Constructing new classes of Strong Linearizations of Rational Matrices

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Given a rational matrix $G(\lambda)$, i.e., a matrix whose entries are rational function in a variable λ the rational eigenvalue problem REP is to find scalars x and constant vectors v such that G(x)v=0. A difficulty of REPs with regard to classical and polynomial eigenvalue problems is that could exist poles close to the eigenvalues.

REPs appear in many different applications in Physics and Engineering and numerical experiments have shown that using strong linearizations of $G(\lambda)$ is the best method for solving REPs numerically nowadays.

Strong linearizations of $G(\lambda)$ are linear polynomial matrices whose eigenvalues are the zeros of $G(\lambda)$ and such that the eigenvalues of a principal submatrix of the linearization are the finite poles of $G(\lambda)$.

Any rational matrix $G(\lambda)$ can be uniquely written as $G(\lambda) = D(\lambda) + G_{sp}(\lambda)$ where $D(\lambda)$ is a polynomial matrix and $G_{sp}(\lambda)$ is a strictly proper rational matrix, i.e., the entries of $G_{sp}(\lambda)$ are strictly proper rational functions. In this talk we present new classes of strong linearizations of a rational matrix constructed from carefully combining strong linearizations of its polynomial part and state-space realizations of its strictly proper part. More precisely, we consider strong linearizations of the polynomial part $D(\lambda)$ that belong to the ansatz spaces $\mathbf{M}_1(D)$ and $\mathbf{M}_2(D)$, recently developed by H. Faßbender and P. Saltenberger [arXiv:1609.09493v3, 2017]. Moreover, if the rational matrix is symmetric, we present strong linearizations that preserve its structure.