Polishing Lead Crystal Glass

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The Team

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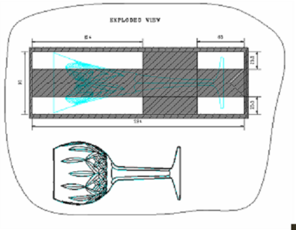
Summary

Introduction

- Irish manifacturer produces lead crystal glasses.
- They become opaque and rough after the cutting process.
- Polishing with immersion in acid.

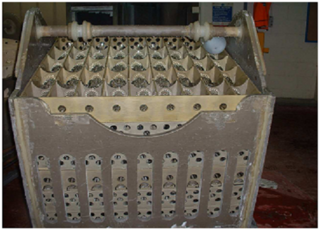
Polishing process

Acid immersion → Rinsing process → Settle down



The glasses are introduced in inserts like the one we can see in the picture.

Those inserts belong to a basket which is immersed in the acid solution.



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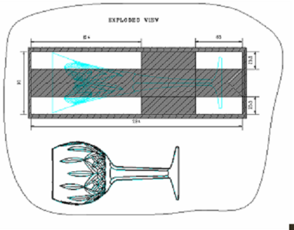

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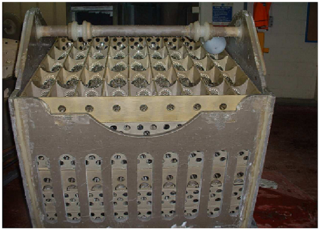

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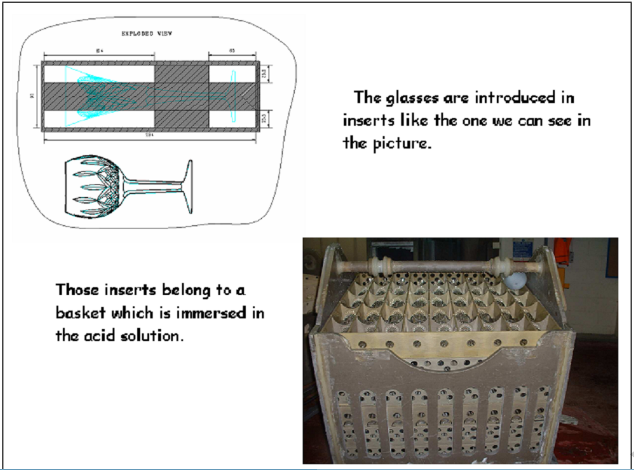
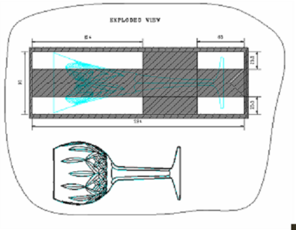


Polishing process

Acid immersion $\longrightarrow$ Rinsing process $\longrightarrow$ Settle down



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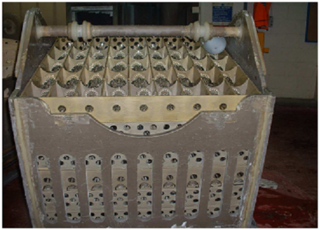


Polishing process

- Reactions.

$$
\begin{aligned}
SiO_2 + 4HF &\longrightarrow SiF_4 + 2H_2O \\
PbO + H_2SO_4 &\longrightarrow PbSO_4 + H_2O \\
K_2O + 2HF &\longrightarrow 2KF + H_2O \\
SiF_4 + 2HF &\longrightarrow H_2SiF_6
\end{aligned}
$$

- Oxid + Acid = Salts.
- Soluble salts disappear in the water.
- Insoluble salts precipitate.

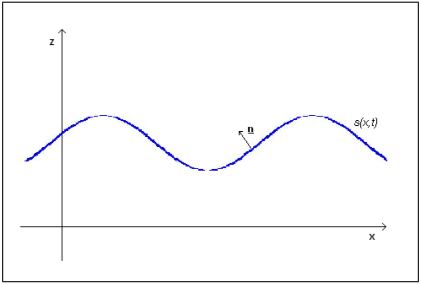
What is the problem?

- How does the process work?
- How long should the glass be immersed?
- Optimising the problem?

General assumptions

- One dimensional problem
- Initial form as the roughness: sinus .
- Homogeneous Neumann conditions.

