# UCM Modelling Week

16th-24th June 2008

Madrid

# New Optimal Strategies for the Station Keeping of Communications Satellites in Geostationary Orbits using Electric Propulsion

Problem proposed by GMV

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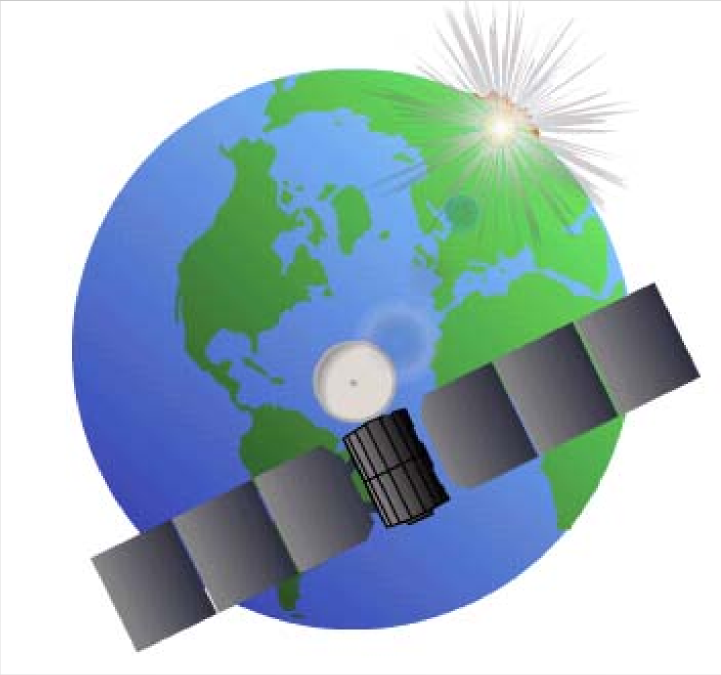
# Work Structure

1. Statement of the problem: optimal control for a dynamical system
2. Model for the dynamical system: second order differential equations
3. Solution with the method of variation of the constants
4. Analysis of the evolution of parameters involved
5. Control linear equations
6. Optimization control minimizing a cost function
7. Assumptions for determining the cost function
8. Definition of the cost function
9. Algorithm for minimize the cost function
10. Analysis of the results
11. Future works

# Geostationary Orbit

To keep a satellite in a nominal longitude above the Earth



$P = 24^h \Rightarrow a_s = 42164.2\,Km$

$i = 0^o$ equatorial

$e = 0$ circular

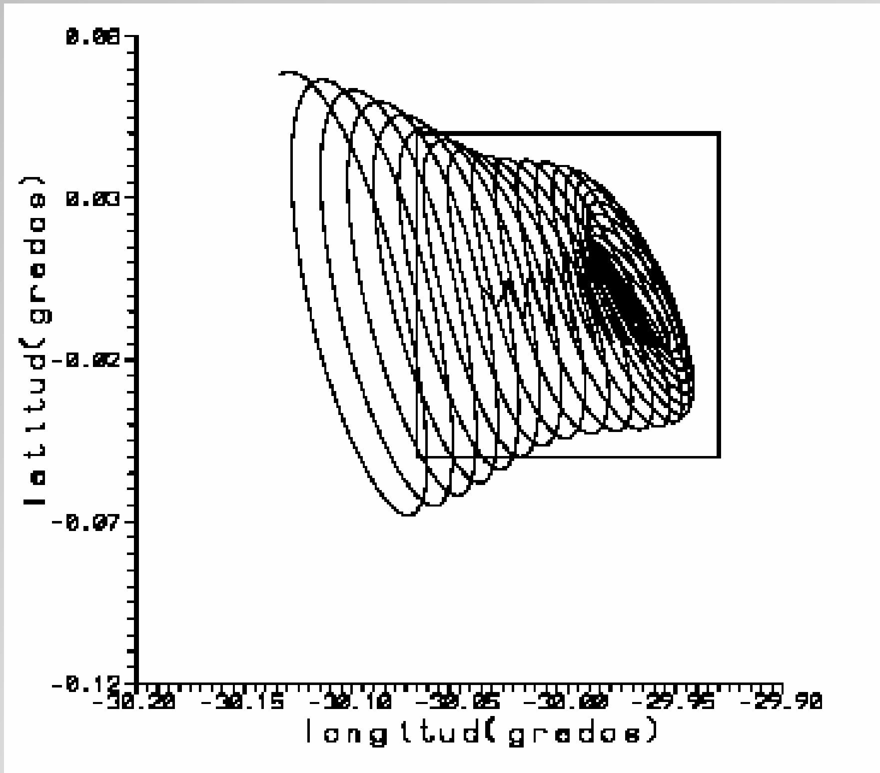
Perturbations tend to shift a geostationary satellite from its nominal station point.

# Problem Specification

The orbit changes with time

Main perturbing forces are:

- Earth Gravitational Field
- Lunisolar Force
- Solar Radiation Pressure




Natural evolution for a month

GENERAL PROBLEM: How to maintain a geostationary satellite within its orbital window.

