III MODELLING WEEK UCM
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Contents

Introduction to the risk measurement problem
Banks activity

- Financial Institutions get big part of their benefit from spread between the income they get from their assets (loans, mortgages, credit lines) and the costs of financing those activities.
- In a nutshell, most banks money comes from their **role as intermediators**.

Assets

Invest the money in contracts to clients and investors

Bank

- Spread or margin
- Fees

Liabilities

Cashflows coming from clients and investors.

- The aim of the *spread* or margin banks charge is twofold:
  - Give profit to the bank's owners
  - Cover their investment againts **possible loss** derived from underlying risk factors.

Why a bank could suffer losses?

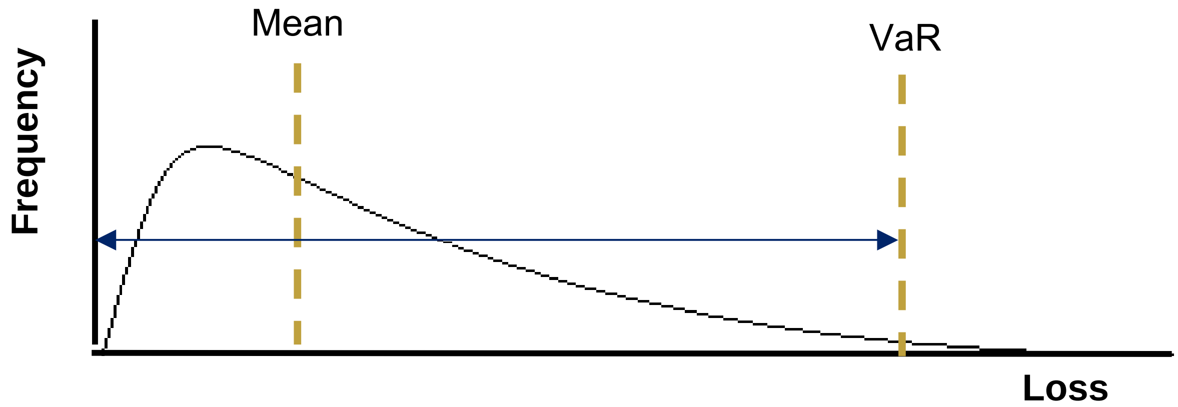
- Risk of not getting your money back due to solvency problems of your counterparty → ***Credit Risk***
- Risk of violent interest rate movement that depreciate your loan → ***Market / ALM risk***.
- Risk of some unethical action of an employee or a failure in systems → ***Operational risk***.
- Risk of a strike in your reputation due to bad practices → ***Reputational risk***.
- Risk of not being able to comply with your commitments due to temporary lack of cash → ***Liquidity Risk***.

Contents

Risk and its quantification

- Risk quantification is essential for financial entities and companies in general. The modelling of possible losses in a time horizon allows them to **take the necessary measures** to guarantee their survivance.
- **Risk quantification** has the following fundamental steps:
  - Define a **time horizon** along which you want to model the possible losses.
  - Identify the **fundamental risk factors**, elements whose evolution one needs to model.
  - Define a **dynamical model** to predict those factors evolution
  - Simulate **possible realizations** of those factors
  - Evaluate the **loss** in each scenario

➢ The output of the above process is a full loss probability distribution:



Relevant concepts:

- Expected loss
- Value at risk
- Confidence level

Contents

Basic elements of the problem

- During this week, we are going to face the problem of measuring the probability distribution of a bank due to Credit Risk.
- In order to do so, we identify the key elements of the loss. For each loan the bank has given to a customer, there are 3 risk elements that determine the possible loss on the loan:
  - **Probability of default: PD**

    Probability of a counterparty not being able to comply with its obligations with your bank.
  - **Exposure at default: EAD**

    The amount of the loan at default moment
  - **Loss given default: LGD**

    Once default has ocurred, the amount of the loan the bank is able to recover.
- All three elements are in principle stochastic.

