School and Workshop on Topics in operator algebras and some applications

Abstracts

School part
Workshop part
Posters

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Quasidiagonality, finite dimensional approximations and operator algebras

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This mini-course will focus on quasidiagonal C*-algebras, their relationship with other classes of algebras (e.g., nuclear or exact) and applications. More precisely, I will present the main theoretical results (e.g., Voiculescu's homotopy invariance theorem); discuss many concrete examples; explore the relationship between quasidiagonality, nuclearity and exactness; and also cover tracial states on quasidiagonal C*-algebras. Along the way, a few applications will fall out. For example, the resolution of Herrero's approximation problem (a question in single operator theory), a unified approach to the various examples of BDF extension semigroups which are not groups, and even connections with theoretical numerical analysis.

Though it will not be possible to present complete proofs of everything, I will take great care in proving all basic facts and do my best to sketch the main ideas of everything else. The only prerequisites for the course are a solid introductory course in C*-algebras, a working knowledge of completely positive maps and familiarity with Voiculescu's Weyl-von Neumann Theorem (though I'll quickly review the necessary results).

Continuous fields of C*-algebras and K-theory invariants

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July 23, 2010

The continuous fields of C*-algebras play the role of topological bundles in the C*-algebra theory. Continuous fields appear naturally as C*-algebras with Hausdorff primitive spectrum but also as versatile tools in a large array of contexts such as index theory and deformation quantization. Unlike the classical bundles most interesting continuous fields of C*-algebras are not locally trivial. We plan to give an introduction to this area and discuss certain generalizations of the of the Dixmier and Douady theory to continuous with fibers nuclear C*-algebras. In particular we will focus on continuous fields whose fibers are Cuntz algebras or more generally Kirchberg algebras.

The Cuntz semigroup of a certain class of C*-algebras

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The Cuntz semigroup is a powerful techical device that can be attached to any C*-algebra and captures a great deal of its structure. This invariant is essentially constructed as a continuous version of the projection semigroup (built out of Murray-von Neumann equivalence classes of projections). Its order, not algebraic except in trivial situations, happened to be the key feature that helped in distinguishing two non-isomorphic C*-algebras with the same Elliott invariant. The fact that, for important classes of algebras, the said invariant can be recovered functorially from the semigroup, brought attention to its possible use in the classification programme. Indeed, this has been put into practice recently by a number of authors, and there is therefore a strong need for computational techniques of the Cuntz semigroup for various classes of algebras, particularly non-simple ones. In this talk we shall mention some of the techniques involved to compute the semigroup of $C(X, A)$, for a variety of spaces X and algebras A that have, most of the time, stable rank one. We shall also dwell on how to deal with amalgamated sums of algebras of that type. This is joint work with Francesc Perera and Luis Santiago (UAB).

C^*-algebras of separated graphs

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In joint work with K.R. Goodearl [2], the construction of the graph C^*-algebra associated to a directed graph E is extended to incorporate a family C consisting of partitions of the sets of edges emanating from the vertices of E. We show that there is a faithful $*$-homomorphism $L_{\mathbb{C}}(E,C) \to C^*(E,C)$, where $L_{\mathbb{C}}(E,C)$ is the *Leavitt path algebra* of the separated graph (E,C) introduced in [1]. To prove this we make use of a *reduced version* $C^*_{\mathrm{red}}(E,C)$ of the graph C^*-algebra of a separated graph. We also compute the K-theory of $C^*(E,C)$ in terms of the adjacency matrix of the separated graph (E,C).

References

1. P. Ara and K. R. Goodearl, Leavitt path algebras of separated graphs, *submitted*, arXiv:1004.4979v1 [math.RA]
2. P. Ara and K. R. Goodearl, C^*-algebras of separated graphs, *in preparation*.

On Følner nets and crossed products

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Many notions in group theory have their counterparts in the context of operator algebras. Følner nets were introduced in the context of operator algebras by Alain Connes in 1976 as an algebraic analogue of Følner's characterization of amenable discrete groups. This concept has been also applied systematically to spectral approximation problems.

Let *M* be a von Neumann algebra that has a Følner net. In this talk we will give conditions that guarantee that the von Neumann crossed product of *M* with an amenable discrete group has a Følner net. The Følner net for the crossed product is given explicitly and the result is applied to the rotation algebra.

