Schedule

	Monday	Tuesday	Wednesday	Thursday	Friday
9:30 10:30	Jean-Pierre Demailly	Jean-Pierre Demailly	Jean-Pierre Demailly	Jean-Pierre Demailly	Jean-Pierre Demailly
10:40 11:10	Almar Kaid	Elena Andreini	10:40 - 11:40 Daniel Huybrechts	Nikola Penev	Lisema Rammea
11:10 11:30	Coffee Break	Coffee Break	11:40 - 12:00 Coffee Break	Coffee Break	Coffee Break
11:30 12:30	Daniel Huybrechts	Daniel Huybrechts	12:00 - 13:00 Angelo Vistoli	Daniel Huybrechts	11:30 - 12:20 Jarod Alper
12:40 13:00	Robert Vollmert	Sylvain Brochard		Filippo Viviani	GAeL
	Lunch break				
15:00 16:00	Angelo Vistoli	Angelo Vistoli	Free afternoon	Angelo Vistoli	Angelo Vistoli
16:10 16:50	Margarida Melo	Evgeny Smirnov		Jacopo Stoppa	16:10 - 16:40 Oliver Petras
16:50 17:10	Tea Break	Tea Break		Tea Break	16:40-17:00 Tea Break
17:10 18:00	Ada Boralevi	Francesco Malaspina		Zahid Raza	17:00 - 17:50 Elisa Tenni
18:00 19:00	Free discussion time	Free discussion time		Free discussion time	18:00 - 18:50 Álvaro Nolla

Géométrie Algébrique en Liberté XVI

Monday 21 - Friday 25 April, 2008

CES Felipe II, Aranjuez, Madrid - Spain

Welcome!

Dear participant,

It is a pleasure to welcome you to the 16th edition of *Géométrie Algébrique en Liberté*, also known as GAeL. As the title suggests, this conference aims to give you an opportunity to discuss algebraic geometry freely amongst other young researchers. Therefore we encourage you to ask questions, no matter how silly you think they might be, just go ahead and ask them.

Apart from the abundance of junior speakers, three senior speakers will each give a mini course. We are very grateful to Jean-Pierre Demailly, Daniel Huybrechts and Angelo Vistoli for agreeing to lecture at GAeL.

This 16th edition of GAeL would not have been possible without our sponsors. We want to thank Foundation Compositio Mathematica, i-MATH Ingenio Mathematica, CES Felipe II, Instituto de Matemática Interdisciplinar, Universidad Complutense de Madrid, Universidad Autónoma de Madrid, Real Sociedad Matemática Española and la Dirección General de Universidades e Investigación de la Consejería de Educación de la Comunidad de Madrid for their generous support.

Last but not least, we thank Frances Kirwan (University of Oxford, United Kingdom) and Frans Oort (Utrecht University, Netherlands), GAeL's scientific committee, for encouraging us and helping us out with good advice.

We hope you will consider this year's GAeL as an interesting conference where you both learned a lot and met new research colleagues.

The organizing committee,

Stephen Coughlan,
Sultan Erdoğan,
Michael Kerber,
Alberto López,
María Pe Pereira,
Sönke Rollenske,
Damiano Testa,
İnan Utku Türkmen and
Tim Wouters

Surface fibrations and their relative canonical algebra
Elisa Tenni (Università degli Studi di Pavia, Italy - University of Warwick, UK)

I will define the relative canonical algebra of a fibred surface and I will show its importance when we are looking for the invariants of the surface.

I will prove that when the genus of the fibre is 5 or 7 the study of the relative canonical algebra leads to an explicit relation between the invariants.