# Station Keeping

Orbital station keeping manoeuvres for a geostationary satellite are performed to compensate for natural perturbations that tends to change the orbit to non geostationary.

Station keeping Modelling:

- Mean orbital elements: obtained by means of linearized Lagrange equations, where the perturbation function contains only those terms causing secular and long period perturbations.
- Linear equations for computing manoeuvres

Classical Approach

- Two thrusters located in normal plane (N/S) and in tangential plane(E/W)

New Model (proposed by GMV):

- One thruster with direction specified by the cant, $\gamma$, and, $\sigma$, slew angles.

# Objectives

Problem definition

Objective function

Equality constraints

Inequality constraints

$$\min \; f(x)$$

$$x \in \Re^n : \begin{array}{ll} g_i(x) = 0 & i = 1,\ldots,m_e \\ g_i(x) \geq 0 & i = m_{e+1},\ldots,m \end{array}$$

Objective function

$$\min \sum_{manoeuvre=1}^{n} Mass_{manoeuvre}$$

Optimisation variables for each manoeuvre:

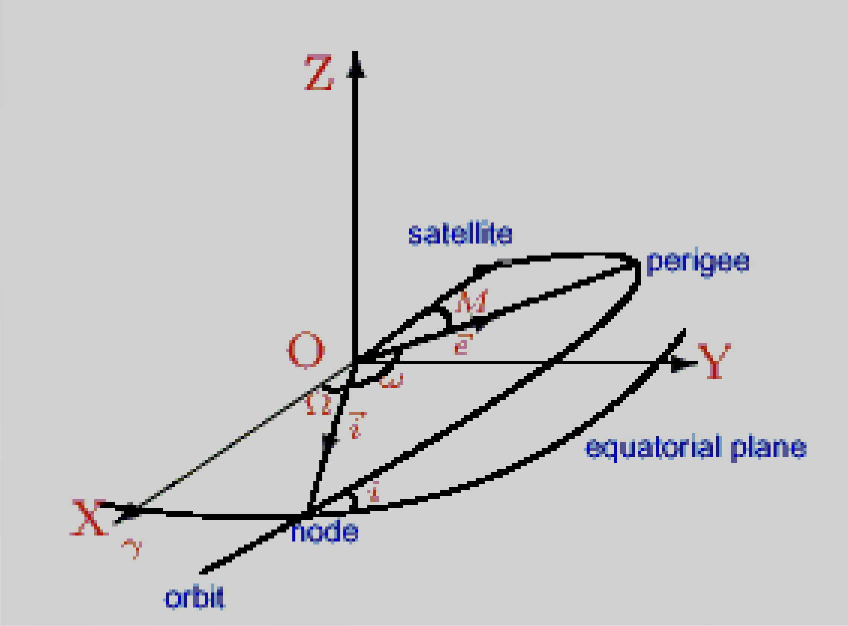
- Mid-point of the manoeuvre
- Duration of the manoeuvre

# Geostationary Orbit

## SYNCHRONOUS ORBITAL ELEMENTS:

Geostationary satellites have e and i values close to zero. To avoid numerical singularities the following orbital elements are considered

- Semimajor axis, a
- Eccentricity vector
  - $e_x = e \cos(\Omega + \omega)$
  - $e_y = e \sin(\Omega + \omega)$
- Inclination vector
  - $i_x = i \cos\Omega$
  - $i_y = i \sin\Omega$
- Mean longitude, $l = \Omega + \omega + M - \theta_G$



# Geostationary Orbit Evolution

$$
\begin{aligned}
\frac{da}{dt} &= \frac{2}{na} \cdot \frac{\partial R}{\partial l}, \\
\frac{dl}{dt} &= n - \Omega_{\oplus} - \frac{2}{na} \cdot \frac{\partial R}{\partial a} + \frac{e(1-e^2)^{1/2}}{na^2\left[1+(1-e^2)^{1/2}\right]} \cdot \frac{\partial R}{\partial e} + \\
&+ \frac{\tan i/2}{na^2(1-e^2)^{1/2}} \cdot \frac{\partial R}{\partial i}, \\
\frac{de_x}{dt} &= -\frac{(1-e^2)^{1/2}}{na^2} \cdot \frac{\partial R}{\partial e_y} - \frac{e_y \tan i/2}{na^2(1-e^2)^{1/2}} \cdot \frac{\partial R}{\partial i} - \\
&- \frac{e_x(1-e^2)^{1/2}}{na^2\left[1+(1-e^2)^{1/2}\right]} \cdot \frac{\partial R}{\partial l}, \\
\frac{de_y}{dt} &= \frac{(1-e^2)^{1/2}}{na^2} \cdot \frac{\partial R}{\partial e_x} + \frac{e_x \tan i/2}{na^2(1-e^2)^{1/2}} \cdot \frac{\partial R}{\partial i} - \\
&- \frac{e_y(1-e^2)^{1/2}}{na^2\left[1+(1-e^2)^{1/2}\right]} \cdot \frac{\partial R}{\partial l}, \\
\frac{di_x}{dt} &= -\frac{1}{na^2(1-e^2)^{1/2}} \cdot \frac{\partial R}{\partial i_y} - \frac{\cos\Omega \tan i/2}{na^2(1-e^2)^{1/2}\,\mathrm{sen}\, i} \frac{\partial R}{\partial \omega}, \\
\frac{di_y}{dt} &= \frac{1}{na^2(1-e^2)^{1/2}} \cdot \frac{\partial R}{\partial i_x} - \frac{\mathrm{sen}\,\Omega \tan i/2}{na^2(1-e^2)^{1/2}} \frac{\partial R}{\partial \omega},
\end{aligned}
$$

