DEPARTAMENTO DE ANÁLISIS MATEMÁTICO Y MATEMÁTICA APLICADA

Instituto de Matemática Interdisciplinar

COLLOQUIUM DE ANÁLISIS MATEMÁTICO

Krzysztof Ciesielski

West Virginia University, USA

Differentiability versus continuity: Restriction and extension theorems and monstrous examples

This talk is based on the first part of a recent Bulletin of the American Mathematical Society work jointly written with Juan B. Seoane Sepúlveda. Its aim is to revisit the centuries old discussion on the interrelations between continuous and differentiable functions from R to R. The new angle of this presentation is influenced by a series of very recent results in this research area. This is presented in an narrative that answers two classical questions: (1) To what extend a continuous function must be differentiable? and (2) How strong is the assumption of differentiability of a function? Question (2) will be interpreted as: To what extent the derivative F' of an F : R → R must be continuous? Here we recall some well known properties of the derivatives (large set of points of continuity, Darboux property) as well as newer (e.g., a finite composition of derivatives from I = [0, 1] to I has fixed point property). We will also provide a very easy new construction of everywhere differentiable nowhere monotone map. Concerning question (1) we indicate a simple new proof that for every continuous f : R → R there is a perfect set Q contained in R such that f restrocted to Q is differentiable; discuss Jarník and Whitney differentiable extension theorems; deduce that for every continuous f : R → R there is a C1 map g : R → R such that f ∩ g is uncountable. We will also present a new seemingly paradoxical example a differentiable function F : R → R (which can be nowhere monotone) and of compact perfect X in R such that F'(x) = 0 for all x in X while F[X] = X; thus, the map f = F restricted to X is shrinking at every point while, paradoxically, not globally.

Organizado por el Departamento de Análisis Matemático y Matemática Aplicada y el Instituto de Matemática Interdisciplinar (IMI)

Fecha: Jueves 8 de noviembre de 2018
Hora: 13:00 horas
Lugar: Aula 222
Facultad de CC Matemáticas, UCM