Cohomological support loci for Abel-Prym curves
Filippo Viviani (Humboldt Universität Berlin, Germany)

Cohomological support loci were introduced by Green-Lazarsfeld to prove some generic vanishing theorems for irregular varieties. Later, Pareschi-Popa used them to define a concept of regularity on principally polarized abelian varieties which resembles many of the property of the Castelnuovo-Mumford regularity on projective spaces. As an application, they studied the cohomological support loci of the Abel-Jacobi curve on a Jacobian, obtaining a cohomological characterization of them. In this talk, based on a joint work with S. Casalaina-Martin and M. Lahoz, we consider the analogous problem for the "next class of curves", namely Abel-Prym curves inside Prym varieties.

Torus actions and deformation theory
Robert Vollmert (Freie Universität Berlin, Germany)

I will describe how toric deformations fit into the language of T-varieties and how they may be generalized in this setting. Here, a T-variety is a normal affine variety with an action by a possible lower-dimensional torus T. It has a partially combinatorial description by a polyhedral divisor on a quotient by T. Toric deformation theory allows the construction of some deformations of toric varieties, mostly those with toric total space. Such a deformation corresponds to a decomposition of a polytope as a Minkowski sum.

Schubert decomposition for double Grassmannians

Evgeny Smirnov (Université Joseph Fourier, Grenoble 1, France)

Classical Schubert calculus deals with orbits of a Borel subgroup $B \subset \mathrm{GL}(V)$ acting on a Grassmann variety $\mathrm{Gr}(k, V)$ of k-planes in a finite-dimensional vector space V. These orbits (Schubert cells) and their closures (Schubert varieties) are very well studied both from the combinatorial and the geometric points of view.

One can go one step farther, considering the direct product of two Grassmannians $\mathrm{Gr}(k, V) \times \mathrm{Gr}(l, V)$ and the Borel subgroup $B \subset \mathrm{GL}(V)$ acting diagonally on this variety. In this case, the number of orbits still remains finite, but their combinatorics and geometry of their closures become much more involved. It would be challenging to extend the whole body of the Schubert calculus to this situation.

I will explain how to index the B-orbit closures in $\mathrm{Gr}(k, V) \times \mathrm{Gr}(l, V)$ combinatorially, describe the inclusion relations between them, and construct their desingularizations, which are analogous to Bott–Samelson desingularizations for ordinary Schubert varieties. If time allows, I will also try to discuss the relations of this situation with geometry of quiver representations; these relations were recently found by Bobinski and Zwara.

Scalar curvature and polystability

Jacopo Stoppa (Università degli Studi di Pavia, Italy - Imperial College London, UK)

S. Donaldson proved that a polarised manifold admitting a constant scalar curvature Kaehler metric is algebraically K-semistable. But the standard analogy with HYM bundles suggests that this can be strengthened to algebraic K-polystability. We present some work in progress in this direction.

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Topology of Plane curves and Disrciminants

Zahid Raza (Abdus Salam School of Mathematical Sciences, GCU Lahore Pakistan)

We explore the relation between the topology of the fibers of a polynomial in two complex variables and the degree of the associated discriminant. This gives, in particular, lower and upper bounds for this degree, and the polynomials realizing these bounds, or even close values, can be described geometrically.

References

[1] E. Artal Bartolo, P. Cassou-Nogues, I. Luengo Velasco, *On polynomials whose fibers are irreducible with no critical points*, Math. Ann. **299** (1994), 477-490.

[2] P. Cassou-Nogues, A. Dimca: *Topology of complex polynomials via polar curves*, Kodai Math. J. **22** (1999),131–139.

[3] A. Dimca, L. Paunescu *On the connectivity of complex affine hypersurfaces II*, Topology **39** (2000) 1035-1043.

[4] A.H. Durfee, *Five definitions of Critical Points at Infinity*, Singularities: the Brieskorn Anniversary Volume, Progress in Mathematics **162**.

Functional equations for polylogarithms in motivic cohomology

Oliver Petras (Johannes-Gutenberg Universität Mainz, Germany)

For an infinite field F, we study the integral relationship between the Bloch group $B_2(F)$ and the higher Chow group $CH^2(F,3)$ by proving some relations corresponding to the functional equations of the dilogarithm. The groups involved in Suslin's exact sequence

$$0 \to Tor_1^{MZ}(F_s, F_s)^{\sim} \to CH^2(F,3) \to B_2(F) \to 0$$

are identified with homology groups of the cycle complex $Z^2(F,\bullet)$ computing higher Chow groups.

