

# Polishing Lead Crystal Glass

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Introduction

Model 1: constant normal velocity

Model 2: linear velocity

Model 3: exponential velocity

Conclusions

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# Summary

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- 4 Model 3: exponential velocity**
- 5 Conclusions**



# Introduction

- Irish manufacturer produces lead crystal glasses.
- They become opaque and rough after the cutting process.
- Polishing with immersion in acid.



## Introduction

Model 1: constant normal velocity

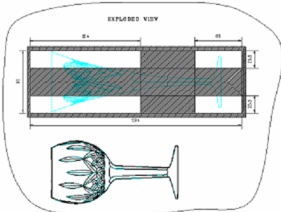
Model 2: linear velocity

Model 3: exponential velocity

Conclusions

# Polishing process

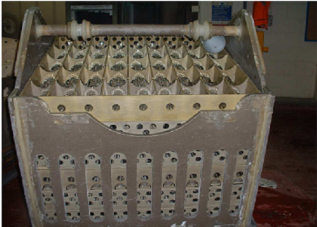
Acid immersion → Rinsing process → Settle down



The diagram, labeled "EXPLODED VIEW", shows a cross-section of a glass insert. It features a central cylindrical section with a textured surface, flanked by two rectangular sections. Dimensions are indicated with letters: 'E' for the length of the central section, 'D' for its diameter, 'G' for the length of the side sections, and 'D1' for their diameter. A blue cone-shaped area is shown within the central section, representing the polishing action. Below the diagram is a photograph of a glass goblet with a similar textured surface on its bowl.

**The glasses are introduced in inserts like the one we can see in the picture.**

**Those inserts belong to a basket which is immersed in the acid solution.**



The photograph shows a large, rectangular, perforated metal basket filled with numerous glass inserts. The basket is positioned in a facility, likely a polishing station, with a horizontal shaft and rollers visible above it.



## Introduction

Model 1: constant normal velocity

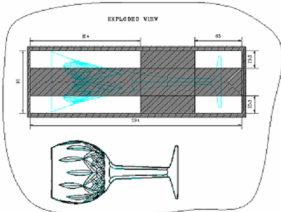
Model 2: linear velocity

Model 3: exponential velocity

Conclusions

# Polishing process

Acid immersion → Rinsing process → Settle down

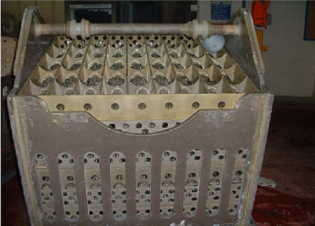


EXPLoded VIEW

The diagram shows an exploded view of a glass insert. It consists of a central cylindrical section with a diameter of 124 and a length of 100. This section is flanked by two shorter sections, each with a diameter of 124 and a length of 25. The total length of the insert is 150. The diagram also shows a glass tumbler with a similar insert inside it.

**The glasses are introduced in inserts like the one we can see in the picture.**

**Those inserts belong to a basket which is immersed in the acid solution.**

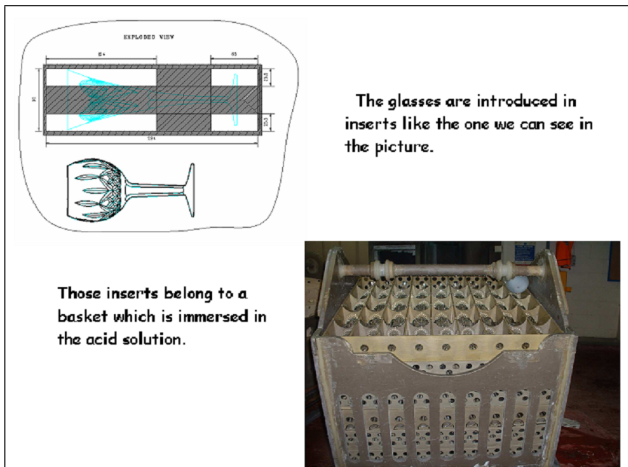


The photograph shows a metal basket filled with many glass inserts. The basket is made of metal and has a perforated bottom. The inserts are arranged in rows and are being processed in the basket.



