Interest Rate Curves Calibration with Monte-Carlo Simulation

Problem presented by Indizen Technologies

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Main purpose

In finance the **yield curve** is the relation between the *interest rate* and the *time to maturity* of the debt for a given borrower in a given currency. The value of an interest rate curve can be known today and can be used to obtain today's values for fixed income securities, futures, derivatives etc. Now, if we wish to know the value of these securities in the future, the interest rate curves must be simulated. Our work deals with this simulation.

Montecarlo simulator

Lognormal model for the risk factors (Simple compounding interest rates)

$$\frac{dX}{X} = \mu dt + \sigma dW$$

where dW is a Wiener process, ie

$$dW = \sqrt{dt} Z$$

where $Z \sim N(0,1)$, has a standard normal distribution.
Explicit solution

$$X(t) = X(t_0) \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)(t - t_0) + \sigma\sqrt{t - t_0} Z\right).$$

Generating correlated samples

$$X_j(t) = X_j(t_0) \exp\left(\left(\mu_j - \frac{\sigma_j^2}{2}\right)(t - t_0) + \sigma_j \sqrt{t - t_0} Z_j\right).$$

The main problem now is how to generate the N normal variables with the correlation implied by the data. This is a standard procedure that can be done in several ways. Here we need to take into account the time scaling factor in the stochastic term of the model.

Generating random samples with the data matrix

R^* is the matrix defined by the centered data:

$$r_{ij}^* = r_{ij} - \mu_j$$

and we have by definition (cf. **??** and **??**) that

$$V_{ml} = \operatorname{cov}(Z_m, Z_l) = \frac{\left\{ (R^*)^T R^* \right\}_{lm}}{(M-1)\,\sigma_l \sigma_m dt}$$

So that, if we rescale the data:

$$C_{ij} = \frac{r_{ij}^*}{\sigma_j \sqrt{(M-1)\,dt}}$$

Then

$$V = C^T C$$

is the covariance matrix of the returns.

Now consider the output of the product

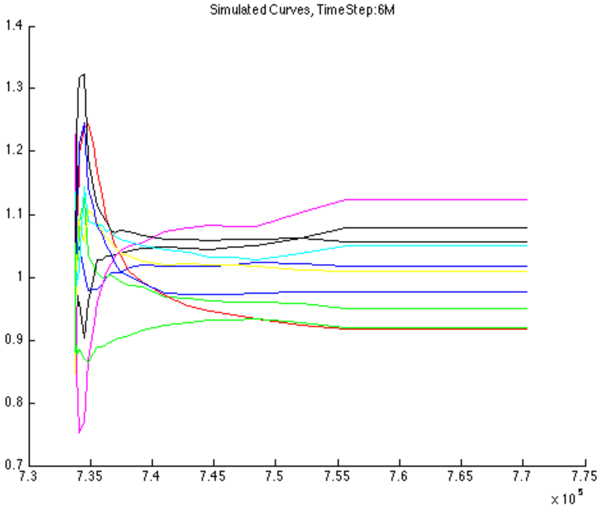
$$Z = \Omega\xi$$

where ξ is an M dimensional vector of independent standard normal variables and $\Omega = C^T$. If we consider the covariance matrix of the variable Z we have that

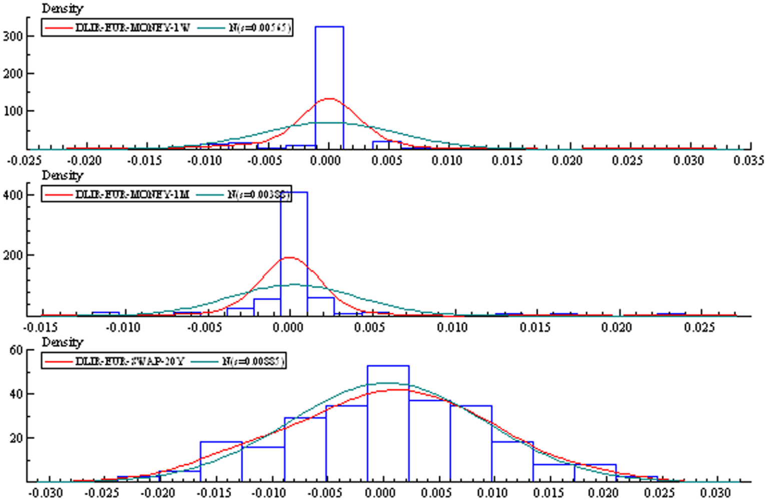
$$E\left[(\Omega\xi)(\Omega\xi)^T\right] = \Omega\Omega^T = C^T C = V$$

