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Workshop of Young Researchers 2015

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Richard M. Aron (Kent State University)

Homomorphisms on $\mathcal{H}^\infty(\mathbb{D})$ and beyond

Let $\mathbb{D} = \{z \in \mathbb{C} \mid |z| < 1\}$ be the complex unit disc. For $n = 1, 2, \ldots$, let $\mathcal{H}^\infty(\mathbb{D}^n)$ denote the set of functions $f : \mathbb{D}^n \to \mathbb{C}$ such that f is holomorphic and bounded, endowed with the norm $f \in \mathcal{H}^\infty(\mathbb{D}^n) \rightsquigarrow \|f\| = \sup_{z\in\mathbb{D}^n} |f(z)|$. It isn't hard to see that $\mathcal{H}^\infty(\mathbb{D}^n)$ is a complete, commutative, unital algebra. The *maximal ideal space* $\mathcal{M}(\mathcal{H}^\infty(\mathbb{D}^n))$ of homomorphisms (= non-trivial, linear, multiplicative functionals $\varphi : \mathcal{H}^\infty(\mathbb{D}^n) \to \mathbb{C}$) is a much-studied, very relevant, very interesting set.

In this *expository!* talk, we hope to justify the previous sentence. We will briefly describe past and present work as well as some intriguing problems for future study.

Alba Segurado (UCM)

Limiting logarithmic interpolation

Interpolation theory plays an important role in the study of function spaces, operator theory and other areas of mathematics. Many of these applications are based on the real method $(A_0, A_1)_{\theta,q}$ introduced by Lions and Peetre (see [1]), where $0 < \theta < 1$.

Logarithmic methods $(A_0, A_1)_{\theta,q,\mathbb{A}}$, defined by means of a broken logarithmic function $\ell^{\mathbb{A}}(t)$, are an important extension of the real method such that, under certain additional assumptions, θ can also take the values 0 and 1. In this talk we will be interested in the limiting cases where $\theta = 0$ or 1, since there were certain natural questions about these spaces that had not been studied before. In particular, we will show that the description of these spaces in terms of the J-functional depends on the relationship between q and $\mathbb{A}$, contrary to the case where $0 < \theta < 1$. As a consequence of these J-descriptions, we will be able to investigate the behaviour of compact and weakly compact operators under logarithmic interpolation methods. The contents of this talk are part of a joint work with Fernando Cobos ([2]).

Referencias

[1] Bergh, Jöran; Löfström, Jörgen; *Interpolation spaces. An introduction.* Grundlehren der Mathematischen Wissenschaften, No. 223. Springer-Verlag, Berlin-New York, 1976.

[2] Cobos, Fernando; Segurado, Alba; Description of logarithmic interpolation spaces by means of the J-functional and applications. J. Funct. Anal. 268 (2015), 2906–2945.

Irina Arévalo (UAM)

Mixed norm spaces of analytic functions

In this talk we will consider the mixed norm spaces, a family of spaces given by the integrability of the integral mean of analytic functions in the unit disk, and that therefore generalize the Hardy and Bergman spaces. We will see results on mean and pointless growth of the functions in the mixed norm spaces, comparing them with their counterpart in the classic spaces, and we will give a complete characterization of the inclusions between different spaces of the family.

Marcos de la Oliva (UAM)

Laminates meet Condition (N)

Let $\Omega \subset \mathbb{R}^n$ be a domain, and $\alpha \in (0,1)$. We will show that there exists a convex function u such that ∇u is a homeomorphism, equals the identity on the border of Ω, and belongs to $C^\alpha(\Omega) \cap W^{1,p}(\Omega)$ for all $p < n$, however $\det(D\nabla u) = 0$ a.e.

This improves previous results of a recent series of papers of Hencl and Cerny. Our approach is strictly different, and based on the theory of laminates.

María López Fernández (UZH Zürich)

Towards time adaptivity for convolution equations arising in wave problems

We address the efficient numerical approximation of time convolution equations. Our applications include the approximation of retarded potentials associated to wave problems. By working in the Laplace domain, we propose a time-stepping method which allows to use variable steps, possibly driven by some error control mechanism. We present a priori error estimates for our method and some ideas for the control of the steps. The implementation is delicate and relies on contour integral techniques in the complex plane. Numerical experiments are presented to show the potential of our approach.