# Polishing process

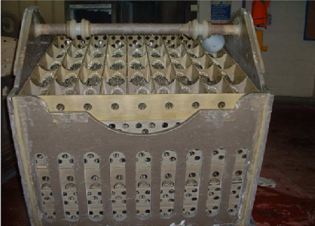
Acid immersion → Rinsing process → Settle down



The diagram shows an exploded view of a glass insert, which is a small, cylindrical component with a central hole and a textured surface. The insert is shown in relation to a larger glass component, likely a lead crystal glass. The diagram is labeled "EXPLODED VIEW" and includes various dimensions and labels such as "E", "G", "H", "I", "J", "K", "L", "M", "N", "O", "P", "Q", "R", "S", "T", "U", "V", "W", "X", "Y", "Z", "AA", "AB", "AC", "AD", "AE", "AF", "AG", "AH", "AI", "AJ", "AK", "AL", "AM", "AN", "AO", "AP", "AQ", "AR", "AS", "AT", "AU", "AV", "AW", "AX", "AY", "AZ", "BA", "BB", "BC", "BD", "BE", "BF", "BG", "BH", "BI", "BJ", "BK", "BL", "BM", "BN", "BO", "BP", "BQ", "BR", "BS", "BT", "BU", "BV", "BW", "BX", "BY", "BZ", "CA", "CB", "CC", "CD", "CE", "CF", "CG", "CH", "CI", "CJ", "CK", "CL", "CM", "CN", "CO", "CP", "CQ", "CR", "CS", "CT", "CU", "CV", "CW", "CX", "CY", "CZ", "DA", "DB", "DC", "DD", "DE", "DF", "DG", "DH", "DI", "DJ", "DK", "DL", "DM", "DN", "DO", "DP", "DQ", "DR", "DS", "DT", "DU", "DV", "DW", "DX", "DY", "DZ", "EA", "EB", "EC", "ED", "EE", "EF", "EG", "EH", "EI", "EJ", "EK", "EL", "EM", "EN", "EO", "EP", "EQ", "ER", "ES", "ET", "EU", "EV", "EW", "EX", "EY", "EZ", "FA", "FB", "FC", "FD", "FE", "FF", "FG", "FH", "FI", "FJ", "FK", "FL", "FM", "FN", "FO", "FP", "FQ", "FR", "FS", "FT", "FU", "FV", "FW", "FX", "FY", "FZ", "GA", "GB", "GC", "GD", "GE", "GF", "GG", "GH", "GI", "GJ", "GK", "GL", "GM", "GN", "GO", "GP", "GQ", "GR", "GS", "GT", "GU", "GV", "GW", "GX", "GY", "GZ", "HA", "HB", "HC", "HD", "HE", "HF", "HG", "HH", "HI", "HJ", "HK", "HL", "HM", "HN", "HO", "HP", "HQ", "HR", "HS", "HT", "HU", "HV", "HW", "HX", "HY", "HZ", "IA", "IB", "IC", "ID", "IE", "IF", "IG", "IH", "II", "IJ", "IK", "IL", "IM", "IN", "IO", "IP", "IQ", "IR", "IS", "IT", "IU", "IV", "IW", "IX", "IY", "IZ", "JA", "JB", "JC", "JD", "JE", "JF", "JG", "JH", "JI", "JJ", "JK", "JL", "JM", "JN", "JO", "JP", "JQ", "JR", "JS", "JT", "JU", "JV", "JW", "JX", "JY", "JZ", "KA", "KB", "KC", "KD", "KE", "KF", "KG", "KH", "KI", "KJ", "KK", "KL", "KM", "KN", "KO", "KP", "KQ", "KR", "KS", "KT", "KU", "KV", "KW", "KX", "KY", "KZ", "LA", "LB", "LC", "LD", "LE", "LF", "LG", "LH