Therefore, we can generate a sample of normal correlated variables with V correlation matrix

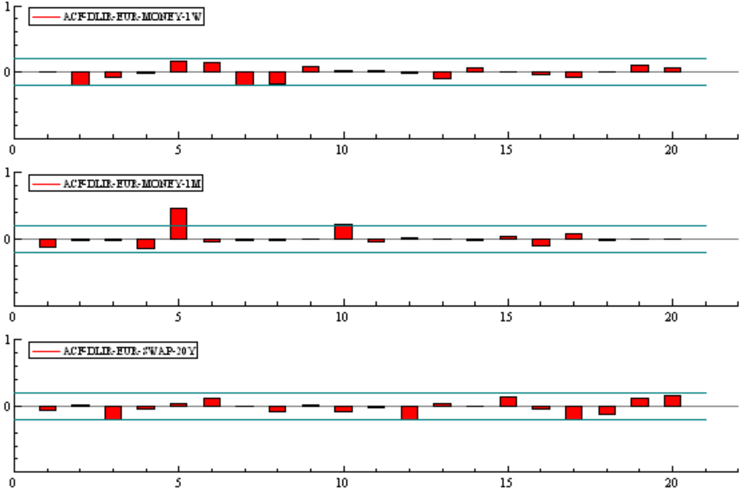
Outcome from the simulator (Indizen's program)



Returns distribution



Autocorrelation of the returns



Conclusion

The Lognormal assumptions are not valid for maturities shorter than two years.

The fitting method of Nelson and Siegel allows to construct the *instantaneous forward yield curve* by a family of functions consisting of a constant and the solutions of a second order differential equation with constant coefficients, when the roots of the associated polinomial are real and equal.

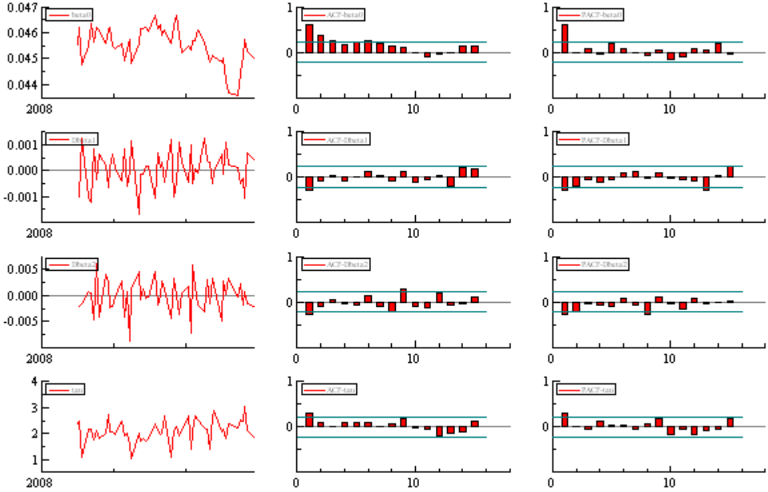
$$R(m) = \beta_0 + \frac{\tau}{m}(\beta_1 + \beta_2)(1 - \exp(-m/\tau)) - \beta_2 \exp(-m/\tau)$$

$R(m)$ are the continous compounding interest rates.
In this case β_0 is the behaviour for $m \to \infty$, $\beta_0 + \beta_1$ is the short term behavior, and $\beta_1 + \beta_2$ defines the mid term behaviour (about two or three years).

Fitting the betas with the data

We estimated the parameters for 102 dates using 31 maturities (between 1 day and 100 years) for each date.
The fitting process has been done with nonlinear least squares optimization using quasi Newton iterative process.
At the end of this estimation stage we have a set of parameters for each date.