Eduardo Fernández (UCM)

Symmetries and Differential Equations

In this work we analyze the point symmetries of partial differential equations from the perspective of the theory of differential forms after Cartan. In particular, we review the Harrison-Estabrook method for the effective computation of the symmetry generators, as well as the algorithmic implementation based on this procedure.

Carlos Buelga Sánchez (UCM)

Factorization of operators in Banach lattices

In this lecture we will expose some important results on factorization of operators in Banach lattices. In particular, we analyze Maurey-Nikishin's theorem which establishes sufficient and necessary conditions for the factorization of L^p-valued operators. We explore conditions that guarantee factorization within the terms of the type and cotype of the involved spaces.

In addition we consider operators on spaces of variable exponent spaces $L^{p(.)}$ .The lattice structure of these non-symmetric spaces is quite different comparing with classical Lebesgue L^p spaces; disjoint characteristic function sequences do not generate subspaces isomorphic to l_p. We study strictly singular endomorphisms in these spaces, providing characterizations which involve the essential range $R_{p(.)}$ of the exponent functions $p(.)$.

Antonio Rubio Calzado (UCM)

Invariant Hypersurfaces by Endomorphisms

In this talk (that is also the presentation of my Master Thesis), we will a general panoramic of the situation of the problem of invariant hypersurfaces under the action of an endomorphisms of complex manifolds. Particularly, we will focus on a theorem written by Serge Cantat in 2010, which uses algebraic geometry techniques to study a problem of complex topological dynamics, which there exists an analogous result for foliated manifolds due to Ghys and Jounolou.

Sergio Fernández Rincón (UCM)

Dynamics of a competition model when diffusion goes zero

When considering a model for two competing species with constant diffusion coefficients in an heterogeneous environment, it arises the question of how is related its dynamics for small diffusion with the one of the kinetic associated model. We show that some properties like persistence or bistability can be obtained for the parabolic model with enought small diffusion although the kinetic does not exhibit them.

Piermarco Cannarsa (Università di Roma “Tor Vergata”)

Control of certain classes of hypoelliptic diffusions

The control of diffusive systems is well developed for uniformly parabolic operators, much less so for degenerate problems. In the latter case, a general theory which cobers controllability and observability issues is still missing. However, we can give a fairly complete analysis of such properties for certain classes of degenerate parabolic equations. This talk will focus on controllability, observability, and Lipschitz stability for diffusions associated with hypoelliptic operators including the Grushin and Heisenberg laplacians.

Nada el Berdan (Poitiers University)

Regularity of solutions and theirs gradients for semi-linear elliptic problem

Study of the existence, uniqueness, regularity properties of solutions and their gradients of semi-linear elliptic problem associated with variable coefficient operators of the form $Lu = F(x, u)$.

Ahmad Makki (Poitiers University)

Study of strongly anisotropic Cahn-Hiliard and Allen-Cahn models

The aim is to study the anisotropic Cahn-Hilliard and Allen-Cahn models which takes into account strong anisotropic effects. In particular, the free energy contains a regularization term, called Willmore regularization.

Angélica Benito (UAM)

Resolution of Singularities and Applications

In this talk we will try to give an intuitive idea of the problem of resolution of singularities. We will also discuss its applications to other branches of mathematics beyond algebraic geometry or algebra. Time permitting, we will expose some of the recent developments in the case of positive characteristic.

Carlos Abad Reigadas (ICMAT)

On the behavior of the multiplicity on algebraic varieties and Rees algebras

Let X be an algebraic variety over a perfect field. The multiplicity on X, say $\mathrm{mult} : X \to \mathbb{N}$, is an upper semi-continuous function. X is regular at a point ξ if and only if $\mathrm{mult}(\xi) = 1$. Otherwise ξ is said to be singular.