Lagrange equations

# Earth Gravitational Field

Acting mainly on the semi major axis and longitude

$$\frac{dl}{dt} = n - \Omega_{\oplus} - \frac{2}{na}\frac{\partial R}{\partial a}$$

$$\frac{da}{dt} = \frac{2}{na}\frac{\partial R}{\partial l}$$

Terrestrial perturbing potential

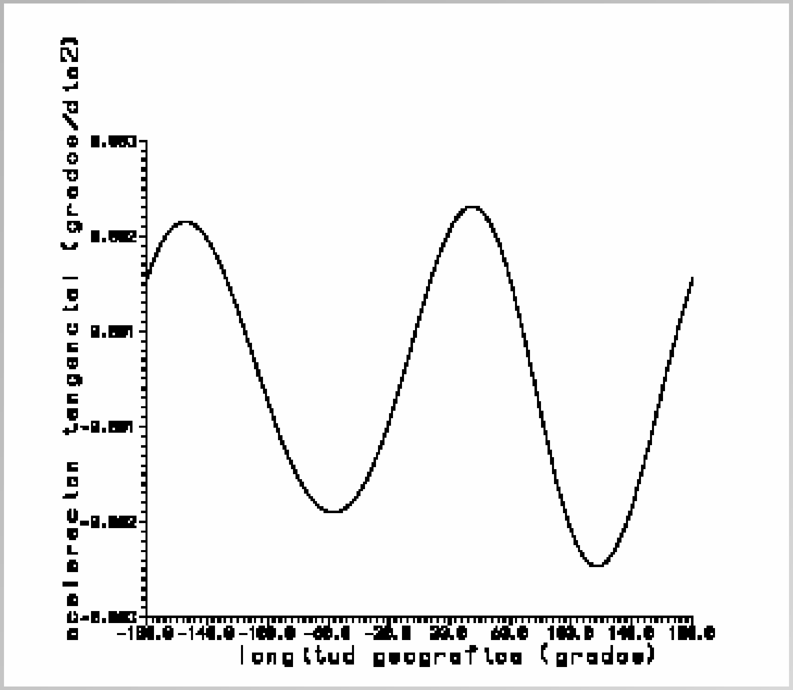
# Earth Gravitational Field

4 equilibrium points depending on l (l″=0):

$l_1$ = 14º.92 W (unstable) $l_2$ = 75º.08 E (stable)

$l_3$ = 104º.92 W (unstable) $l_4$ = 165º.08 E (stable)

# Earth Gravitational Field




| $l$ (grados) | $\ddot{l}$ $(m/s^2)$ | $\ddot{l}$ (grados/día$^2$) |
|---|---|---|
| 73.00 | -0.0000000501 | 0.000152 |
| 74.00 | -0.0000000261 | 0.000079 |
| 75.00 | -0.0000000020 | 0.000006 |
| 76.00 | 0.0000000222 | -0.000068 |
| 77.00 | 0.0000000464 | -0.000141 |
| 78.00 | 0.0000000706 | -0.000215 |

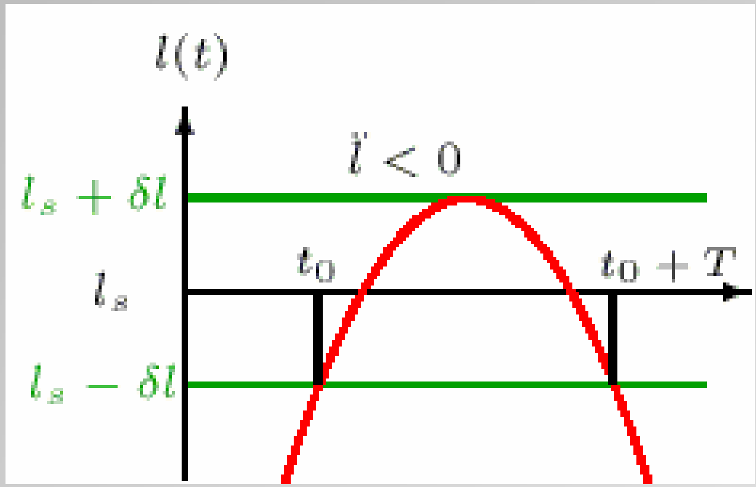
The longitude describes a parabola in time:

$$l(t) = l(t_0) + \dot{l}(t_0)(t - t_0) + \frac{\ddot{l}}{2}(t - t_0)^2$$

# Earth Gravitational Field

Maximum time within the orbital window:

$$T = 4\sqrt{|\delta l / \ddot{l}|}$$




| $l$ (grados) | $\ddot{l}$ (grados/día$^2$) | $T$ (días) |
|---|---|---|
| 75 E | 0.000006 | 365 |
| 76 E | -0.000068 | 108 |
| 83 E | -0.000577 | 37 |
| 117 W | -0.001968 | 20 |
| 30 W | -0.000887 | 30 |
| 105 W | 0.000006 | 365 |