", "LI", "LJ", "LK", "LL", "LM", "LN", "LO", "LP", "LQ", "LR", "LS", "LT", "LU", "LV", "LW", "LX", "LY", "LZ", "MA", "MB", "MC", "MD", "ME", "MF", "MG", "MH", "MI", "MJ", "MK", "ML", "MM", "MN", "MO", "MP", "MQ", "MR", "MS", "MT", "MU", "MV", "MW", "MX", "MY", "MZ", "NA", "NB", "NC", "ND", "NE", "NF", "NG", "NH", "NI", "NJ", "NK", "NL", "NM", "NN", "NO", "NP", "NQ", "NR", "NS", "NT", "NU", "NV", "NW", "NX", "NY", "NZ", "OA", "OB", "OC", "OD", "OE", "OF", "OG", "OH", "OI", "OJ", "OK", "OL", "OM", "ON", "OO", "OP", "OQ", "OR", "OS", "OT", "OU", "OV", "OW", "OX", "OY", "OZ", "PA", "PB", "PC", "PD", "PE", "PF", "PG", "PH", "PI", "PJ", "PK", "PL", "PM", "PN", "PO", "PP", "PQ", "PR", "PS", "PT", "PU", "PV", "PW", "PX", "PY", "PZ", "QA", "QB", "QC", "QD", "QE", "QF", "QG", "QH", "QI", "QJ", "QK", "QL", "QM", "QN", "QO", "QP", "QQ", "QR", "QS", "QT", "QU", "QV", "QW", "QX", "QY", "QZ", "RA", "RB", "RC", "RD", "RE", "RF", "RG", "RH", "RI", "RJ", "RK", "RL", "RM", "RN", "RO", "RP", "RQ", "RR", "RS", "RT", "RU", "RV", "RW", "RX", "RY", "RZ", "SA", "SB", "SC", "SD", "SE", "SF", "SG", "SH", "SI", "SJ", "SK", "SL", "SM", "SN", "SO", "SP", "SQ", "SR", "SS", "ST", "SU", "SV", "SW", "SX", "SY", "SZ", "TA", "TB", "TC", "TD", "TE", "TF", "TG", "TH", "TI", "TJ", "TK", "TL", "TM", "TN", "TO", "TP", "TQ", "TR", "TS", "TT", "TU", "TV", "TW", "TX", "TY", "TZ", "UA", "UB", "UC", "UD", "UE", "UF", "UG", "UH", "UI", "UJ", "UK", "UL", "UM", "UN", "UO", "UP", "UQ", "UR", "US", "UT", "UU", "UV", "UW", "UX", "UY", "UZ", "VA", "VB", "VC", "VD", "VE", "VF", "VG", "VH", "VI", "VJ", "VK", "VL", "VM", "VN", "VO", "VP", "VQ", "VR", "VS", "VT", "VU", "VV", "VW", "VX", "VY", "VZ", "WA", "WB", "WC", "WD", "WE", "WF", "WG", "WH", "WI", "WJ", "WK", "WL", "WM", "WN", "WO", "WP", "WQ", "WR", "WS", "WT", "WU", "WV", "WW", "WX", "WY", "WZ", "XA", "XB", "XC", "XD", "XE", "XF", "XG", "XH", "XI", "XJ", "XK", "XL", "XM", "XN", "XO", "XP", "XQ", "XR", "XS", "XT", "XU", "XV", "XW", "XX", "XY", "XZ", "YA", "YB", "YC", "YD", "YE", "YF", "YG", "YH", "YI", "YJ", "YK", "YL", "YM", "YN", "YO", "YP", "YQ", "YR", "YS", "YT", "YU", "YV", "YW", "YX", "YY", "YZ", "ZA", "ZB", "ZC", "ZD", "ZE", "ZF", "ZG", "ZH", "ZI", "ZJ", "ZK", "ZL", "ZM", "ZN", "ZO", "ZP", "ZQ", "ZR", "ZS", "ZT", "ZU", "ZV", "ZW", "ZX", "ZY", "ZZ".