The Cuntz Semigroup and Comparison of Open Projections

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We show that a number of naturally occurring comparison relations on positive elements in a C*-algebra are equivalent to natural comparison properties of their corresponding open projections in the bidual of the C*-algebra. In particular we show that Cuntz comparison of positive elements corresponds to a comparison relation on open projections, that we call Cuntz comparison, and which is defined in terms of-and is weaker than-a comparison notion defined by Peligrad and Zsidó. The alter corresponds to a well-known comparison relation on positive elements defined by Blackadar. We show that Murray-von Neumann comparison of open projections corresponds to tracial comparison of the corresponding positive elements of the C*-algebra. We use these findings to give a new picture of the Cuntz semigroup.

This is a joint work with Mikael Rørdam and Hannes Thiel.

Square and maximal functions in noncommutative L_p spaces

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We will provide a brief survey on the recent theory of noncommutative martingale inequalities. It extends to noncommutative L_p spaces most of classical martingale L_p inequalities, which involve martingale square and maximal functions. If time permits, we will also relate these results with the noncommutative Khintchine inequalities, maximal ergodic theorems and applications in operator space theory.

Graph algebras: an overview

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July 23, 2010

We will present some excerpts on graph C^*-algebras. We will start by doing a brief historical introduction to the topic. We will present the definition of a graph C^*-algebra, illustrating it with some examples. Then, we will briefly explain how tighly related are the properties of such algebras with those of the underlying graphs. We will end by sampling this fact through the explicit computation of the K-Theoretical invariants of a graph C^*-algebra, which also would highlight some of the techniques used in this topics.

Orthogonality preserving operators between C*-algebras

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July 28, 2010

We shall survey the results describing the structure of those (bounded or unbounded) linear mappings preserving orthogonal elements between C*-algebras. We recall that two elements, a, b in a C*-algebra A are said to be *orthogonal* ($a \perp b$ for short) if $ab^* = b^*a = 0$. An *orthogonality preserving* mapping T between two C*-algebras A and B is a map satisfying that

$$T(a) \perp T(b) \text{ whenever } a \perp b.$$

The study of orthogonality preserving operators between C*-algebras began with the paper [1], where W. Arendt established a completed description of all separating or orthogonality preserving bounded linear operators between $C(K)$-spaces. In [4], K. Jarosz extended the study to the setting of orthogonality preserving (not necessarily continuous) linear mappings between abelian C*-algebras. Among the consequences derived from Jarosz result, it follows that every separating linear bijection between $C(K)$ spaces is automatically continuous.

In the setting of general C*-algebras, M. Wolff classified the symmetric orthogonality preserving bounded linear operators from a unital C*-algebra to another C*-algebra. The complete description of all orthogonality preserving bounded linear operators between C*-algebras was obtained by M.J. Burgos, F.J. Fernández-Polo, J.J. Garcés, J. Martínez and A.M. Peralta in [2] and [3].

We shall survey these results and we explore the open problems on automatic continuity of biorthogonality preserving operators between general C*-algebras.

References

[1] W. Arendt, Spectral properties of Lamperti operators, *Indiana Univ. Math. J.* **32**, no. 2, 199-215 (1983).

[2] M. Burgos, F. J. Fernández-Polo, J. J. Garcés, J. Martínez Moreno, and A. M. Peralta, Orthogonality preservers in C*-algebras, JB*-algebras and JB*-triples, *J. Math. Anal. Appl.*, **348**, 220-233 (2008).

[3] M. Burgos, F. J. Fernández-Polo, J. J. Garcés, and A. M. Peralta, Orthogonality preservers Revisited, *Asian-European Journal of Mathematics* **2**, No. 3, 387-405 (2009).

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[5] M. Wolff, Disjointness preserving operators in C^*-algebras, *Arch. Math.* **62**, 248-253 (1994).

A brief introduction to noncommutative L_p spaces

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July 22, 2010

Noncommutative integration is now living a period of stimulating new developments because of its interactions with other fields such as operator spaces, quantum probability, noncommutative harmonic analysis and quantum information. In this lecture we will give a brief introduction to noncommutative L_p-spaces. We will start from the definition and then present fundamental properties of these spaces. We will also emphasize some remarkable differences between them and the usual L_p-spaces. The lecture will end up with some important examples.

