# Non linear and non local problems: from the theory to the applications

Seminario Internacional Complutense

Madrid, 6-8 February 2008

This Meeting will gather national and international experts together on the study of applied problems whose modelization involves nonlinear integrodifferential equations. Such systems appear in a wide range of applications from Mathematics and Physics to Biology and Engineering. In fact, most of the physical dynamics are implicitly non linear by nature and, often, non local due to some temporal or spatial long-range memory and interactions.

The conference programme includes a number of plenary talks and several short communications. The participation will be free and open to all interested researchers and students.

All lectures will be held at Facultad de CC. Matemáticas of Universidad Complutense de Madrid, "Aula Miguel de Guzmán" S-118 (Floor $-1$).

# PLANNED SCHEDULE

## Wednesday February 6

**11:30 Arrival and Opening Ceremony**

**12:00 Lecture: G. Zaslavsky** . Courant Institute, New York University, New York, USA.
*"Origin of fractional dynamics in systems with long-range memory and interaction"*.

**12:50 Lecture: J.I. Díaz**. Universidad Complutense de Madrid, Spain.
*"Global controls to stabilize the chemical turbulence: a non local complex Ginzburg-Landau equation"*.

**13:40 Lunch**

**15:10 Lecture: J. M. Mazón**. Universitat de Valencia, Spain.
*"The limit as $p \to \infty$ in a nonlocal $p$-Laplacian evolution equation. A non local approximation of a model for sand piles"*.

**16:00 Lecture: R. Vilela Mendes**. Technical University Lisbon, Portugal.
*"Stochastic solutions of nonlinear partial differential equations"*.

**16:50 Coffee Break**

**17:30 Communication: T. Pierantozzi**. Universidad Complutense de Madrid, Spain.
*"On the finite time extinction phenomenon for some nonlinear fractional evolution equations"*.

**18:00 Communication: N. Alibaud**. Institut de Mathematiques et de Modelisation de Montpellier, France.
*"Fractional semi-linear parabolic equations with unbounded data"*.

# Thursday February 7

**10:40 Communication: S. Antontsev**. CMAF, University of Lisbon, Portugal.
*"Finite speed of propagation for non local viscoelastic medium"*.

**11:15 Coffee Break**

**12:00 Lecture: S. Bonaccorsi**. Universitá di Trento, Italy.
*"Volterra integro-differential equations with completely monotone kernels"*.

**12:50 Lecture: H. Gómez Díaz**. Universidad de A Coruña, Spain, and University of Texas at Austin, USA.
*"Non-local phase-field models in science and engineering: from the Cahn-Hilliard equation to strain-gradient hyperelasticity"*.

**13:40 Lunch**

**15:10 Lecture: J. F. Padial**. Universidad Politécnica de Madrid, Spain.
*"Some non local problems arising in the mathematical modelling of the nuclear fusion"*.

**16:00 Lecture: J. Trujillo**. Universidad de La Laguna, Tenerife, Spain.
*"On Fast Fractional Fourier Transform and open problems"*.

**16:50 Coffee Break**

**17:30 Communication: E. Cuesta**. Universidad de Valladolid, Spain.
*"Runge-Kutta convolution quadrature methods for well-posed equations with memory"*.

**18:00 Communication: D. Usero**. Universidad Complutense de Madrid, Spain.
*"Non-Local Model for Nonlinear Dark Solitary Waves"*.

# Friday February 8

**10:00 Communication: M. Sauvageout**. Université Paris 6, Laboratoire Jacques-Louis Lions, Paris, France.
*"Euler's best column: a non linear and non local reformulation"*.

**10:30 Lecture: B. Vinagre**. Universidad de Extremadura, Spain.
*"Some challenges in modelling and control of fractional dynamic systems. An engineering approach"*.

**11:20 Coffee Break**

**12:00 Lecture: S. Salsa**. Politecnico di Milano, Italy.
*"Obstacle problem for the fractional Laplacian"*.

**12:50 Lecture: D. Cordoba**. IMAFF-Instituto de Matemática Aplicada y Física Fundamental, CSIC, Spain.
*"On the existence of solutions of the Surface Quasi-geostrophic equation"*.