Using these results, we can give explicit cycles in motivic cohomology generating the integral motivic cohomology groups of number fields and determine whether a given cycle in the Chow group already lives in one of the other subgroups of Suslin's sequence.

At last, we give some indications about similar computations in codimension three.

Non-General Type Surface in Weighted $\mathbb{P}^4$

Lisema Rammea (University of Bath, UK)

Looking at a particular example carefully chosen so that the weights are pairwise coprime, we discuss construction of smooth surfaces not of general type in weighted four dimensional projective space. Using our example we shall show how to write the Riemann-Roch formula and application of Beilinson (1978) Theorem.

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Group picture of GAeL XV at Sabancı Üniversitesi, İstanbul
(June 18th - June 22nd, 2007)

Around the McKay correspondence with G-Hilb
Álvaro Nolla de Celis (University of Warwick, UK)

I this talk I will give an introduction to the McKay correspondence established in the early 80's and explain various ways of looking at it, giving special attention to the G-Hilb approach. Since is the last talk of the conference, to entertain the audience I promise lots of pictures and examples, which I hope will show the beauty of this correspondence.

Perfect stratifications and the Chow ring of $\mathcal{M}_g$ for $3 \leq g \leq 6$
Nikola Penev (Stanford University, USA)

Finding the rational Chow ring $\mathcal{A}^*_{\mathbb{Q}}(\mathcal{M}_g)$ of the coarse moduli space of smooth genus g curves is, in general, a very difficult task. For small genera however, one can fully describe the ring: for $g = 3, 4, 5$ one can prove that $\mathcal{A}^*_{\mathbb{Q}}(\mathcal{M}_g) \simeq \mathbb{Q}[\lambda_1]/(\lambda_1^{g-1})$ where λ_1 is the first Chern class of the Hodge bundle as proved in the 1990's by Faber for $g = 3, 4$ and Izadi for $g = 5$.

In a recent preprint Fontanari and Looijenga give significantly shorter proofs of the same results by presenting $\mathcal{M}_g$ as a disjoint union of affine subvarieties with trivial rational Chow ring, one for each codimension $\leq g - 2$. Each stratum has a nice geometric construction: for example in genus 4 one has $\mathcal{M}_4 \supset \mathcal{M}'_4 \supset \mathcal{H}_4$ where $\mathcal{M}_4 \setminus \mathcal{M}'_4$ is the set of curves whose canonical model is a complete intersection of a smooth quartic and a cubic and $\mathcal{M}'_4 \setminus \mathcal{H}_4$ is the set of curves whose canonical model is a complete intersection of a cone and a cubic while $\mathcal{H}_4$ is the set of hyperelliptic curves.

In my talk I will introduce the general strategy, discuss briefly Fontanari and Looijenga's stratifications and give idea how one can find a similar, though slightly more complicated stratification of $\mathcal{M}_6$ and potentially use it to describe the rational chow ring of that moduli space.

Regularity and Splitting Criteria for Vector Bundles on Projective Varieties
Francesco Malaspina (Politecnico di Torino, Italy)

A classical result by Horrocks characterizes the vector bundles without intermediate cohomology on a projective space as direct sum of line bundles. A very simple proof of this criterion uses Castelnuovo-Mumford regularity.

It was soon clear that Mumfords definition of Castelnuovo-Mumford regularity was a key notion and a fundamental tool in many areas of algebraic geometry and commutative algebra. Several extensions of this notion were proposed to handle different situations.

In this talk we will introduce suitable denitions of regularity on quadric hypersurfaces, multiprojective spaces and Grassmannians of lines in order to obtain splitting criteria for vector bundles.

