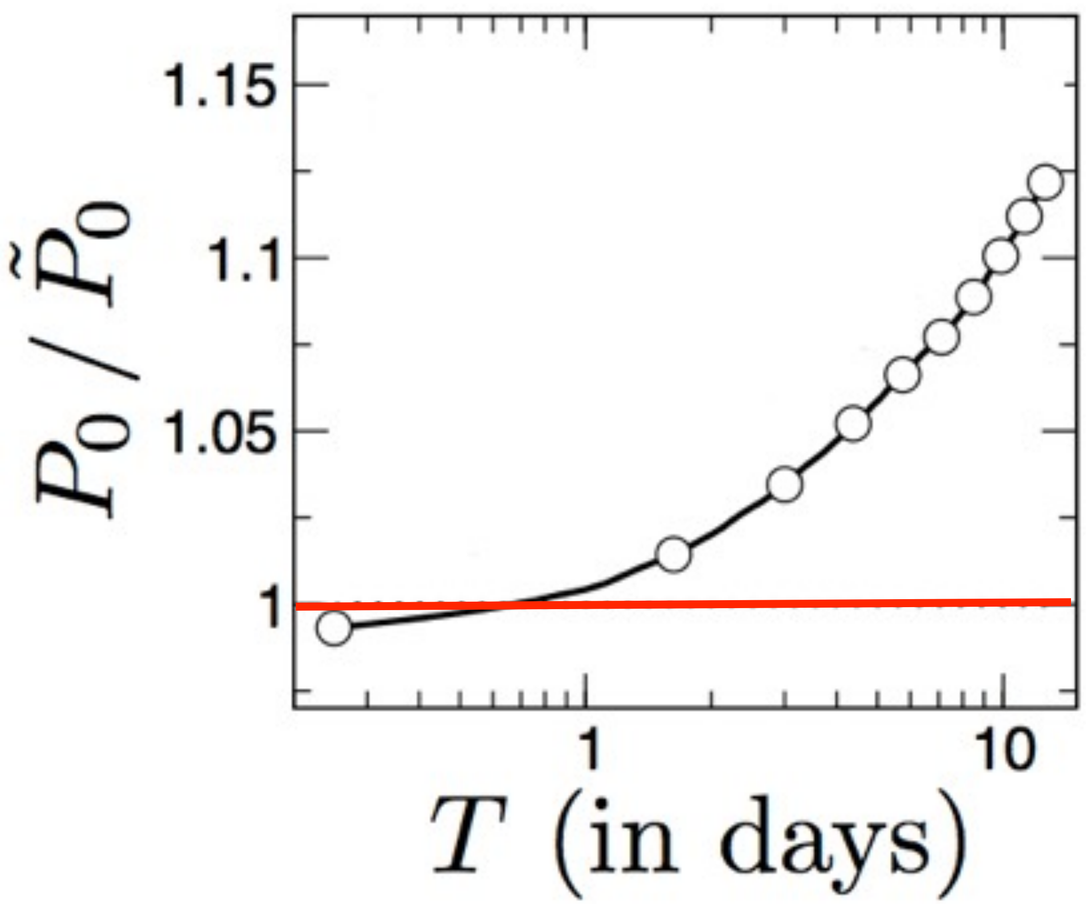
$$\mathcal{T}_{ij} = 1 - P^0_{ij}$$

$$P^0_{ij} = \int_T^\infty P(\tau_{ij})d\tau_{ij}$$



$$P^0_{ij} > \widetilde{P}^0_{ij}$$

$$\mathcal{T}_{ij} < \widetilde{\mathcal{T}}_{ij}$$



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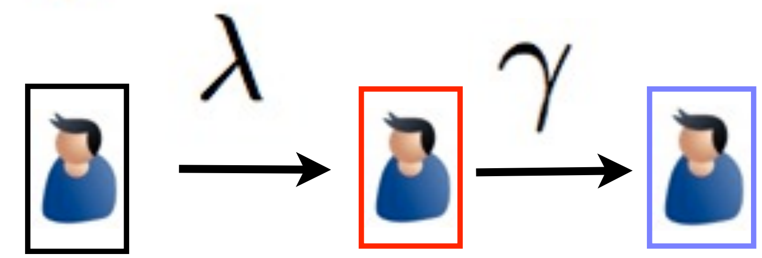
Implications in information spreading