**13:40 Lunch**

**15:10 Communication: P. J. Miana**. Universidad de Zaragoza, Spain.
*"Hermite Matrix-Valued Functions Associated to Matrix Differential Equations"*.

**15:40 Communication: Yandong Jiao**. Academy of Mathematics & Systems Science, Chinese Academy of Sciences, China, and UCM, Spain.
*"Symmetric Hamiltonian Algorithms with Application to Nonlinear Schrödinger System"*.

**16:00 Communication: Quandong Feng**. Academy of Mathematics & Systems Science, Chinese Academy of Sciences, China, and UCM, Spain.
*"Implementing Arbitrarily High-Order Symplectic Methods via Krylov Deferred Correction Technique"*.

**16:20 Lecture: L. Vázquez**. Universidad Complutense de Madrid, Spain.
*"From the Nonlocal Problems to Fractional Differential Equations"*.

**17:00 Closing Ceremony**

# Abstracts of Contributions

## Fractional semi-linear parabolic equations with unbounded data

N. Alibaud and C. Imbert

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We investigate the study of semi-linear parabolic equations whose principal term is fractional, *i.e.* is integral and eventually singular. A typical example is the fractional Laplace operator. We present an original case: if the associated first order Hamilton-Jacobi equation is such that perturbations propagate at finite speed, then the semi-linear parabolic equation somehow keeps memory of this property. By using such a result, locally bounded initial data that are merely integrable at infinity can be handled. Next, we discuss regularity of the solution and, eventually, strong convergence of gradients as the fractional term disappears for strictly convex non-linearity.

## Finite speed of propagation for non local viscoelastic medium

S. Antontsev

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The finite speed of propagation (FSP) was established for certain viscoelastic materials at the 70's by the american school (Gurtin, Dafermos, Nohel, etc.) for the special case of the presence of memory effects. A different approach can be applied by the construction of suitable super and subsolutions (Crandall, Nohel, Díaz-Gómez, etc.). In this talk we present an alternative method to prove (FSP), which only uses some energy estimates, following the presentation made in the book Antontsev- Díaz-Shmarev (Birkhäuser 2002).

## Volterra integro-differential equations with completely monotone kernels

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We consider a linear abstract Volterra integrodifferential equation in a Hilbert space, perturbed by a random forcing term. The equation involves a completely monotone convolution kernel with a singularity at $t = 0$ and a sectorial linear spatial operator. Existence and uniqueness of a weak solution is established. Our method consists in a state space setting so that the corresponding solution process is Markovian, and the tools of linear analytic semigroup theory can be utilized. In particular we analyze long time behaviour of the solution. The talk is based on a joint paper with W. Desch (University of Graz, Austria).

## On the existence of solutions of the surface quasi-geostrophic equation

D. Cordoba

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The 2D inviscid surface quasi-geostrophic equation serves as a lower dimensional model of the 3D Euler equations in additional to its geophysical relevance. Recently there has been considerable renewed interests and important progress on various topics related to the fundamental issue of global existence and smoothness. In this talk I will give a survey of results on the well-posedness of the surface quasi-geostrophic equations.

## Runge–Kutta convolution quadrature methods for well–posed equations with memory

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In this work Runge–Kutta convolution quadrature methods are considered for the time discretization of well–posed, linear, and homogeneous Volterra equations.
Our first contribution is the formulation of the Runge–Kutta convolution quadratures in a general framework extending the formulation given by Lubich and Ostermann in the framework of the sectorial operators. We also formulate and study numerical methods based on these quadratures for the Volterra equations mentioned above.
The second contribution is a representation of the numerical solution obtained as an integral of the continuous solution respect to a measure which does not depend on the equation but only on the Runge–Kutta methods considered.
Finally the error and stability analysis of the numerical method is based on the mentioned representation, and, in the particular case of the backward Euler method, it can be shown that numerical solution inherits some properties of the continuous one such as the positivity or the contractivity.