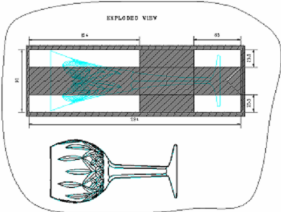
The glasses are introduced in inserts like the one we can see in the picture.

Those inserts belong to a basket which is immersed in the acid solution.



# Polishing process

Acid immersion → Rinsing process → Settle down

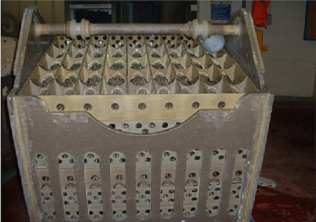


EXPLoded VIEW

The diagram shows an exploded view of a glass insert. The insert is a rectangular block with a central channel. Dimensions are labeled:  $E_1$ ,  $E_2$ ,  $E_3$ ,  $E_4$ ,  $E_5$ ,  $E_6$ ,  $E_7$ ,  $E_8$ ,  $E_9$ ,  $E_{10}$ ,  $E_{11}$ ,  $E_{12}$ ,  $E_{13}$ ,  $E_{14}$ ,  $E_{15}$ ,  $E_{16}$ ,  $E_{17}$ ,  $E_{18}$ ,  $E_{19}$ ,  $E_{20}$ ,  $E_{21}$ ,  $E_{22}$ ,  $E_{23}$ ,  $E_{24}$ ,  $E_{25}$ ,  $E_{26}$ ,  $E_{27}$ ,  $E_{28}$ ,  $E_{29}$ ,  $E_{30}$ ,  $E_{31}$ ,  $E_{32}$ ,  $E_{33}$ ,  $E_{34}$ ,  $E_{35}$ ,  $E_{36}$ ,  $E_{37}$ ,  $E_{38}$ ,  $E_{39}$ ,  $E_{40}$ ,  $E_{41}$ ,  $E_{42}$ ,  $E_{43}$ ,  $E_{44}$ ,  $E_{45}$ ,  $E_{46}$ ,  $E_{47}$ ,  $E_{48}$ ,  $E_{49}$ ,  $E_{50}$ ,  $E_{51}$ ,  $E_{52}$ ,  $E_{53}$ ,  $E_{54}$ ,  $E_{55}$ ,  $E_{56}$ ,  $E_{57}$ ,  $E_{58}$ ,  $E_{59}$ ,  $E_{60}$ ,  $E_{61}$ ,  $E_{62}$ ,  $E_{63}$ ,  $E_{64}$ ,  $E_{65}$ ,  $E_{66}$ ,  $E_{67}$ ,  $E_{68}$ ,  $E_{69}$ ,  $E_{70}$ ,  $E_{71}$ ,  $E_{72}$ ,  $E_{73}$ ,  $E_{74}$ ,  $E_{75}$ ,  $E_{76}$ ,  $E_{77}$ ,  $E_{78}$ ,  $E_{79}$ ,  $E_{80}$ ,  $E_{81}$ ,  $E_{82}$ ,  $E_{83}$ ,  $E_{84}$ ,  $E_{85}$ ,  $E_{86}$ ,  $E_{87}$ ,  $E_{88}$ ,  $E_{89}$ ,  $E_{90}$ ,  $E_{91}$ ,  $E_{92}$ ,  $E_{93}$ ,  $E_{94}$ ,  $E_{95}$ ,  $E_{96}$ ,  $E_{97}$ ,  $E_{98}$ ,  $E_{99}$ ,  $E_{100}$ . Below the diagram is a drawing of a glass with the insert inside.

The glasses are introduced in inserts like the one we can see in the picture.

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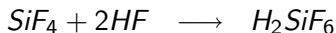
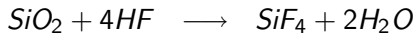
A photograph showing a metal basket filled with many small, rectangular glass inserts. The basket is made of perforated metal and has a handle on top. The inserts are arranged in a grid pattern.





# Polishing process

- Reactions.



- Oxid + Acid = Salts.
- Soluble salts disappear in the water.
- Insoluble salts precipitate.