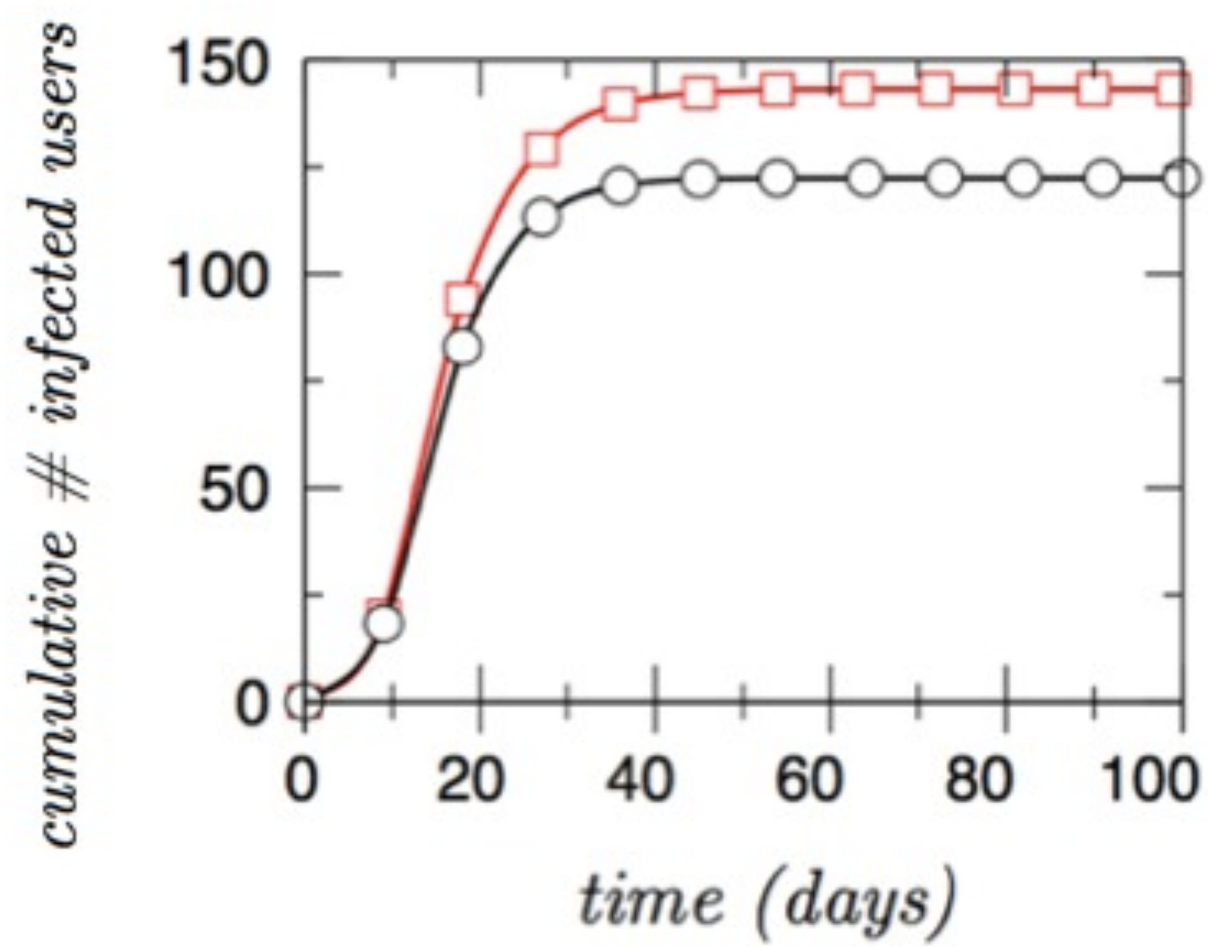
$$P^0_{ij} > \widetilde{P}^0_{ij} \quad \longrightarrow \quad T_{ij} < \widetilde{T}_{ij}$$

SIR MODEL on Real Data



Assumptions

- in any call infection happens with probability $\lambda = 1$
- an infected node recovers after a fixed $T = 1/\gamma$



The bursty real dynamics makes the reach narrower!

Summary

- The way humans communicate is very heterogeneous - bursty behavior - (hurry up and wait)

Barabasi, Bursts: The Hidden Pattern Behind Everything We Do, Dutton Books (2010)

- The observed behavior can be modelled and understood by means of mathematical tools

Daley and Kendall , Nature, vol. 204, no. 4963 (1964)
Murray, Math. Biol, vol.17, Springer, NY (2002)

M.E.Newman, Phys. Rev. E 66, 16128 (2002)

- Heterogeneity affects dynamical real processes e.g. information spreading

Miritello, Moro & Lara, PRE Rapid Comm. (2011)
http://arxiv.org/abs/1011.5367