## Global controls to stabilize the chemical turbulence: a non local complex Ginzburg-Landau equation

J.I. Díaz

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We show how to stabilize the uniform oscillations of the complex Ginzburg-Landau equation with periodic boundary conditions by means of some global delayed feedback. The proof is based on an abstract pseudo-linearization principle and a careful study of the spectrum of the linearized operator.

## Implementing Arbitrarily High-Order Symplectic Methods via Krylov Deferred Correction Technique

Q.D. Feng, J.F. Huang, Y.F. Tang

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An efficient numerical procedure is presented to implement the Gaussian Runge-Kutta (GRK) methods, which are symplectic and symmetric. When applying these methods to systems of ODE, since the unknowns at different collocation points are coupled in the discretized system, direct solution of the resulting algebraic equations is in general inefficient. Instead, we use the Krylov deferred correction (KDC) method in which the spectral deferred correction (SDC) scheme is applied as a preconditioner to decouple the original system, and the resulting preconditioned nonlinear system is solved efficiently using Newton-Krylov schemes such as Newton-GMRES method. Numerical results show the accuracy, stability, and efficiency of these methods, especially for Hamiltonian systems of high nonlinearity.

## Non-local phase-field models in science and engineering: from the Cahn-Hilliard equation to strain-gradient hyperelasticity

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Phase-field methodology has emerged as a powerful tool for modeling the evolution of microstructures and the interactions of defects in a wide range of materials and physical processes.
Phase-field methodology typically leads to partial-differential equations of higher order. For example, the Cahn-Hilliard equation, involves fourth-order spatial partial differential operators. In the context of finite element methods, fourth-order operators necessitate basis functions that are piecewise smooth and globally C1-continuous. There are a very limited number of two-dimensional finite elements possessing C1-continuity applicable to complex geometries, but none in three-dimensions. As a result, a number of different devices have been employed over the years to deal with higher-order operators. All represent theoretical and computational complexities of one degree or another. It may be said that after 50 years of finite element research, no general, elegant and efficient solution of the higher-order operator problem exists.
Recently, a new methodology, isogeometric analysis, has been introduced into computational mechanics that may open the door to the approximation of higher-order operators in complex geometries. In this talk, I will present our initial efforts at applying isogeometric analysis to phase-field models. Most of our work has focused on the Cahn- Hilliard equation, but preliminary results for a strain-gradient hyperelasticity theory and the Navier-Stokes-Korteweg system for liquid-vapor flows are also presented. I will go into the details of the physics of those models.

## Symmetric Hamiltonian Methods for the Nonlinear Schrödinger Equation

Y.D. Jiao, Q.D. Feng, Y.F. Tang

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Using several kinds of coordinate transformations, we standardize the noncanonical symplectic structure of the Ablowitz-Ladik model (**A-L model**) of Nonlinear Schrödinger Equation (**NLSE**), then we employ some symplectic scheme to simulate the solitons motion and test the evolution of the discrete invariants of the **A-L model** and also the conserved quantities of the original **NLSE**. In comparison with a higher-order non-symplectic scheme applied directly to the **A-L model**, we show the overwhelming superiorities of the symplectic method. We also compare the implementation of the same symplectic scheme to different standardized Hamiltonian systems resulting from different coordinate transformations, and show that the symmetric coordinate transformation improves the numerical results obtained via the asymmetric one, in preserving the invariants of the **A-L model** and the original **NLSE**.

## Hermite Matrix-Valued Functions Associated to Matrix Differential Equations

P. J. Miana

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Some sequences of matrix polynomials have been introduced recently as solutions of certain second-order differential equations, which can be seen as appropriate generalizations, to the matrix setting, of classical orthogonal polynomials. In this talk, we consider families (in a complex parameter) of matrix-valued special functions of Hermite type, which arise as natural extensions of the aforementioned matrix polynomials of the same type. We show that such families are solutions of corresponding differential equations and enjoy several structural properties. In particular, they satisfy a Rodrigues'formula expressed in terms of the Weyl fractional calculus. We also show that, unlike the scalar case, a second-order differential operator having such a family as a set of joint eigenfunctions need not be unique.