An extension of Fourier-Stieltjes algebras

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July 28, 2010

This article grew from an effort to define an operator-valued Fourier-Stieltjes algebra for a locally compact group. It was quickly realized that a far more general results are possible, and we can obtain completely contractive Banach algebras of matrices $M_I(A)$ for any completely contractive Banach algebra A. In particular, we get interesting matrix-valued measure algebras. In the case that G is a locally compact group, we identify bounded matrices with entries in the Fourier-Stieltjes algebra $M_I(B(G))$ with the space of completely bounded $B(\ell^2(I))$-valued functions which we denote $\mathcal{CB}(G, B(\ell^2(I)))$. We show that this space is a natural generalization of the scalar-valued Fourier-Stieltjes algebra of Eymard

A SYMBOL CALCULUS FOR A C*-ALGEBRA OF SINGULAR INTEGRAL OPERATORS WITH SHIFTS

CLAUDIO ANTÓNIO FERNANDES AND M.A. BASTOS AND YU. KARLOVICH

Consider the C^*-algebra $\mathfrak{B} := \text{alg}(\mathcal{A}, U_G) \subseteq \mathcal{B}(L^2(\mathbb{T}))$ generated by the C^*-subalgebra $\mathcal{A}$, of singular integral operators with piecewise slowly oscillating coefficients, and the range of a unitary representation of an amenable discrete group of diffeomorphisms $g : \mathbb{T} \to \mathbb{T}$. Under same conditions over the nature of the fixed points of the elements of G, namely if G acts freely on T, the C^*-algebra $\mathfrak{B}^\pi = \mathfrak{B}/\mathcal{K}$, with $\mathcal{K}$ the ideal of the compact operator, is isomorphic to a crossed product $\mathcal{A}^\pi \otimes G$. Using representation theory for crossed products, summarized in the local-trajectory method, we can construct an Operator Fredholm Symbol for the C^*-algebra $\mathfrak{B}$ when the action of the group G on $\mathbb{T}$ is free. In this poster we show how we can use the previous ideas to get a Symbol calculus and Fredholm criteria for the operators in $\mathfrak{B}$, when the action of G is more general, namely when set of the common fixed points of the elements of G have nonempty interior.

References

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[2] M.A. Bastos, C.A. Fernandes, and Yu.I. Karlovich, A C^*-algebras of functional operators with shifts having a nonempty set of periodic points, *Proceedings of the 6th Isaac International Congress proceedings* (2009), 111–121.

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Analytic Properties of Laguerre-type orthogonal polynomials

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In the last years some attention has been paid to the so called canonical spectral transformations of measures supported either on the real line or on the unit circle. This work is intended to provide a summary of analytic properties of orthogonal polynomials associated with the Uvarov's canonical linear spectral transformation on the Laguerre's classical measure supported on the real line.

Semicrossed Products of Operator Algebras and Their C^*-envelopes

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July 21, 2010

Let $\mathcal{C}$ be a C^*-algebra and let α be a *-homomorphism of $\mathcal{C}$. Using a convolution through the action of α on $\mathcal{C}$, we can define various (non-selfadjoint) operator algebras, the *semicrossed products of* $\mathcal{C}$ *by* α. Here, we show that the C^*-envelope of a semicrossed procuct is (a full corner of) an appropriate crossed product.

By contrast, if we begin with a (non-selfadjoint) operator algebra $\mathcal{A}$, instead of $\mathcal{C}$, and α is a (completely isometric) automorphism of $\mathcal{A}$, then this may not be the case, as an example shows. Nevertheless, there is a vast category of pairs $(\mathcal{A}, \alpha)$ for which the C^*-envelope of a semicrossed product is a crossed product; for instance, this happens when $\mathcal{A}$ is the tensor algebra $\mathcal{T}_{\mathcal{X}}^+$ of a C^*-correspondence and α is a completely isometric automorphism of $\mathcal{T}_{\mathcal{X}}^+$ that fixes its diagonal elementwise.

References

[1] Evgenios T.A. Kakariadis, Elias G. Katsoulis, *Semicrossed Products of Operator Algebras and their C^*-envelopes*, preprint.

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σ-Contractible and σ-biprojective Banach Algebras

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The notion of σ-amenability for Banach algebras and its related notions were introduced and extensively studied by M. S. Moslehian and A. N. Motlagh. We develop these notions parallel to the amenability of Banach algebras introduced by B. E. Johnson. Briefly, we investigate σ-contractibility and σ-biprojectivity of Banach algebras, which are extensions of usual notions of contractibility and biprojectivity, respectively, where σ is a bounded endomorphism of the corresponding Banach algebra. We also give the notion σ-diagonal. Then we verify relations between σ-contractibility, σ-biprojectivity and the existence of a σ-diagonal for a Banach algebra, when σ has dense range or is an idempotent. Moreover, we obtain some hereditary properties of these concepts.