# Lunisolar Force

Acting mainly on the inclination vector

$$\frac{di_x}{dt} = -\frac{1}{na^2}\frac{\partial R}{\partial i_y}$$

$$\frac{di_y}{dt} = \frac{1}{na^2}\frac{\partial R}{\partial i_x}$$

R Lunisolar Perturbing Potential $R = R_L + R_S$

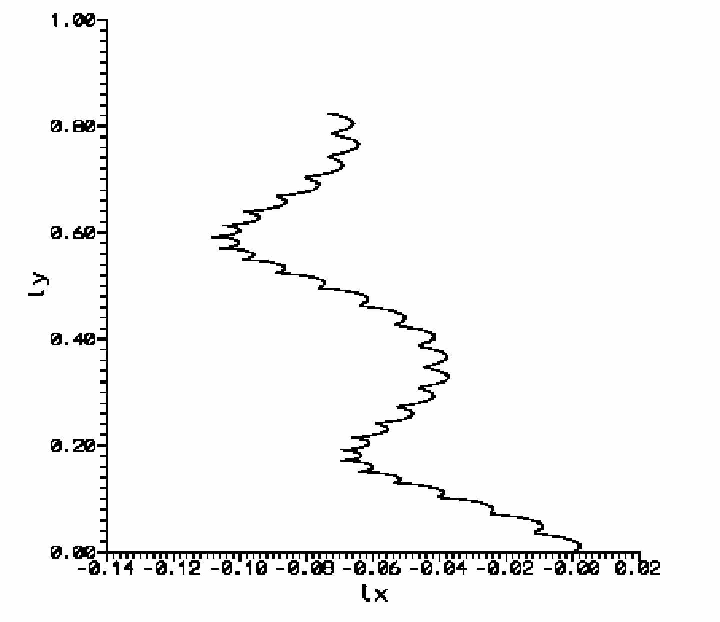
# Lunisolar Force

- The inclination vector is modified:

$$i_x = -0^{\circ}.3895\cos\Omega_L - 0^{\circ}.00457\cos 2(\omega_L + \upsilon_L) - 0^{\circ}.02331\cos 2\lambda_{\odot}$$

$$i_y = 0^{\circ}.8475t - 0^{\circ}.2903\sin\Omega_L - 0^{\circ}.004\sin 2(\omega_L + \upsilon_L) - 0^{\circ}.02139\sin 2\lambda_{\odot}$$

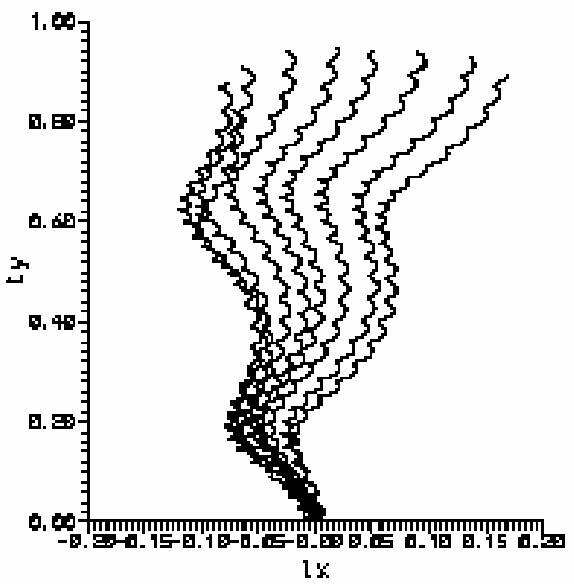
- Periodical perturbations and secular drift



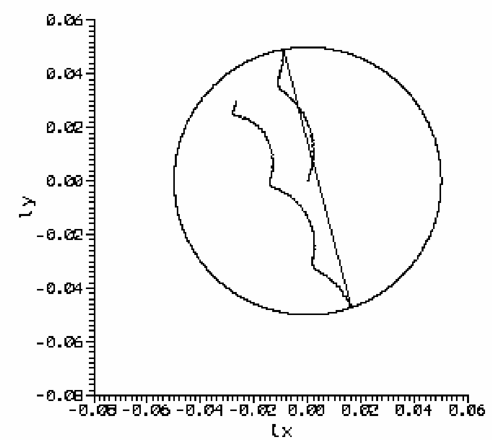
# North/South Station keeping

## Mean Secular Line Strategy

| Year | $\Delta i_{sec}$ (deg) | $\Omega_{sec}$ (deg) |
|---|---|---|
| 2003 | 0.9186 | 96.08 |
| 2004 | 0.9358 | 93.83 |
| 2005 | 0.9445 | 91.25 |
| 2006 | 0.9442 | 88.57 |
| 2007 | 0.9350 | 86.01 |
| 2008 | 0.9173 | 83.79 |
| 2009 | 0.8922 | 82.13 |
| 2010 | 0.8617 | 81.23 |
| 2011 | 0.8286 | 81.26 |
| 2012 | 0.7967 | 82.34 |
| 2013 | 0.7703 | 84.45 |
| 2014 | 0.7537 | 87.39 |
| 2015 | 0.7498 | 90.73 |
| 2016 | 0.7595 | 93.96 |
| 2017 | 0.7808 | 96.57 |
| 2018 | 0.8102 | 98.25 |




Annual evolution of the inclination vector for different years.



