Covering theorems, generalizations and applications

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Covering theorems are useful tools in real analysis. We will focus on Vitali's and Besicovitch's covering theorems. The first one is valid in the context of doubling spaces and it will allow us to prove some basic results of analysis in these spaces, such as Lebesgue's differentiation theorem, bounds for the Hardy-Littlewood's maximal operator or John-Niremberg inequality for BMO. The Besicovitch's covering lemma doesn't depend on the measure. From this result we deduce Vitali's covering theorem for any Radon measure on ($\mathbf{R}^n$; ||.||). We also study a covering lemma which is equivalent to Lebesgue's differentiation theorem for L^p functions and we introduce gaussian measures in Hilbert spaces, because some of them exemplify that the Vitali property is a sufficient but not necessary condition for the Lebesgue's differentiation theorem.