Fractional Dynamical Systems

International Workshop on fractional models

June 20 & 21, 2011, Madrid, Spain.
ETSI Telecomunicación, Room A-123,
Universidad Politécnica de Madrid.

Programme:

Monday

14h00: Registering and welcome

14h30: Luis Vázquez, UCM:
Fractional Calculus as a modelling tool

15h30: Tang Yi-Fa, CAS:
Discrete Analysis of a Kind of Fractional Differential Operators

16h30: coffee break

17h00: Jorge A. González, IVIC and UPM:
Dynamics of solitons in nonlocal Klein-Gordon equations

18h00: Nie Ning-ming, CAS:
Solving Fractional Diffusion Equations by Spectral Methods

Tuesday

10h30: Carlos Aguirre, UAM:
Tomographic methods for spatial data

11h30: Huang Jianfei, CAS:
Convergence Analysis of Block-by-block Methods for Fractional Differential Equations

12h30: coffee break

13h00: María Pilar Velasco, UCM:
Fractional dynamics of populations

14h00: lunch

15h30: David Usero, UCM:
Time-Fractional Quantum Systems

16h30: Salvador Jiménez, UPM:
Fractional Duffing's equation

17h30: Round Table: *modelling with fractional tools*
Tang Yi-Fa, Luis Vázquez, Jorge A. González, Nie Ning-ming,
David Usero, Huang Jianfei, Salvador Jiménez

Confirmed Speakers:

Carlos Aguirre	Dept. Ingeniería informática, EPS, Universidad Autónoma de Madrid
Jorge A. González	Centro de Física, Instituto Venezolano de Investigaciones Cinetíficas, Venezuela, and Dept. Matemática Aplicada a las TT.II., ETSIT, Universidad Politécnica de Madrid, Spain
Huang Jianfei	ICMSEC, Academy of Mathematics & Systems Sciences, Chinese Academy of Sciences, China
Salvador Jiménez	Dept. Matemática Aplicada a las TT.II., ETSIT, Universidad Politécnica de Madrid, Spain
Nie Ning-ming	CNIC, Chinese Academy of Sciences, China
Tang Yi-Fa	LSEC,ICMSEC, Academy of Mathematics & Systems Sciences, Chinese Academy of Sciences, China
David Usero	Dept. Matemática Aplicada, Universidad Complutense de Madrid, Spain
Luis Vázquez	Dept. Matemática Aplicada, Universidad Complutense de Madrid, Spain
María Pilar Velasco	Dept. Matemática Aplicada, Universidad Complutense de Madrid, Spain

Attendance is free but previous registration is required

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Abstracts

Carlos Aguirre *Tomographic methods for spatial data:*

Jorge A. González *Dynamics of solitons in nonlocal Klein-Gordon equations:*

We investigate the nonlocal Klein-Gordon equation in the presence of spatiotemporal perturbations. This system is used as a mathematical model for Josephson junctions in very thin films. We analyze the dynamics of solitons, which will behave as extended deformable particles moving in an effective multi-well potential perturbed by time-dependent forces.
Under certain conditions, the interaction between the solitons will have long-range character. We have found new mechanisms for the excitation of soliton internal degrees of freedom. We will show the connections between nonlocality, nonlinearity, internal degrees of freedom, long-range interactions and power-law behaviors.

Huang Jianfei *Convergence Analysis of Block-by-block Methods for Fractional Differential Equations*:

In this talk, we devote to systematically study of the block-by-block methods for a class of fractional differential equations (FDEs). Based on the equivalent integral form of FDEs, error representations and estimates for the truncation are derived. By means of the analysis of the weights, the high order convergence of the block-by-block methods for FDEs is analytically proved. And numerical experiments are given to verify our theoretical results.

Salvador Jiménez *Fractional Duffing's equation:*

We investigate the Fractional Duffing equation in the presence of non-harmonic external perturbations and study the Geometrical Resonance of this equation. Some considerations on the characterization of chaos is presented.

Nie Ning-ming *Solving Fractional Diffusion Equations by Spectral Methods:*

The phenomenon of anomalous diffusion could be described more precisely by fractional diffusion equations than classic diffusion one, due to the potential nonlocal properties of fractional derivatives. In this talk, the spectral methods are applied to two types of fractional diffusion equations. First, we deal with the steady state fractional advection dispersion equation by spectral method numerically and analyze the scheme's stability. Second, we treat an initial-boundary value problem of spatial-fractional partial differential diffusion equation which is obtained from the application in groundwater pollution. The equation is transformed into an equivalent weak form, and the existence and uniqueness of the weak solution are proved. Then the weak form is approximated by an implicit Galerkin spectral fully discrete scheme, which is unconditional stable and convergent. Finally, we check our theoretical results by numerical experiments.

Tang Yi-Fa

Discrete Analysis of a Kind of Fractional Differential Operators

Abstract. In fractional analysis, the eigenvalue problem of the differential equation

$$lu = \frac{1}{\Gamma(1-\rho)}\frac{d}{dx}\int_0^x \frac{u'(t)}{(x-t)^\rho}dt = \lambda u$$

with boundary conditions

$$u(0) = u(1) = 0$$

is of great importance. It is known that this differential equation is equivalent to the integral equation

$$u(x) = \frac{\lambda}{\Gamma(\rho^{-1})}\left[\int_0^x (x-t)^{\frac{1}{\rho}-1}u(t)dt - \int_0^1 x^{\frac{1}{\rho}-1}(1-t)^{\frac{1}{\rho}-1}u(t)dt\right].$$

The kernel of this integral equation is $K(x,t) = \theta(x,t)(x-t)^\mu - x^\mu(1-t)^\mu$, where $\mu = \rho^{-1} - 1$ and $\theta(x,t) = \begin{cases} 1, & \text{if } t < x; \\ 0, & \text{if } t \geq x. \end{cases}$

In this talk, we discuss the total nonnegativity of the matrix $T(\mu) = (K_{ij}) = \left(K(\frac{i}{n}, \frac{j}{n})\right)$, $1 \leq i, j \leq n-1$. We prove that the eigenvalues of $T(2)$ are simple and positive, and those of $T(1+\epsilon)$ are simple and real with ϵ being small enough.

David Usero

Time-Fractional Quantum Systems

Copenhague interpretation of quantum physics relates wave-functions with probability densities, Somehow it can be interpreted that the involved equations are in some sense, probabilistic equations. In this work we analyse different possibilities to use time-fractional Caputo derivative in quantum systems, and the properties derived from this modification of the temporal dynamic of such systems. Two different equations are studied: the classical Schrödinger equation, and the Dirac equation for 1/2 spin particles. We obtain the free-particle solutions and the infinite well solutions and study the fractional analogous of some properties of such solutions.

Luis Vázquez

Fractional Calculus as a modelling tool:

We present a panoramic of the foundation of the Fractional Calculus and its ability to generate new mathematical modelling scenarios. The new families of equations and functions present a natural framework to model phenomena associated to nonlocal effects in space and to memory effects in time.

María Pilar Velasco

Fractional dynamics of populations:

Nature often presents complex dynamics, which cannot be explained by means of ordinary models This is the motivation to establish an approach to certain fractional dynamic systems using only deterministic arguments. In these cases the behavior of the trajectories of fractional non-linear autonomous systems around the corresponding critical points

in the phase space is studied and it is possible to observe that the order of fractional derivation is an excellent controller of the velocity how the mentioned trajectories approach to (or away from) the critical point. As applications, this approach could be applied in classical models, for example populations in competition.