Compactified Picard stacks over $\bar{\mathcal{M}}_g$.
Margarida Melo (Università degli Studi Roma Tre, Italy)

Let X be a projective curve of genus g. The generalized jacobian of X, $J(X)$, parameterizing isomorphism classes of invertible sheaves on X having degree 0 on every irreducible component of X, is projective if and only if X is a curve of compact type. There are several constructions of compactifications of $J(X)$, differing from one another in various aspects such as the geometric interpretation or the functorial properties.

In this talk we will explain how to construct geometrically meaningful algebraic (Artin) stacks $\bar{\mathcal{P}}_{d,g}$ over the moduli stack of stable curves, $\bar{\mathcal{M}}_g$, giving a functorial way of compactifying the relative degree d Picard variety for families of stable curves. The functorial property of these stacks consists on the fact that, giving a family of stable curves $f : \mathcal{X} \to S$, the fiber product of its moduli map onto $\overline{M}_g$ by $\bar{\mathcal{P}}_{d,g}$ is either isomorphic to Caporaso's compactification of the degree d Picard variety of $\mathcal{X}$ over S or has a canonical map onto it.

Differential equations and hyperbolicity of algebraic varieties
Jean-Pierre Demailly (Université de Grenoble I, France)

The goal of the lectures is to provide an introduction to the theory of hyperbolic varieties, which are interesting for their geometric properties (e.g., partially negative curvature) as well as for their conjectured diophantine properties.

A complex variety is said to be hyperbolic in the sense of Kobayashi if the holomorphic mappings from the unit disk sending the origin to any given point form a normal family. In the compact case - actually our main interest is the case of projective varieties - this is equivalent to the non existence of non constant entire holomorphic curves defined on the whole complex line, and in particular this property implies the non existence of rational or elliptic curves.

Although hyperbolicity looks like an analytic property, it is expected that it can be expressed in purely algebraic terms, at least as far as projective varieties are concerned. One of the deepest conjectures states that hyperbolicity should be equivalent to the fact that all subvarieties are of general type. On the other hand, one of the main tools is the study of algebraic differential equations, namely sections of certain jet bundles and their cohomology.

In the last two decades various new techniques have been developped, which borrow tools from quite central parts of algebraic geometry : vanishing / non vanishing theorems, Riemann Roch calculations, Morse inequalities, meromorphic connections, meromorphic vector fields ... We will try to present a fair amount of these fundamental techniques.

References

- A good reference is the recent survey written by Erwan Rousseau (*http://arxiv.org/abs/0709.3882* - however the notes are written in French, so a few people might find this unsuitable.)
- Older (very detailed, still useful) notes of mine are on my web page http://www-fourier.ujf-grenoble.fr/ demailly/research.html

  see especially :

  [50] *Algebraic criteria for Kobayashi hyperbolic projective varieties and jet differentials*, Proceedings of Symposia in Pure Math., Vol. **62.2** (AMS Summer Institute on Algebraic Geometry, Santa Cruz, July 1995), ed. J. Kollar, R. Lazarsfeld, (1997), 285-360 [hyperbolic.pdf]

  [55] (in collaboration avec J. El Goul) *Hyperbolicity of generic surfaces of high degree in projective 3-space*, math.AG/9804129, Amer. J. Math. **122** (2000) 515-546. [hyp_generic.pdf]
- See also the recent papers of my PhD student Simone Diverio :
  - *Differential Equations on Complex Projective Hypersurfaces of Low Dimension*, arXiv:0706.1031,
  - *Existence of Global Invariant Jet Differentials on Projective Hypersurfaces of High Degree*, arXiv:0802.0045.

Algebraic stacks and the Picard functor

Sylvain Brochard (Université de Versailles, France)

The Picard functor of a scheme, classifying invertible sheaves on it, has been studied extensively in the 60's. However, the work of Giraud, Deligne, Mumford and Artin gave birth in the 70's to the notion of an *algebraic stack*, which generalizes that of a scheme. The following question arises then: does the Picard functor of an algebraic stack behave like that of a scheme?