On this talk we will see that it is possible find an immersion of X into a regular variety, say $X \subset V$, a set of functions on V, say $f_1, \ldots, f_r$, and integers $N_1, \ldots, N_r$, which describe the maximum multiplicity locus of X. Namely,

$$\underline{\mathrm{Max}}\, \mathrm{mult}(X) = \bigcap_i \{\xi \in V \mid \mathrm{ord}_\xi(f_i) \geqslant N_i\},$$

and this relation is preserved by blow-ups along regular centers, and smooth morphisms. These functions naturally induce a Rees algebra:

$$\mathcal{G} = \mathcal{O}_V[f_1 W^{N_1}, \ldots, f_r W^{N_r}] \subset \mathcal{O}_V[W].$$

In the case of characteristic zero, there is an algorithm to construct from $\mathcal{G}$ a sequence of blow-ups so that the maximum multiplicity of X drops. This procedure differs from Hironaka's method of resolution of singularities, which is based on the Hilbert-Samuel function.

Finally we will see that using $\mathcal{G}$ one can attach a canonical Rees algebra to the maximum multiplicity stratum of X. This algebra does not depend on the immersion $X \subset V$, or the functions $f_1, \ldots, f_r$.

María Ramírez-Nicolás

Analysis of dynamic processes on Mars

In the frame of the project MeigaMetNet Precurssor were developed different studies related to three fundamental aspects in the dynamics of Mars. In this sense have been analyzed: the trajectories of charged particles from the solar wind with the magnetic field, the interaction between the solar wind and the Martian ionosphere and the electric field values obtained inside a dust devil by the interaction of dust particles from a new method.

Ana Portilla Ferreira - UC3M

Gromov Hyperbolicity in Mycielskian Graphs

Since the characterization of Gromov hyperbolic graphs seems a too ambitious task, there are many papers studying the hyperbolicity of several classes of graphs. In this paper we show the Gromov hyperbolicity of every Mycielskian graph. Furthermore, we prove that the hyperbolicity constant of the Mycielskian G^M of any graph G verifies $5/4 \leqslant \delta(G^M) \leqslant 5/2$. Graphs whose Mycielskian have hyperbolicity constant 5/4 or 5/2 are characterized.

Verónica Hernández - UC3M

On an Erdös theorem on the diameter and minimum degree of a graph

Denote by $G = (V, E)$ a simple connected graph such that every edge has length equal to 1. Here $V = V(G)$ denotes the set of vertices of G and $E = E(G)$ the set of edges of G. The degree of $v \in V(G)$ is the number of edges incident to the vertex and is denoted $\deg(v)$. The maximum degree of a graph G, denoted by Δ, is defined as $\Delta := \text{máx}\{\deg(v) \mid v \in V(G)\}$. Similarly, the minimum degree of a graph G, denoted by δ_0, is defined as $\delta_0 := \text{mín}\{\deg(v) \mid v \in V(G)\}$. The diameter of a graph is defined as $\text{diam}(G) := \text{máx}\{d(x, y) \mid (x, y) \in G\}$. In this work, we obtain good upper bounds for the diamteter of any graph in terms of its minimum degree and its order, improving a classical theorem due to Erdös, P., Pach, J., Pollack, R. and Tuza, Z. (see [1]). On the other hand, we deal with hyperbolic graphs in the Gromov sense. If X is a geodesic metric space and $x_1, x_2, x_3 \in X$, a geodesic triangle $T = \{x_1, x_2, x_3\}$ is the union of the three geodesics $[x_1x_2]$, $[x_2x_3]$ and $[x_3x_1]$ in X. The space X is δ-hyperbolic in the Gromov sense if any side of T is contained in a δ-neighborhood of the union of the two other sides, for every geodesic triangle T in X. If X is hyperbolic, we denote by $\delta(X)$ the sharp hyperbolicity constant of X, i.e. $\delta(X) = \text{ínf}\{\delta \geqslant 0 : \quad X \text{ is } \delta\text{-hyperbolic}\}$. To compute the hyperbolicity constant is a very hard problem. Then it is natural to try to bound the hyperbolicity constant in terms of some parameters of the graph. Let $\mathcal{H}(n, \delta_0)$ be the set of graphs G with n vertices and minimum degree δ_0, and $\mathcal{J}(n, \Delta)$ be the set of graphs G with n vertices and maximum degree Δ. In this work we estimate $a(n, \delta_0) := \text{mín}\{\delta(G) \mid G \in \mathcal{H}(n, \delta_0)\}$, $b(n, \delta_0) := \text{máx}\{\delta(G) \mid G \in \mathcal{H}(n, \delta_0)\}$, $\alpha(n, \Delta) := \text{mín}\{\delta(G) \mid G \in \mathcal{J}(n, \Delta)\}$ and $\beta(n, \Delta) := \text{máx}\{\delta(G) \mid G \in \mathcal{J}(n, \Delta)\}$. In particular, we compute the precise value of $a(n, \delta_0)$, $\alpha(n, \Delta)$ and $\beta(n, \Delta)$ for all values of n, δ_0 and Δ, respectively.