# Solar Radiation Pressure

Acting mainly on the eccentricity vector

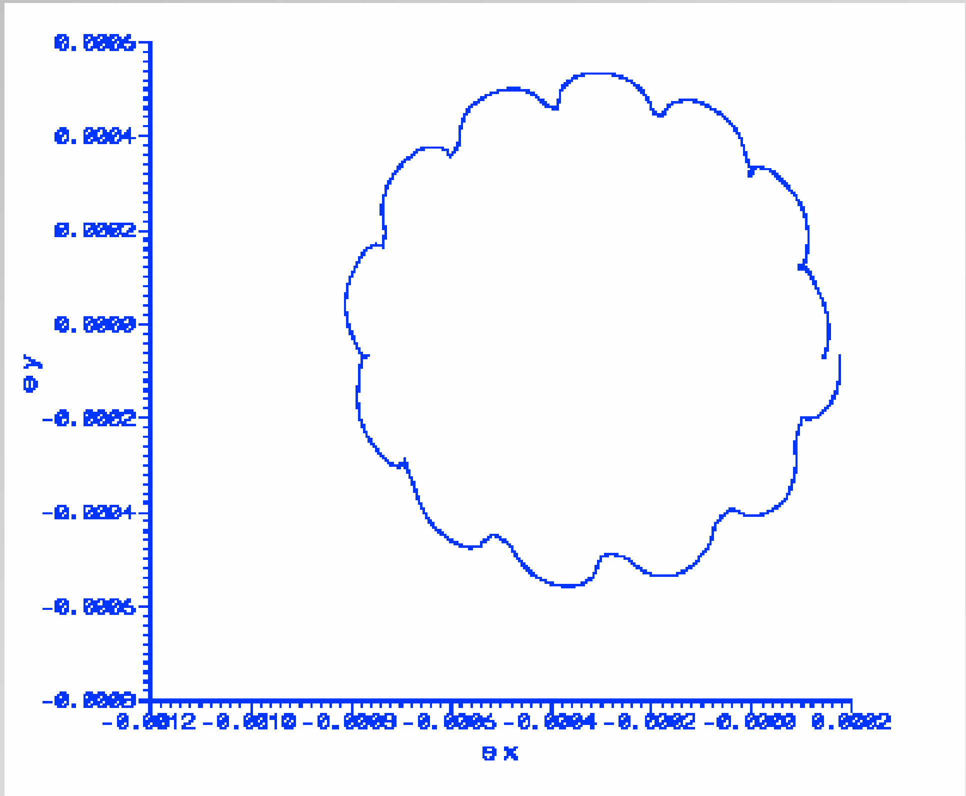
$$\frac{de_x}{dt} = -\frac{1}{na^2}\frac{\partial R}{\partial e_y}$$

$$\frac{de_y}{dt} = \frac{1}{na^2}\frac{\partial R}{\partial e_x}$$

R perturbing potential depends on satellite mass, reflectivity and surface area, as well as shielding (Like the sail of a sailboat).

# Solar Radiation Pressure

Eccentricity vector describes a circle with one year period




$$e_x(t) = e_x(t_0) + R_e(\cos s_\odot(t) - \cos s_\odot(t_0)),$$

$$e_y(t) = e_y(t_0) + R_e(\sin s_\odot(t) - \sin s_\odot(t_0)),$$

# Model for the GEO Orbit Evolution

We consider the evolution of mean orbital elements when the perturbing function only contains those terms causing long period perturbations. Thus,

- The evolution of the mean longitude is parabolic
- The evolution of the mean inclination vector has a secular drift in a direction (varying each year) with periodic components superimposed
- The annual evolution of the mean eccentricity vector can be approximated by a circle.

# Linear Manoeuvres

The effects on the synchronous elements due to a velocity increment $\Delta\vec{V}$, (split into the radial, $\Delta\vec{V}_r$, tangential, $\Delta\vec{V}_t$, and normal, $\Delta\vec{V}_n$, directions), from a manoeuvre trust in a sidereal time in the satellite of $s_b$, are given by the following linearized equations:

$$
\left.
\begin{aligned}
\Delta e_x &= \frac{2V_t}{V}\cos(s_b) + \frac{V_r}{V}\sin(s_b) \\
\Delta e_y &= \frac{2V_t}{V}\sin(s_b) - \frac{V_r}{V}\cos(s_b) \\
\Delta i_x &= -\frac{V_n}{V}\cos(s_b) \\
\Delta i_y &= -\frac{V_n}{V}\sin(s_b)
\end{aligned}
\right\}
\Rightarrow
\left\{
\begin{aligned}
\Delta V_t &= V\frac{\left|\Delta\vec{e}\right|}{2} \\
\Delta V_n &= V\left|\Delta\vec{i}\right| \\
\Delta V_r &= V\left|\Delta\vec{e}\right|
\end{aligned}
\right.
$$