Statement of the problem
Basic elements of the problem

- In general, a bank classifies its investments according to their fundamental features (e.g. credit cards, mortgages, unguaranteed loans etc). Such classification gives rise to the concept of "*portfolio*" or *risk unit* (RU in what follows).
- Each RU has a defined number of elements (contracts), each having a value of PD, LGD and EAD.
- Assuming EAD and LGD are deterministic, in a **given horizon**, each risk unit has a loss distribution given by:

$$L = \sum_{i=1}^{N} Ber\left(PD_i^z\right) \cdot EAD_i \cdot LGD_i$$

  - N is the number of contracts in the risk unit.
  - PD_i^z is the probability of default along the horizon, conditionned to a **state of the economy**. Such state of the economy is the **risk factor** that codifies the evolution of the losses.

- Loss distribution of *M risk units* of a bank will be given by:

$$L = L_1 + \ldots + L_M$$

- In order to properly measure the risk, it is essential to take into account the **correlation** between contracts within the same risk unit and the correlation between different risk units.

Contents

- Complubank has two businesses: credit cards and mortgages. Therefore, it has two RU, each composed of a number of contracts.
- IT staff at CompluBank are clever enough to keep in their database each contract of the bank with its LGD and EAD.
- For the sake os simplicity, along the problem LGD and EAD will not be considered stochastic but deterministic variables. That's to say, along the horizon of risk measurement they will not change.
- However, they did not keep track of the PD of each contract. It is an *unknown parameter*. All we know is that, at the moment of measure, all contracts in a given RU have **same PD**, which corresponds to the probability of default within a **1 year horizon**.
- This PD should be an **unconditional default probability**. This concept means that this probability should not be conditionned to any particular state of the economy realization.

Statement of the problem
Presentation of the problem

- Moreover, for each RU we have a time series of conditional PD^z for a whole economic cycle (from 1990 to 2007). In each of these series, for each three month period, we have the PD of the RU (PD of each contract of the unit) conditionned to the macroeconomic situation ocurred at that period.
- We assume the relation between quarterly known PD^z and the unconditionnal PD (unknown) is given by the Vasicek function:

$$pd^Z = N\left(\frac{N^{-1}(pd)-\sqrt{\rho}z}{\sqrt{1-\rho}}\right)$$

  - Z is assumed to be a N(0,1), common to all contratcs in a RU. It's called **systemic factor**, and represents the state of the economy in given time horizon. It is the fundamental risk driver of the loss.
  - ρ is an unknown parameter between 0 and 1.
- Correlation *Θ* between the Z factors of risk units is also an ***unknown parameter*** that can be derived from the correlation between systemic factors of each RU.

- Tha aims of the week are:
  - Be able to stablish a causal relation between the systemic factor os each RU and macroeconomic factors.
  - Be able to **compute the loss distribution** of both A (credit cards) and B (mortgage loans) risk units at CompluBank.
  - Be able to compute aggregated loss distribution of CompluBank, taking into account the diversification / correlation of both risk units.
- Once the above is reached:
  - Analyze the sensibility of percentile of loss distribution of both RUs A and B and global, to changes in PD, ρ and Θ.
  - Estimate losses in macroeconomic stress scenarios.

Presentation of the problem

- The following scheme can support during the week:
  - 1^{st} Day:
    - Find an estimator for the unconditional PD and ρ for each unit risk. Find an estimator for *Ɵ.* Apply the estimators to the data.
  - 2^{nd} Day:
    - Establish a causal relationship between macro variables and systemic factors Z for each unit risk.
  - 3^{rd} Day:
    - Simulate the loss distributions of each unit risk and the portfolio.
  - 4^{th} Day:
    - Study the sensitivity of the VaR of the loss distributions of each unit risk and the global portfolio to the risk parameters PD, ρ and *Ɵ*.
  - 5^{th} Day:
    - Estimate values of PD for certain conditional macro scenarios and conduct sensitivity analysis.
  - 6^{th} Day:
    - Conclusions and report elaboration

The data provided

- The next data set will serve for experimental work:
  - For each of the two units risk, a table with the ID, EAD and LGD for each operation.
  - For each of the two units risk, the time series of conditional PD.
  - A table with macro variables.