Referencias

[1] Erdös, P., Pach, J., Pollack, R. and Tuza, Z., Radius, Diameter and Minimum Degree, *Journal of Combinatorial Theory* **47** (1989), 73-79.

Amauris de la Cruz Rodríguez (UC3M)

On hyperbolicity in the Cartesian Sum of Graphs

If X is a geodesic metric space and $x_1, x_2, x_3 \in X$, a *geodesic triangle* $T = \{x_1, x_2, x_3\}$ is the union of the three geodesics $[x_1x_2]$, $[x_2x_3]$ and $[x_3x_1]$ in X. The space X is δ-*hyperbolic* (in the Gromov sense) if any side of T is contained in a δ-neighborhood of the union of the two other sides, for every geodesic triangle T in X. If X is hyperbolic, we denote by $\delta(X)$ the sharp hyperbolicity constant of X, i.e. $\delta(X) = \text{ínf}\{\delta \geqslant 0 : X \text{ is } \delta\text{-hyperbolic}\}$. Some previous works characterize the hyperbolic product graphs (for the Cartesian, strong, join, corona and lexicographic products) in terms of properties of the factor graphs. In this paper we characterize the hyperbolic product graphs for the Cartesian sum $G_1 \oplus G_2$: $G_1 \oplus G_2$ is always hyperbolic, unless either G_1 or G_2 is the trivial graph (the graph with a single vertex); if G_1 or G_2 is the trivial graph, then $G_1 \oplus G_2$ is hyperbolic if and only if G_2 or G_1 is hyperbolic, respectively. Besides, if $t \notin \{5/4, 3/2\}$ we characterize the Cartesian sums with $\delta(G_1 \oplus G_2) = t$ in a very simple way; also, we characterize the Cartesian sums with $\delta(G_1 \oplus G_2) = 5/4$ and with $\delta(G_1 \oplus G_2) = 3/2$. We obtain the sharp inequalities $1 \leqslant \delta(G_1 \oplus G_2) \leqslant 3/2$ for every non-trivial graphs G_1, G_2. Furthermore, we obtain simple formulae for the hyperbolicity constant of the Cartesian sum of many graphs. Finally, we prove the inequalities $3/2 \leqslant \delta(\overline{G_1 \oplus G_2}) \leqslant 2$ for the complement graph of $G_1 \oplus G_2$ for every G_1, G_2 with $\text{mín}\{\text{diam}V(G_1), \text{diam}V(G_2)\} \geqslant 3$.

Álvaro de Vicente-Retortillo Rubalcaba (UCM)

Characterization of solar radiation on Mars with the Monte-Carlo method

The solar radiation that reaches the Martian surface has a strong impact on the atmospheric dynamics and climate of Mars and it has important implications for the habitability of the planet. A description of a radiative transfer model that has been developed relying on the Monte-Carlo method is provided, and some important results for the characterization of the radiative environment are shown. The results provided by the model can greatly enhance the scientific return of present and future missions to Mars, such as Mars Science Laboratory and MetNet.