We will explain how the study of deformations of invertible sheaves allows us to prove that the Picard functor is representable by an algebraic space (under suitable hypothesis). We will also try to see, through a few examples, what is the effect on the Picard scheme, when we modify a scheme by adding some "stacky structure".

Topology of Plane curves and Disrciminants

Almar Kaid (University of Sheffield, UK)

Let R be an integral domain of finite type over $\mathbb{Z}$ and let $f : X \to Spec\, R$ be a smooth projective morphism of relative dimension $d \geq 1$. We investigate, for a vector bundle E on the total space X, under what arithmetical properties of a sequence $(p_n, e_n)_{n\in\mathbb{N}}$, consisting of closed points p_n in $Spec\, R$ and Frobenius descent data $E_{p_n} \cong F^{e_n *}(F)$ on the closed fibers X_{p_n}, the bundle E_0 on the generic fiber X_0 is semistable.

Good moduli spaces for Artin stacks
Jarod Alper (Stanford University, USA)

I will develop an intrinsic theory for associating schemes or algebraic spaces with nice geometric properties to arbitrary Artin stacks. This theory offers a stack-theoretic approach to geometric invariant theory. I will define the notion of a good moduli space which simultaneously generalizes the existing notions of good GIT quotients and tame stacks. I will give the fundamental properties of good moduli spaces and discuss applications to the geometry of certain moduli spaces.

Quantum Cohomology of Root Gerbes
Elena Andreini (Universität Zürich, Switzerland)

I will give a short introduction to quantum cohomology and to gerbes over smooth projective varieties. For gerbes which are obtained as roots of line bundles I will explain how to compute the genus zero orbifold Gromov-Witten theory in terms of the genus zero Gromov-Witten theory of the base variety.

Homogeneous vector bundles on flag manifolds and quiver representations
Ada Boralevi (Università degli studi di Firenze, Italy)

The category of homogeneous vector bundles on a flag manifold $X = G/P$ of ADE-type is equivalent to the category of representations of a certain quiver Q_X with relations. This equivalence was found in some cases by Bondal, Kapranov and Hille. In the particular case of Hermitian symmetric varieties, Ottaviani and Rubei used this equivalence as a tool to compute the cohomology of homogeneous vector bundles, thus generalizing the well-known Bott Theorem.

I will show results holding in the general case. If time permits, I will also show an interesting application to simplicity of tangent bundles.

K3 surfaces: Cycles, Chow groups, and derived categories.
Daniel Huybrechts (Universität Bonn, Germany)

In these lectures I will try to approach the geometry of K3 surfaces by using three fundamental invariants: Chow groups, Hodge structures, derived categories.

The cohomology of a K3 surface together with the intersection pairing and the Hodge structure is an important invariant. The non-algebraic part, the transcendental lattice, is a weight two Hodge structure whose automorphisms (over $\mathbb{Q}$) form a field of a very particular type. See Zarhin: *Hodge group of K3 surface*. J. Reine Angew. Math. **341** (1983), 193-220.

The Chow group of a K3 surface has 'infinite rank' (result of Mumford), but contains a natural subring studied by Beauville and Voisin in:
On the Chow ring of a K3 surface. J. Algebraic Geom. **13** (2004), 417-426.

The bounded derived category of the abelian category of coherent sheaves is a triangulated category which in the case of a K3 surface has an interesting group of autoequivalences. I will discuss examples of such autoequivalences and explain how they act on cohomology.
See Mukai's paper: *On the moduli space of bundles on K3 surfaces*, in Vector bundles on algebraic varieties. Bombay (1984)
and my book *Fourier–Mukai transforms in algebraic geometry*, OUP.