# What is the problem?

- How does the process work?
- How long should the glass be immersed?
- Optimising the problem?

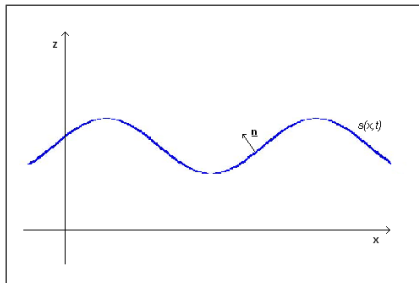


# General assumptions

- One dimensional problem
- Initial form as the roughness: sinus .
- Homogeneous Neumann conditions.



# Model 1: constant normal velocity



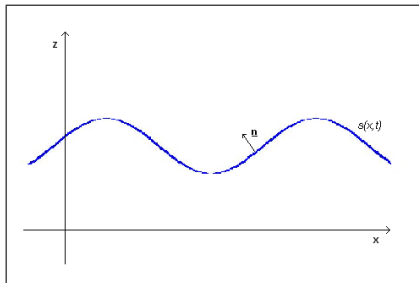
- $s(x, t)$  surface.
- $F(x, z, t) = z - s(x, t) = 0$ .
- $\underline{n} = \frac{\nabla F}{\|\nabla F\|} = \frac{(-s_x, 1)}{\sqrt{1+s_x^2}}$ .
- Material Derivative  $\frac{\partial F}{\partial t} + v_n \|\nabla F\| = 0$   
 $v_n$  rate removal surface

## First Model Equation

$$s_t = -v \sqrt{1 + s_x^2}$$



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 $\frac{\partial F}{\partial t} + v_n \|\nabla F\| = 0$   
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$$s_t = -v \sqrt{1 + s_x^2}$$



# Charpit Method

## Non-dimensionalised equation

$$s_t = -\sqrt{1 + s_x^2}$$

$$F(x, t, s, p, q) = q + \sqrt{1 + p^2} = 0, \quad p = s_x, \quad q = s_t$$

### Problem

$$\begin{cases} \dot{x} = F_p \\ \dot{t} = F_q \\ \dot{s} = pF_p + qF_q \\ \dot{p} = -F_x - pF_s \\ \dot{q} = -F_t - qF_s \end{cases}$$



# Problem and Solution

$$\left\{ \begin{array}{l} \dot{x} = \frac{p}{\sqrt{1+p^2}} \\ \dot{t} = 1 \\ \dot{s} = -\frac{1}{\sqrt{1+p^2}} \\ \dot{p} = 0 \\ \dot{q} = 0 \end{array} \right.$$

$$S(X(\xi, t), t) = S_0(\xi) - \frac{1}{\sqrt{1+S_0'^2}} t$$

$$\left\{ \begin{array}{l} x = \xi \\ t = 0 \\ s = S_0(\xi) = A \sin(\xi) \\ p = S_0'(\xi) \\ q = -\sqrt{(1 + S_0'(\xi)^2)} \end{array} \right.$$

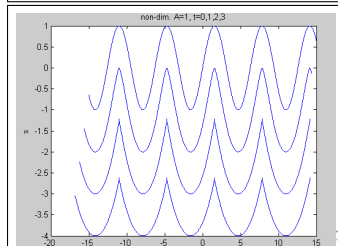
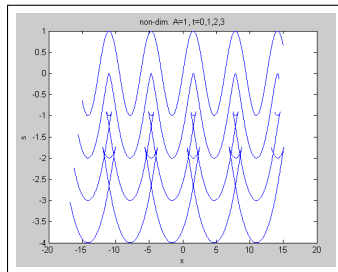
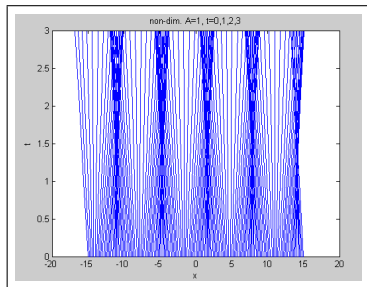
$$X(\xi, t) = \xi + \frac{S_0'}{\sqrt{1+S_0'^2}} t.$$