# Assumptions for modelling the cost function

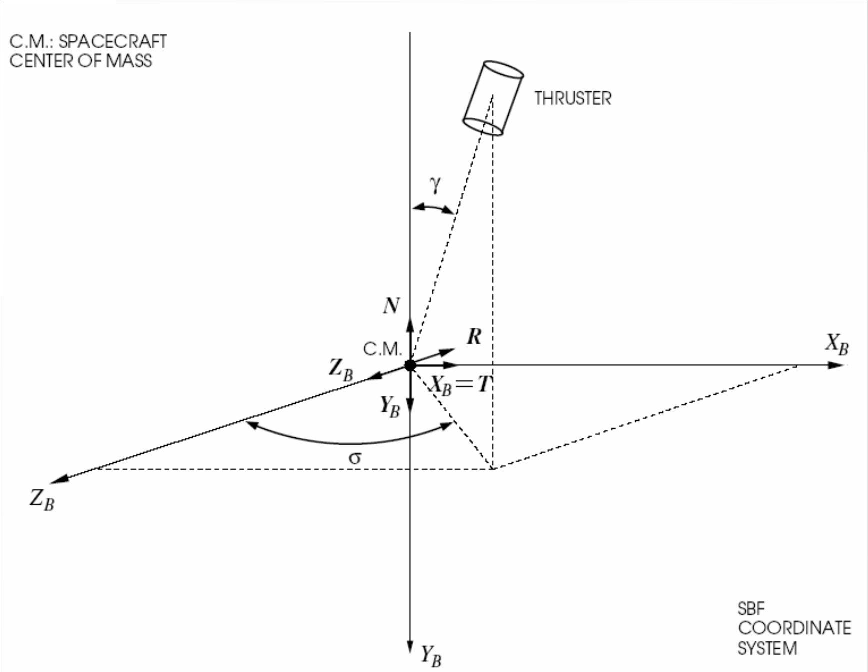
- Fix the longitude $l_s$=30ºW

$$\Longrightarrow \quad \ddot{l} = -0.000887 \deg/ day^2$$

- Fix the longitude and latitude dead-bands to be ±0.05º.
- Fix the year to be 2008. So Δi=0.9173, $\Omega_{sec}$=83.79º
- Consider each day separately and assume that 21 march corresponds to $s_\odot$=0º and $n_\odot$=0.9856 deg/day in order to model the solar radiation pressure effect.

# More assumptions

- Assume only 1 thruster, whose direction is defined by a *cant angle*, γ, and *slew angle*, σ.
- Assume the thruster is a *Stationary Plasma Thruster,* which gives $F=61.5 \times 10^{-3}$ N.
- Assuming the mass of the satellite is 4000kg we get an acceleration of $a=1.537 \times 10^{-5}$ m/s$^2$



# Cost function

Define:

c1=a sinγ cosσ

c2=-a sinγ sinσ

c3= -a cosγ

$$\Rightarrow \quad f(T_d(s_b,\gamma,\sigma)) = 3T_d = \frac{\Delta V_r}{c_1} + \frac{\Delta V_t}{c_2} + \frac{\Delta V_n}{c_3}$$

$$\left|\Delta \vec{i}\,\right| = ? \qquad \left|\Delta \vec{e}\right| = ?$$

# Constraints

Equality constraints:

$$\frac{\Delta V_t}{c_2} - \frac{\Delta V_n}{c_3} = 0 \qquad \frac{\Delta V_r}{c_1} - \frac{\Delta V_t}{c_2} = 0$$

Inequality constraints:

$$c_1, c_2, c_3 > 0$$

$$\left(\left|c_4 \cos s_b\right| + \left|c_4 \cos \Omega\right| - i_x\right)^2 + \left(\left|c_4 \sin s_b\right| - \left|c_4 \sin \Omega\right| + i_y\right)^2 - i_c^{\,2} \le 0$$

Where $c_4 = \dfrac{\left|\Delta i_{\mathrm{sec}.year}\right|}{365}$

# Eccentricity correction:

$$\vec{e}^{\,-} = e_0{}' + \mathrm{R}_{\mathrm{e}} \begin{Bmatrix} \cos s_{\odot}(t) \\ \sin s_{\odot}(t) \end{Bmatrix}$$

$$\vec{e}^{\,+} = e_0 \begin{Bmatrix} -\cos s_b \\ -\sin s_b \end{Bmatrix} + \mathrm{R}_{\mathrm{e}} \begin{Bmatrix} \cos s_{\odot}(t+1) \\ \sin s_{\odot}(t+1) \end{Bmatrix}$$