Gerbes and Essential Dimension
Angelo Vistoli (Scuola Normale Superiore Pisa, Italy)

A basic invariant of an algebraic group is its *essential dimension*, defined by Buhler and Reichstein in [BR97] (see also [Rei00]). Subsequently the notion of essential dimension has been generalized to a much wide context by Merkurjev ([BF03]).

Fix a base field k, which we always going to assume to have characteristic 0. Suppose that ξ is a geometric, or algebraic object, defined over a field extension K of k. A basic question that one can ask is the following: how many paramaters does one need to write ξ?

Here is the precise definition. Let k be a field. We will write Fields_k for the category of field extensions K/k. Let $F\colon \mathrm{Fields}_k \to \mathrm{Sets}$ be a covariant functor.

Definition (Merkurjev)**.** *Let $\xi \in F(K)$, where K is an object of* Fields_k. *A* field of definition *for ξ is an intermediate field $k \subseteq L \subseteq K$ such that ξ is in the image of the induced function $F(L) \to F(K)$.*

The essential dimension $\mathrm{ed}\,\xi$ *of ξ (with respect to L) is the minimum of the transcendence degrees* $\mathrm{tr\,deg}_k L$ *taken over all fields of definition of ξ.*

The essential dimension $\mathrm{ed}\,F$ of the functor F *is the supremum of* $\mathrm{ed}\,\xi$ *taken over all $\xi \in F(K)$ with K in* Fields_k.

When G is an algebraic group over k, the essential dimension of G is defined to be the essential dimension of the functor that associates to K the set of isomorphism classes of G-torsors over K. For example, when G is the orthogonal group O_n, the G-torsors correspond to non-degenerate n-dimensional quadratic forms over K; hence the essential dimension of O_n is the exact upper bound for the number of independent parameters needed to write an arbitrary non-degenerate n-dimensional quadratic form, up to isomorphism. Since every such form can be written in the diagonal form $a_1x_1^2 + \cdots + a_nx_n^2$, the essential dimension of O_n is at most n. It is a non-trivial fact, first shown by Reichstein, that the essential dimension of O_n is exactly n.

So far almost all of the work on essential dimension has been about essential dimension of algebraic groups. However, Merkurjev's more general setup generates very natural questions. For example, given a non-negative integer g, what is the essential dimension of the functor of isomorphism classes of smooth curves of genus g over $\mathbb{Q}$, or over $\mathbb{C}$? For example, when $g = 0$, every smooth curve of genus 0 over a field K is a conic, and every conic is isomorphic to one with equation $ax^2 + by^2 + z^2 = 0$, hence its essential dimension is at most 2. It follows from Tsen's theorem that the essential dimension of a the generic conic, that of equation $ax^2 + by^2 + z^2 = 0$, where a and b are independent variables, is in fact 2.

The question is best viewed from the point of view of algebraic stacks. Brosnan, Reichstein and myself have introduced some machinery for studying the essential dimension of algebraic stacks in [BRV07]. A key point is the study of essential dimension of gerbes (gerbes may be though of as point-like algebraic stacks). Using this, we have answer the question above completely for arbitrary genus g. The answer is the following.

Theorem. *Let k be an arbitrary field of characteristic* 0. *The essential dimension of the functor of isomorphism classes of smooth curves of genus g over extensions of k is*

$$\begin{cases} 2 & \text{for } g = 0, \\ +\infty & \text{for } g = 1, \\ 5 & \text{for } g = 2, \\ 3g-3 & \text{for } g \geq 3. \end{cases}$$

This theory is also useful in the "classical" case of algebraic dimension of algebraic groups: for example, we have been able to show the surprising result that the essential dimension of the group Spin_n grows exponentially with n.

In my lectures I will give an introduction to the theory sketched above, assuming no familiarity with algebraic stacks. I will also give a self-contained treatment of gerbes.

References

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[BRV07] Patrick Brosnan, Zinovy Reichstein, and Angelo Vistoli, *Essential dimension and algebraic stacks*, arXiv: math.AG/0701903, 2007.

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