# Plotting with Matlab

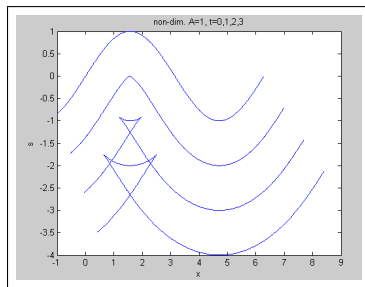
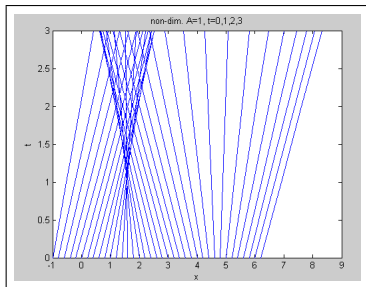
$$t \in [0, 3]$$

$$x \in [-15, 15]$$





# Plotting with Matlab

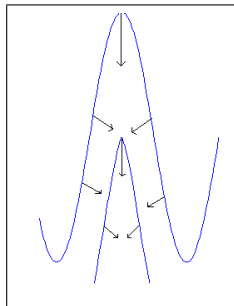
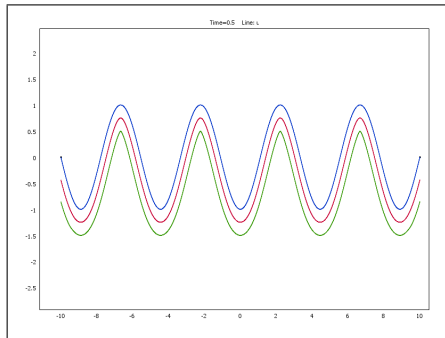


$$t \in [0, 3], x \in [0, 2\pi]$$

*time step* = 1, *space step* = 1



# Plotting with COMSOL Multiphysics



## Model 2: linear velocity

- Linear relationship between velocity and surface curvature  $k$ .
- $v = v_0 + v_1 k$ .
- $\kappa = -\frac{s_{xx}}{(1+s_x^2)^{3/2}}$ .

### Second Model Equation

$$s_t = -v_0 \sqrt{1 + s_x^2} + v_1 \frac{s_{xx}}{1 + s_x^2}$$



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### Second Model Equation

$$s_t = -v_0 \sqrt{1 + s_x^2} + v_1 \frac{s_{xx}}{1 + s_x^2}$$



# Numerical simulations

## 1. Non-dimensionalization

$$s_t = - \left(1 + s_x^2\right)^{\frac{1}{2}} + \epsilon \frac{s_{xx}}{1 + s_x^2}$$

$$\epsilon = \frac{v_1}{l v_0}$$

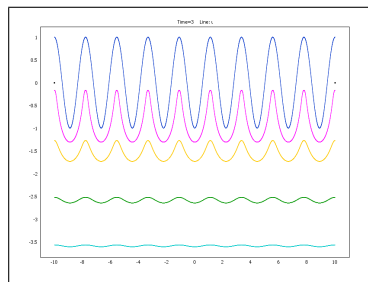
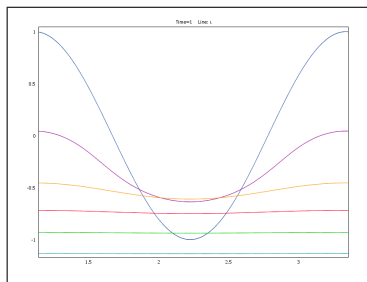
## 2. Finite elements method (COMSOL).

(video)



# Critical $\epsilon$ value

$\epsilon$ values	Results
0	Previous model.
$\epsilon > 0.141$	The surface goes up at the beginning.
$0 < \epsilon \leq 0.141$	The surface always goes down.