By

$$\left|\vec{e}^{\,+}\right| = e_c$$

$$\left|\Delta\vec{e}\right| = \left|\vec{e}^{\,+} - \vec{e}^{\,-}\right|$$

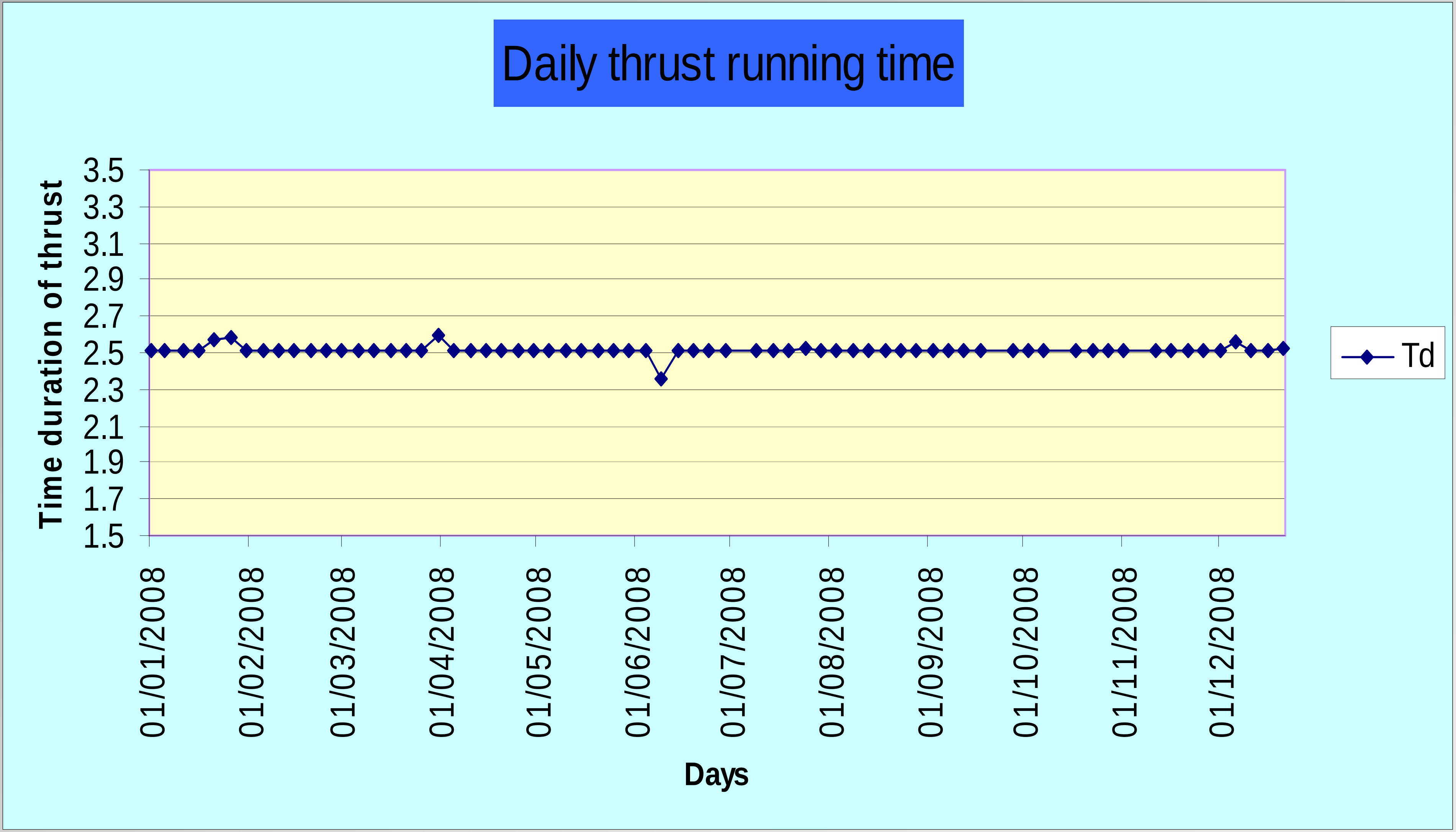
# Program

```
function returns= func_a_mini(variable)
% This is the cost function that returns the engine's time to correct the
% orbit. This function is wich we have to minimize.
Sb=variable(1);
slew=variable(2);
cant=variable(3);
global day Sun_day;
parameters;
returns = abs(Delta_Vr/c1+Delta_Vt/c2+Delta_Vn/c3);
```

This is the cost function in Matlab.