Javier Fresán (ETH Zürich)

Gamma values: regular and irregular

The values of the gamma function at rational numbers remain quite mysterious, one of the reasons being that, conjecturally, they are not periods in the usual sense of algebraic geometry. However, the theory of regular singular connections allows to show that suitable products of them are periods of Hodge structures with complex multiplication, as predicted by Gross and Deligne. To deal with single gamma values, one needs to consider irregular singular connections instead. After a brief exposition of my results on the Gross-Deligne conjecture, I will explain how irregular singular connections may shed some light on the arithmetic nature of these numbers.

Cecilia Lancien (Lyon University)

When Alice & Bob meet Banach

This talk aims at presenting some recent (fruitful!) uses of the so-called local theory of Banach spaces in quantum information theory. More specifically, I will explain how techniques and results from asymptotic geometric analysis may be employed to describe the "typical" properties of high-dimensional multipartite quantum systems (with a particular focus on their amount of entanglement). We will see some familiar random matrix models popping up, and the concentration of measure phenomenon will be ubiquitous. But (hopefully!) no special knowledge in any of these subjects will be needed to understand what I will be talking about.

Ignacio Pascual Deocón - UC3M

Modeling Paradigm Shifts in Cultural Evolution

Cultural transmission is a process of innovation and interaction between individuals that brings about changes in societies. These changes can occur either smoothly and gradually or suddenly and abruptly. Abrupt changes are collective phenomena referred to as 'paradigm shifts'. The goal of this work is to present a simple mathematical model of cultural paradigm shifts. Using population dynamics and mechanisms of cultural transmission, we explore how the interaction between cultural elements changes the dynamics of cultural transmission to the extent that it can explain the occurrence of paradigm shifts.

Juan Carlos García Ardila - UC3M

Christoffel Transformations of Matrix of Measures supported in the Real Line

We consider the matrix bi-orthogonal polynomials with respect to the bilinear form $\langle \cdot, \cdot \rangle_W$

$$\langle P(t), Q(t) \rangle_W = \int P(t) W(t) dM Q(t)^T, \quad P, Q \in \mathbb{P}^{p \times p}(\mathbb{R}),$$

where M is a symmetric, positive definite matrix of measures supported in some infinite subset of the real line, and $W(t)$ is a matrix polynomial with nonsingular leading coefficient. We obtain a connection formula between the sequences of matrix bi-orthogonal polynomials with respect to $\langle \cdot, \cdot \rangle_W$ and μ, and a relation between the corresponding block Jacobi matrices. A symmetric bilinear form is also considered.

Luis Miguel Anguas (UC3M)

Eigenvalue Condition Numbers of Polynomial Eigenvalue Problems under Möbius Transformations

We study the effect of Möbius transformations on the sensitivity of polynomial eigenvalues problems (PEP). More precisely, we compare eigenvalue condition numbers for a PEP and the correspondent eigenvalue condition numbers for the same PEP modified with a Möbius transformation. We bound this relationship with factors that depends on the condition number of the matrix that induces the Möbius transformation and on the eigenvalue whose normwise condition number we consider, and establish sufficient conditions where Möbius transformations do not alter significantly the condition numbers.

Fernando Lledó (UC3M - ICMAT)

Amenability aspects around Roe C*-algebras

There is a classical mathematical theorem (based on the work by Banach and Tarski) that implies the following shocking statement: "An orange can be divided into finitely many pieces, these pieces can be moved and rearranged in such a way to yield two oranges of the same size as the original one." In 1929 J. von Neumann recognized that one of the reasons underlying the Banach-Tarski paradox is the fact that on the unit ball there is an action of a discrete subgroup of isometries that fails to have the property of amenability ("promediabilidad" in Spanish or "Mittelbarkeit" in German). Since then, the notion of amenability has become ubiquitous in mathematics.

In this talk we will approach the concept of amenability from very different perspectives: we will analyze metric, purely algebraic and operator theoretic aspects. Finally, we will present Roe C*-algebras associated to discrete metric spaces with bounded geometry as an example where all these approaches unify.

(Joint work with P Ara (UAB), K. Li (U. Copenhagen), J. Wu (U. Muenster)).