(video)



# Finite Difference Method (Matlab)

Numerical discretisation:

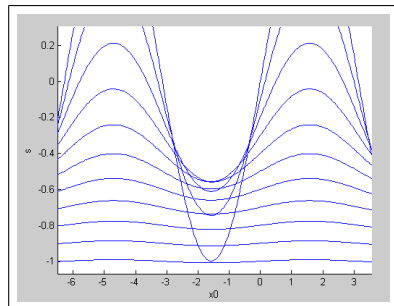
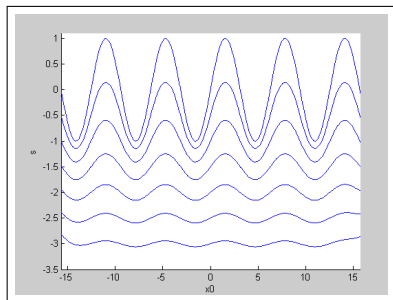
$$S_t = \frac{S_{n+1} - S_n}{\tau}$$

$$S_x = \frac{S_n(x+h) - S_n(x-h)}{2h}$$

$$S_{xx} = \frac{S_{n+1}(x-h) - 2S_{n+1}(x) + S_{n+1}(x+h)}{h^2}$$



# Solutions



Critical  $\epsilon$  value.

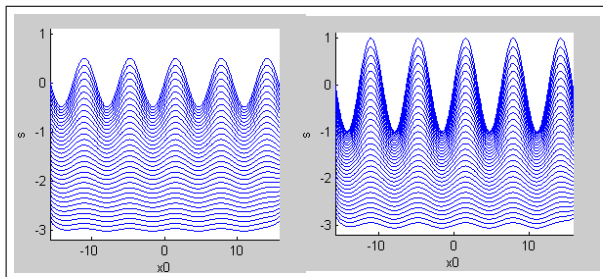




# About initial conditions

$$A = \frac{a}{l}$$

where  $a = \text{height}$  and  $l = \text{length}$ .



$A = 0.5$

$A = 1$

The bigger  $A$  is, the slower velocity goes.

## Model 3: exponential velocity

Exponential relationship between normal velocity and surface curvature.

$$\Downarrow$$
$$v = v_0 + v_1 k = v_0 \left( 1 + \frac{v_1}{v_0} k \right) \cong v_0 \exp \left( \frac{-v_1 s_{xx}}{v_0 (1 + s_x^2)^{\frac{3}{2}}} \right)$$



# Conclusions

1. Model 1,  $v$  as a constant. Hamilton-Jacobi non-linear equation:

$$s_t = -v\sqrt{1 + s_x^2}$$

2.  $t^* = \frac{1}{v}t_c^*(A)$
3. Model 2,  $v$  linearly dependent on  $k$  ( $v = v_0 + v_1k$ ). Diffusion equation:

$$s_t = -\left(1 + s_x^2\right)^{\frac{1}{2}} + \epsilon \frac{s_{xx}}{1 + s_x^2}$$

4.  $\epsilon = \frac{v_1}{v_0}$  critical value, if it is too large it becomes unphysical.
5. Next step: Exponential problem.
6. Not as easy finding a proper velocity rate when several acids appear. New research?



# Bibliography



Fritz John, *Partial differential equations* p21, Springer-Verlag New York Inc. ISBN 0387906096



Lawrence C. Evans, *Partial differential equations, chapter 3.2*, American Mathematical Society. ISBN 0821807722



Aris and Amundson, *First Order Partial Differential equations with Applications Volume 2* p49, Prentice Hall. ISBN 0135610923



P. Ferster, S.C. Muller, B. Hesso, *Critical size and curvature of wave formation in an excitable chemical medium*, Proc. Nail. Acad. Sci. USA vol 86 p 6831-6834, September 1989



Web of knowledge→Web of Science <http://www.accesowok.fecyt.es/>



Google <http://www.google.es/>



Wikipedia [http://en.wikipedia.org/wiki/Main\\_Page](http://en.wikipedia.org/wiki/Main_Page)



MACSI (Mathematics applications Consortium for Science and Industry) <http://www.macsi.ie>



Infante, Juan Antonio, *Numerical Methods Notes*





...questions?