Model 1: constant normal velocity

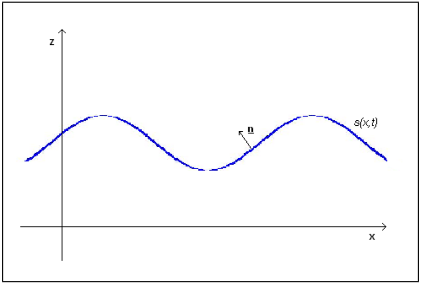


- $s(x,t)$ surface.
- $F(x,z,t) =$ $z - s(x,t) = 0$.
- $\underline{n} = \frac{\nabla F}{\|\nabla F\|} = \frac{(-s_x,1)}{\sqrt{1+s_x^2}}$.
- Material Derivative
  $\frac{\partial F}{\partial t} + v_n \|\nabla F\| = 0$
  v_n *rate removal surface*

First Model Equation

$$s_t = -v\sqrt{1+s_x^2}$$

Model 1: constant normal velocity



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Charpit Method

Non-dimensionalisated equation

$$s_t = -\sqrt{1 + s_x^2}$$

$$F(x, t, s, p, q) = q + \sqrt{1 + p^2} = 0, \quad p = s_x, \quad q = s_t$$

Problem

$$\begin{cases} \dot{x} = F_p \\ \dot{t} = F_q \\ \dot{s} = pF_p + qF_q \\ \dot{p} = -F_x - pF_s \\ \dot{q} = -F_t - qF_s \end{cases}$$

Problem and Solution

$$\begin{cases} \dot{x} = \frac{p}{\sqrt{1+p^2}} \\ \dot{t} = 1 \\ \dot{s} = -\frac{1}{\sqrt{1+p^2}} \\ \dot{p} = 0 \\ \dot{q} = 0 \end{cases}$$

$$\begin{cases} x = \xi \\ t = 0 \\ s = S_0(\xi) = A\sin(\xi) \\ p = S_0'(\xi) \\ q = -\sqrt{(1 + S_0'(\xi)^2)} \end{cases}$$

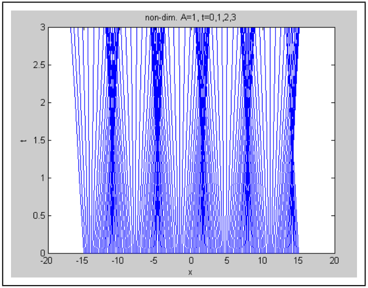
$$S(X(\xi,t),t) = S_0(\xi) - \frac{1}{\sqrt{1+S_0'^2}}t$$

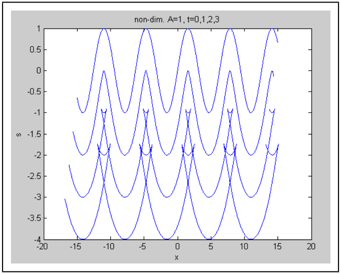
$$X(\xi,t) = \xi + \frac{S_0'}{\sqrt{1+S_0'^2}}t.$$

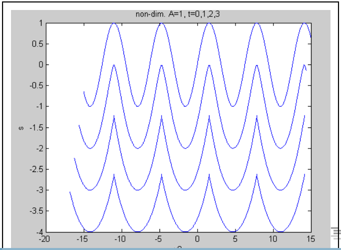
Plotting with Matlab

$$t \in [0, 3]$$

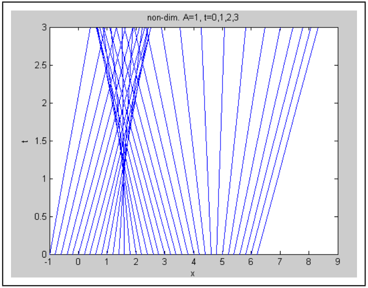
$$x \in [-15, 15]$$

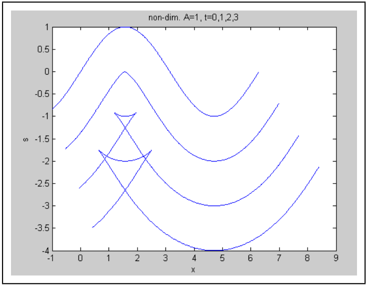






Plotting with Matlab

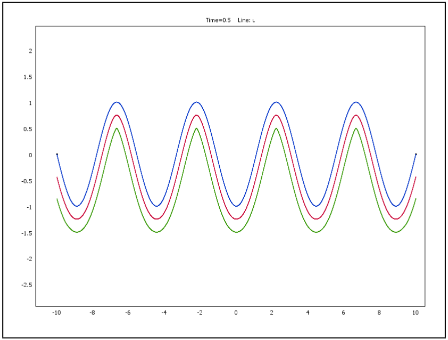


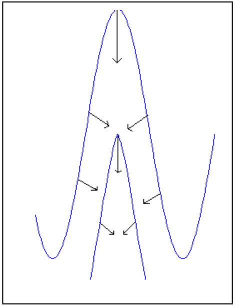


$$t \in [0, 3]\,, x \in [0, 2\pi]$$

$$time\ step = 1,\ space\ step\ = 1$$

Plotting with COMSOL Multiphysics





Model 2: linear velocity

- Linear relationship between velocity and surface curvature k.
- $v = v_0 + v_1 \kappa$.
- $\kappa = -\frac{s_{xx}}{(1+s_x^2)^{3/2}}$.