## The limit as $p \to \infty$ in a nonlocal $p-$Laplacian evolution equation. A nonlocal approximation of a model for sand piles

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We study the nonlocal $\infty-$Laplacian type diffusion equation, obtained as the limit as $p \to \infty$ of solutions to the nonlocal analogous to the $p-$Laplacian evolution,

$$u_t(t,x) = \int_N J(x-y)|u(t,y)-u(t,x)|^{p-2}(u(t,y)-u(t,x))\,dy.$$

We prove existence and uniqueness of a limit solution that verifies an equation governed by the subdifferential of a convex energy functional associated to the indicator function of the set $K = \{u : |u(x)-u(y)| \leq 1, \text{ when } x-y \in supp(J)\}$. We also find some explicit examples of solutions to the limit equation.
If the kernel $J$ is rescaled in an appropriate way, we show that the solutions to the corresponding nonlocal problems converge strongly in $L^\infty(0,T;L^2(\Omega))$ to the limit solution of the local evolutions of the $p-$laplacian, $v_t = \Delta_p v$. This last limit problem has been proposed as a model to describe the formation of a sandpile.
Moreover, we also analyze the collapse of the initial condition when it does not belong to $K$ by means of a suitable rescale of the solution that describes the initial layer that appears for $p$ large. Finally, we give an interpretation of the limit problem in terms of Monge-Kantorovich mass transport theory.

## Some non local problems arising in the mathematical modelling of the nuclear fusion

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We carry out mathematical analysis of some bidimensional non linear problems satisfied by the averaged poloidal flux of the magnetic field in the magnetic confinement of a plasma in the nuclear fusion. We show some mathematical models related to the stationary and evolution regime of a plasma in Tokamak and Stellarotor devices. The models can be formulated as an inverse problems since several nonlinear terms of the partial differential equation are not a priori known (non local terms). Using the current balance within each flux magnetic and the notion of relative rearrangement we can reformulate the problem as a non local one but having a direct formulation. We introduce the notions of relative rearrangement and some properties. Finally, by using a Galerkin type method, we prove the existence of solution for a model which can be expressed as a non local inverse thin obstacle problem.

## On the finite time extinction phenomenon for some non linear fractional evolution equations

J.I.Díaz, T.Pierantozzi and L.Vázquez

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The finite time extinction phenomenon (the solution reaches an equilibrium after a finite time) is peculiar to certain nonlinear problems whose solutions exhibit an asymptotic behavior entirely different from the typical behavior of solutions associated to linear problems. The main goal of this work is twofold. Firstly, we extend some of the results known in the literature to the case in which the ordinary time derivative is considered jointly with a fractional time differentiation. Secondly, we consider the limit case when only the fractional derivative remains. The latter is the most extraordinary case, since we prove that the finite time extinction phenomenon still appears, even with a non-smooth profile near the extinction time. Some concrete examples of quasi-linear partial differential operators are proposed. The results can also be applied in the framework of suitable nonlinear Volterra integro-differential equations.

## Obstacle problem for the fractional Laplacian

S. Salsa

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We consider the obstacle problem for the fractional Laplace operator and related problems (Signorini type problems). We present some recent results on optimal regularity of the solution and the analysis of the free boundary (joint papers with Athanasopoulos, Caffarelli and Silvestre).

## Euler's best column: a non linear and non local reformulation

M. Sauvageot

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In 1757, Leonhart Euler asked what would be the maximum of height of a stable column for a prescribed volume of material, and what would be the corresponding shape. Since then many authors have contributed to the subject, such as J.B. Keller, but the problem is not yet completely solved. In a joint work with J.I. Díaz (Univ. Complutense Madrid), we show that, when the load at the top of the column is large enough, the problem as reformulated by J.B. Keller and F.I. Niordson admits a solution, then that this solution is unique and that it provides actually a unique solution to Euler's original problem.

