Carlos Abad Reigadas (ICMAT)

On the behavior of the multiplicity on algebraic varieties and Rees algebras

Let X be an algebraic variety over a perfect field. The multiplicity on X, say $\mathrm{mult} : X \to \mathbb{N}$, is an upper semi-continuous function. X is regular at a point ξ if and only if $\mathrm{mult}(\xi) = 1$. Otherwise ξ is said to be singular.

On this talk we will see that it is possible find an immersion of X into a regular variety, say $X \subset V$, a set of functions on V, say $f_1, \ldots, f_r$, and integers $N_1, \ldots, N_r$, which describe the maximum multiplicity locus of X. Namely,

$$\underline{\mathrm{Max}}\,\mathrm{mult}(X) = \bigcap_i \{\xi \in V \mid \mathrm{ord}_\xi(f_i) \geqslant N_i\},$$

and this relation is preserved by blow-ups along regular centers, and smooth morphisms. These functions naturally induce a Rees algebra:

$$\mathcal{G} = \mathcal{O}_V[f_1 W^{N_1}, \ldots, f_r W^{N_r}] \subset \mathcal{O}_V[W].$$

In the case of characteristic zero, there is an algorithm to construct from $\mathcal{G}$ a sequence of blow-ups so that the maximum multiplicity of X drops. This procedure differs from Hironaka's method of resolution of singularities, which is based on the Hilbert-Samuel function.

Finally we will see that using $\mathcal{G}$ one can attach a canonical Rees algebra to the maximum multiplicity stratum of X. This algebra does not depend on the immersion $X \subset V$, or the functions $f_1, \ldots, f_r$.