Second Model Equation

$$s_t = -v_0\sqrt{1 + s_x^2} + v_1 \frac{s_{xx}}{1 + s_x^2}$$

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Numerical simulations

1. Non-dimensionalization

$$s_t = -\left(1 + s_x^2\right)^{\frac{1}{2}} + \epsilon \frac{s_{xx}}{1 + s_x^2}$$

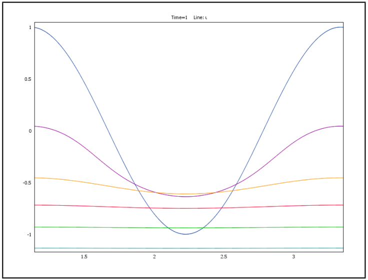
$$\epsilon = \frac{v_1}{l\, v_0}$$

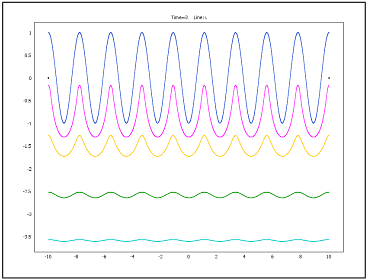
2. Finite elements method (COMSOL).

(video)

Critical ϵ value

ϵ **values**	**Results**
0	Previous model.
$\epsilon > 0.141$	The surface goes up at the beginning.
$0 < \epsilon \leq 0.141$	The surface always goes down.





(video)

Finite Difference Method (Matlab)

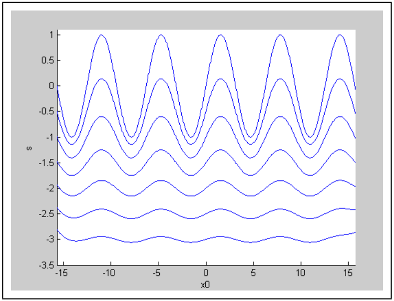
Numerical discretisation:

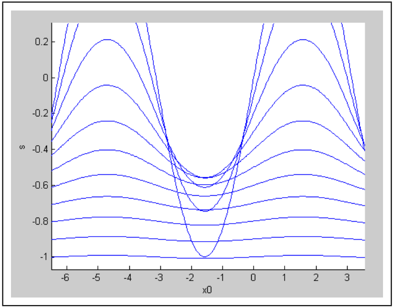
$$S_t = \frac{S_{n+1} - S_n}{\tau}$$

$$S_x = \frac{S_n(x+h) - S_n(x-h)}{2h}$$

$$S_{xx} = \frac{S_{n+1}(x-h) - 2S_{n+1}(x) + S_{n+1}(x+h)}{h^2}$$

Solutions



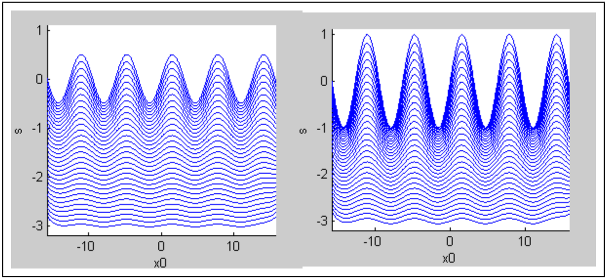


Critical ϵ value.

About initial conditions

$$A = \frac{a}{l}$$

where $a = height$ and $l = length$.



$A = 0.5$ $A = 1$

The bigger A is, the slower velocity goes.

Model 3: exponential velocity

Exponential relationship between normal velocity and surface curvature.

$$\Downarrow$$

$$v = v_0 + v_1 k = v_0 \left(1 + \frac{v_1}{v_0} k\right) \cong v_0 \exp\left(\frac{-v_1 s_{xx}}{v_0 (1+s_x^2)^{\frac{3}{2}}}\right)$$

Conclusions

1. Model 1, v as a constant. Hammilton-Jacobi non-linear equation:

$$s_t = -v\sqrt{1 + s_x^2}$$

2. $t^* = \frac{l}{v} t_c^*(A)$
3. Model 2, v linearly dependent on k ($v = v_0 + v_1 k$). Diffusion equation:

$$s_t = -\left(1 + s_x^2\right)^{\frac{1}{2}} + \epsilon \frac{s_{xx}}{1 + s_x^2}$$

4. $\epsilon = \frac{v_1}{l\, v_0}$ critical value, if it is too large it becomes unphysical.
5. Next step: Exponential problem.
6. Not as easy finding a proper velocity rate when several acids appear. New research?

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...questions?