## On fast fractional Fourier transform and open problems

J. Trujillo

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In this lecture, we introduce the notion of the fractional Fourier transform, which may be considering as a fractional power of the classical Fourier transform. We introduce the definition, the properties and approaches to the continuous fractional Fourier transform. It has been intensely studying during the last decade, an attention it may have partially gained because of the vivid interest in time-frequency analysis methods of signal processing. Like the complex exponentials are the basic functions in Fourier analysis, the chirps (signals sweeping through all frequencies in a certain interval) are the building blocks in the fractional Fourier analysis. In addition, we will pay special attention in the generalization to the fractional case the well-known Fast Fourier Transform through the connection of the Fractional Fourier Transform and the Ordinary Fourier Transform. At the end, we will remark some mistake often find in the literature of Fractional Models and possible solutions. Some of such mistake cam be solved using some new fractional extension of the Fourier transform.

## Non local model for non linear dark solitary waves

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I analyze dark solitary wave solution to an certain nonlocal nonlinear Schrödinger Equation with nonlocal dispersive term of Kac-Baker type. Main purpose is to investigate such solutions with negative nonlinear term with the presence of integral dispersive terms. This waves has a Hamiltonian given by

$$H = \frac{1}{2}\int \mathrm{d}x \int \mathrm{d}y\, G(|x-y|)(\Psi_x^*(x)\Psi_y(y) + \Psi_x(x)\Psi_y^*(y)) - \frac{g}{2}\int \mathrm{d}x\, |\Psi|^4$$

where $\Psi(x,t)$ is the wave function, $g$ is the nonlinear constant and $G(\rho)$ is the kernel that includes this nonlocal effect.

I first present the model and study the properties of the fundamental solution known as Continuous Wave. In the context of NLS equations, Dark solitary waves are perturbations of this plane wave that connect asymptotically two of those Continuous Waves with the same amplitude and different phase.

The study is divided two different cases: Grey and Dark solitary waves which possess different analytical properties. I also study range of existence of such solutions, their stability and energy, finding unusual behavior of these nonlinear solutions under nonlocal dispersive terms.

## From the nonlocal problems to fractional differential equations

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The nonlocal wave equations either in space or time are a window to fractional differential equations. At the same time, a natural way to construct families of fractional differential equations is by considering the different powers of well established ordinary differential equations under certain symmetric conditions. The above contexts allow us to obtain fractional evolution equations with internal degrees of freedom, as the spin in Quantum Mechanics, and whose solutions satisfy certain limit conditions.

## Stochastic solutions of nonlinear partial differential equations

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The stochastic construction of solutions for linear and nonlinear partial differential equations is discussed. The technique provides new exact solutions and also yields efficient numerical codes for localized solutions and parallel computing. A review and some new results for the Poisson-Vlasov and a fractional version of the KPP equation are included.

## Some challenges in modelling and control of fractional dynamic systems. An engineering approach

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Fractional calculus is a field of applied mathematics that deals with derivatives and integrals of arbitrary order. During the last decades, it has became an interesting mathematical tool in many scientific and engineering fields. However, some mathematical issues remain unsolved. In this talk, some of these open problems concerning the modelling and control of fractional order systems will be presented. In the introduction, the fundamentals of fractional calculus, fractional order systems dynamics, and feedback control will be outlined. Next, some important open problems will be considered, ranging from theoretical concepts as initial conditions, state variables or geometrical interpretations of the integrodifferential operators when dealing with fractional systems, to the practical requirements for the efficient implementations. These problems will be presented from an engineering point of view or, in other words, having in mind their real world applications. Some ways will be suggested for interdisciplinary research.