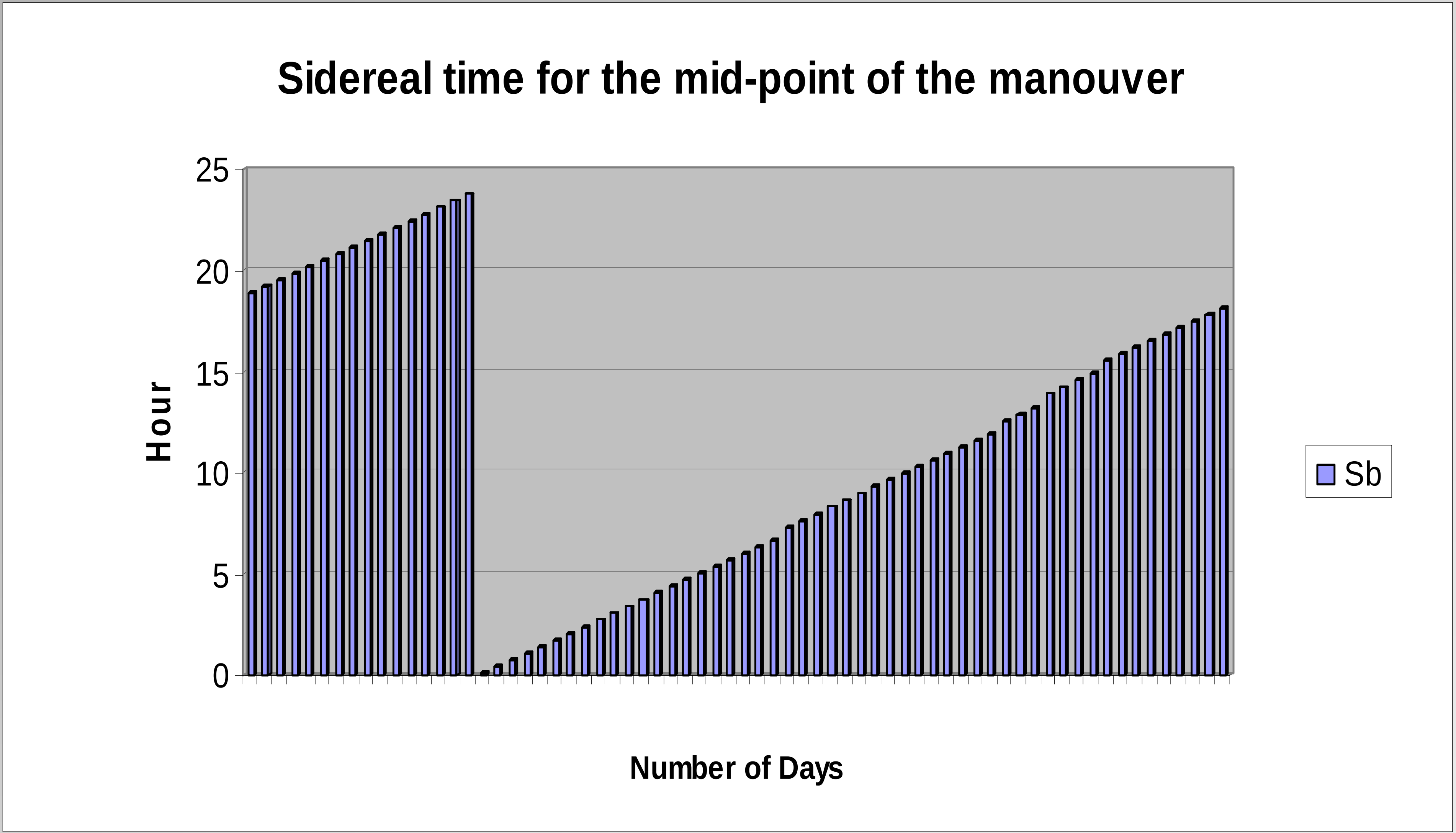
```
function [c,ceq] = restricciones(variable)
% Number of variables: 3
% Functions
%  Objective:                             func_a_mini
%  Gradient:                              finite-differencing
%  Hessian:                               finite-differencing (or Quasi-Newton)
%  Nonlinear constraints:                 restricciones
%  Gradient of nonlinear constraints:     finite-differencing
%Constraints
%  Number of nonlinear inequality constraints: 5
%  Number of nonlinear equality constraints:   2
%  Number of linear inequality constraints:    0
%  Number of linear equality constraints:      0
%  Number of lower bound constraints:          0
%  Number of upper bound constraints:          0
% Algorithm selected
%    medium-scale
Sb=variable(1);
slew=variable(2);
cant=variable(3);
global day Sun_day;
parameters;
ceq(1)=Delta_Vr/c1-Delta_Vt/c2;
ceq(2)=Delta_Vt/c2 - Delta_Vn/c3;
c=[];
c(1) = abs(c4*cos(Sb))+abs(c4*cos(Omega_sec))-ic+ix;
c(2) =- abs(c4*sin(Sb))+abs(c4*sin(Omega_sec))-ic+iy;
c(3) = -c1;
c(4) = -c2;
c(5) = -c3;
```

# Results



Daily thrust running time
Time duration of thrust
3.5
3.3
3.1
2.9
2.7
2.5
2.3
2.1
1.9
1.7
1.5
01/01/2008
01/02/2008
01/03/2008
01/04/2008
01/05/2008
01/06/2008
01/07/2008
01/08/2008
01/09/2008
01/10/2008
01/11/2008
01/12/2008
Days
Td

# More results



# Final conclusions

Linear relationship between $s_b$ and $s_\odot$ due to the prominence of Δi, brought about by the lunisolar perturbation.

$T_d$ more or less constant at 2.5 hours because we don´t consider eclipse effects (during which, manouvres are forbidden).

# Further Work

- Eclipse Effects – When the Earth is between the satellite and the sun, manouvres are forbidden and more correction is needed subsequently
- Complexify the model by removing assumptions:

  Consider different longitudes;

  consider more than one thruster.