Parameter calibration with historical data



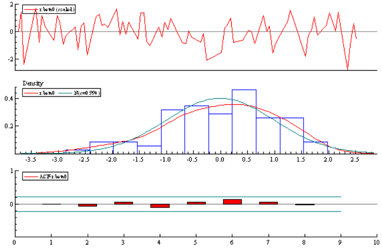
Stochastic processes for the Betas

$$(\beta_{0,t}-0.045) = 0.616(\beta_{0,t-1}-0.045)+\epsilon_{0,t} \text{ with } \epsilon_{0,t} \text{ i.i.d. } N(0, 0.0005)$$

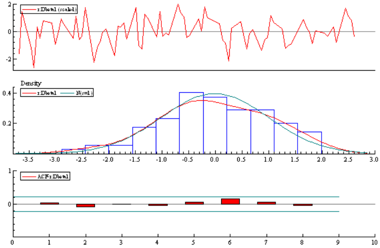
$$\Delta\beta_{1,t} = 3.25510^{-5} + \epsilon_{1,t} - 0.4028\epsilon_{1,t-1} \text{ with } \epsilon_t \text{ i.i.d. } N(0, 0.0006)$$

$$\Delta\beta_{2,t} = 0.000108 + \epsilon_{2,t} - 0.416\epsilon_{2,t-1} \text{ with } \epsilon_t \text{ i.i.d. } N(0, 0.0028)$$

$$\tau_t = 2.1004 + \epsilon_{3,t} + 0.268\epsilon_{3,t-1} \text{ with } \epsilon_t \text{ i.i.d. } N(0, 0.604)$$

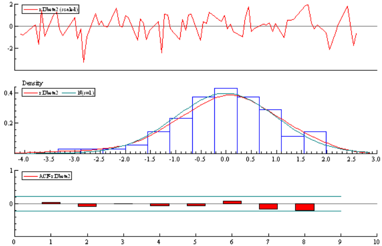


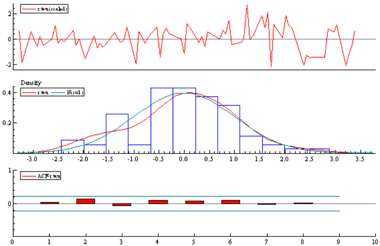
r:beta0 (scaled)
Density
r:beta0
N(s=0.994)
ACF-r:beta0



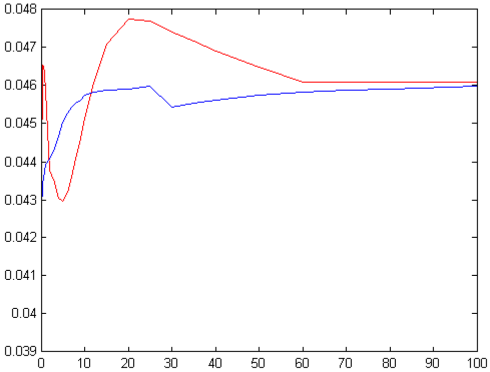
r:Dbeta1 (scaled)
Density
r:Dbeta1
N(s=1)
ACF-r:Dbeta1

Residuals for the fitting model

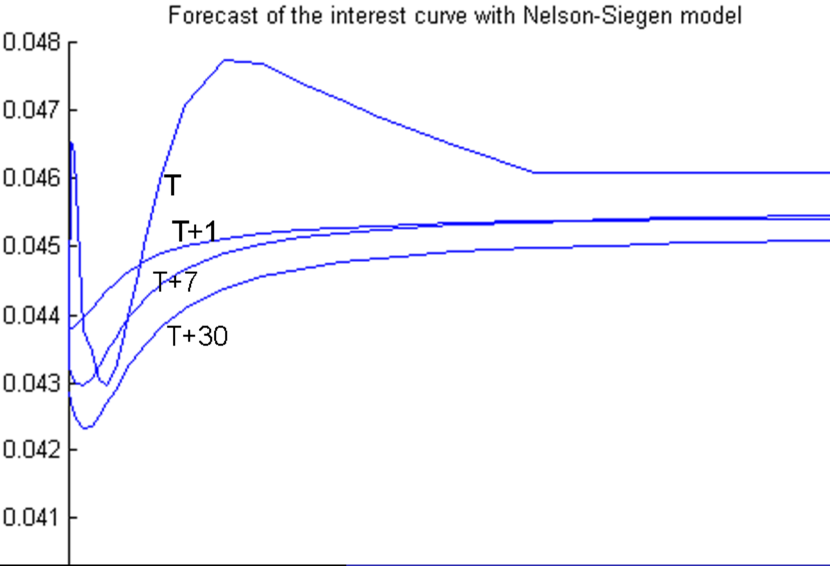




Fitted curves



Forecasted curves



Conclusions

- The lognormal assumption is only valid for the long term interest rates.
- The Monte Carlo simulator based on Geometric Brownian motion shows irregularities on the forecasted curves.
- The Nelson Siegel model generates smoother curves for the time structure of interest rates.
- The evolution of the parameters calibrated for the Nelson Siegel model are better described by time series models such as ARIMA models.