## Origin of fractional dynamics in systems with long-range memory and interaction

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The concept of fractional kinetics (FK) appeared in chaotic particle dynamics, particle advection in fluids, plasma physics, quantum optics, condensed matter physics, kinetics of chemical reactions, biological objects, and many others [1], [2]. Amid the new features of the FK, the most important are anomalous transport, weak mixing, and multi-scaling. The fractional kinetics does not look as the usual one since the moments of the p.d.f., often starting from the 2d one, can be infinite and fluctuations of the observables do not have a finite time of relaxation. Different important physical phenomena such as cooling of particles and signals, particle and wave trapping, dynamical models of Maxwell's Demon, represent some areas where the fractional kinetics proves to be valuable. The notion of FK was introduced in 1992 in our paper in order to describe the superdiffusion that was observed in chaotic advection and in particle acceleration in magnetic field along the stochastic webs. Fractional dynamics (FD) has a long term memory and a large scale interaction between degrees of freedom and it is a basic element of the FK.
Some known reasons to appear of FK are stickiness of orbits and intermittency that lead to a long lasting quasi-regular parts of the system orbits associated with zero or small value of the Lyapunov exponent. The corresponding domains in phase space can be considered as dynamical traps or quasi-traps, where particles can spend a fairly long time with not a small probability. We will demonstrate dynamical models that show how the FK leads to the Maxwell's Demon phenomenon A more specific dynamical property, that leads to FD and FK, is pseudochaos, i.e. randomness of the dynamics in systems with zero Lyapunov exponent. Systems with FK have persistent fluctuations, i.e. p.d.f. of fluctuations has power-wise tails and infinite mean values of some observables. Important application of FK and FD is turbulence of field lines in fluids and tokomak plasmas [3] since the lines can be considered as trajectories of Lagrangian particles of the velocity field or magnetic field. Two issues will be discussed with more specific comments due to their importance for applications: duality of a set of coupled objects ( for example nonlinear oscillators) with long-range-interaction and the corresponding description of the system by the fractional difference equation; sensitivity of the regime of the dynamics of the system ( from synchrony to turbulence) to small variations of the exponent of interaction.
In conclusion we discuss the prospective applications of FD and FK in physics and close areas ( networks evolution, chemical reactions, search dynamics, and others).

[1]. G.M. Zaslavsky, Chaos, fractional kinetics, and anomalous transport, Phys. Rep. 371, 46-580, December 2002.

. G.M. Zaslavsky, Hamiltonian Chaos and Fractional Dynamics, Oxford University Press, New York, 2005.

. G.M. Zaslavsky, B.A. Carreras, V.E. Lynch, and M. Edelman, Topological instability along invariant surfaces and pseudochaotic transport, Phys. Rev. E 72, 026227 (2005).

# USEFUL INFORMATION

- **Conferences**: Aula Miguel de Guzmán Room S118. Basement 1 (floor -1).

- **Coffee Breaks**: Basement 1. In front of the Aula Miguel de Guzmán Room S118.

- **Lunch**: Physical Science Faculty (Basement 1). In front of Mathematics Faculty.

- **Wi-Fi**: there is a Wi-Fi connection available in the Aula Miguel de Guzmán and in the surrounding area. Please, follow the next directions:
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- **Internet room**: S216 Room (Basement 2). The password is: mates.

- **Lounge**: S117 Room (Basement 1).

- **Social Dinner**: Thursday 7 at 21:00. Will be held at: ***El Cisne*** Restaurant, C/ Ventura Rodríguez 4, Madrid. See the map here included:

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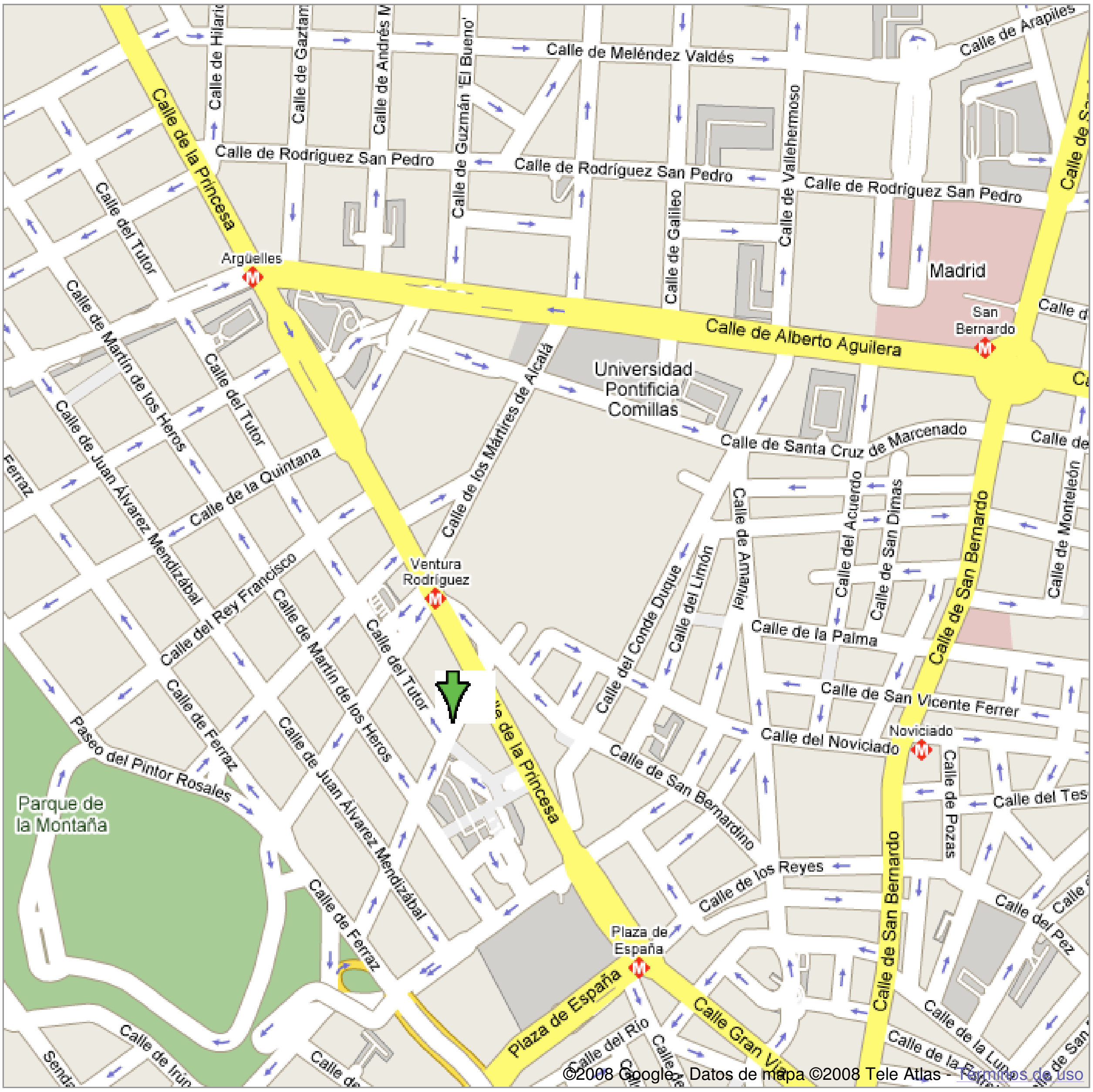

**The Proceedings of the Congress will appear in the special issue: *Non Linear and Non Local Problems: From the Theory to the Applications* (edited by J.I. Díaz, T. Pierantozzi and L. Vázquez) of the Electronic Journal Differential Equations (http://ejde.math.txstate.edu/)**

## Tex and Pdf files must be sent to the organizers before June 30 th, 2008.

Note: Bibliographical references to articles in this proceedings should be written as follows.
Author's name, *Title of the article*, International Conference on Non Linear and Non Local Problems, Electron. J. Diff. Eqns., Conf. 17 (2008), pp. ##-##.

## Instructions to Authors

All topics related to differential equations and their applications (ODE's, PDE's, integral equations, functional differential equations, etc.) will be considered for publication. Research articles must contain correct, new, and nontrivial results. Please emphasize what ideas, methods, or results are new in your manuscript. Articles will be refereed under the same standards used by the finest-quality printed journals.

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