DEPARTAMENTO DE ÁLGEBRA

GOBIERNO DE ESPAÑA MINISTERIO DE EDUCACIÓN

Instituto de Matemática Interdisciplinar

Seminario

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"A counter-example to the Hirsch conjecture"

ABSTRACT:

The Hirsch conjecture, stated in 1957, said that if a polyhedron is defined by *n* linear inequalities in *d* variables then its combinatorial diameter should be at most *n - d*. That is, it should be possible to travel from any vertex to any other vertex in at most n-d steps (traversing an edge at each step). The unbounded case was disproved by Klee and Walkup in 1967. In this talk I will describe the construction of the first counter-example to the bounded case (a polytope).

The conjecture was posed and is relevant in the context of the simplex method in linear programming. The simplex method, after all, finds the optimal solution by moving from vertex to vertex along the edges of the feasibility polyhedron. Experimentally, the simplex method usually finishes in a linear number of steps, but examples where certain choices of pivot rules lead to exponentially long paths exist, and no pivot rule is known that can be proved to always finish in a polynomial number of steps. Even if other methods for linear programming are proved to be polynomial (Karmakar, Khachiyan), the simplex method remains one of the methods most often used in practice.

From a complexity theory point of view, it is also significant that the known methods are polynomial in the "bit complexity" model, but a polynomial pivot rule for the simplex method would provide a "strongly polynomial" algorithm, that is, one that is polynomial also in the "real machine" model. The question whether such a "strongly polynomial" method exists for linear programming was included by S. Smale in his list of "Mathematical problems for the next century" (AMS, 2000). Of course, a polynomial pivot rule can only exist if the combinatorial diameter is polynomially bounded.

Organizado por el Departamento de Álgebra, el ICMAT y el IMI.

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