Modelling of Geothermal Reservoirs

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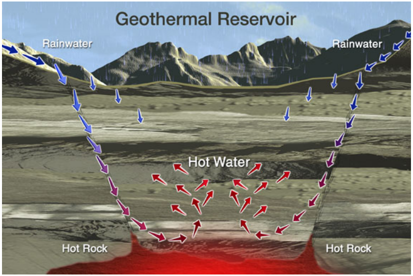
Outline

Geothermal energy

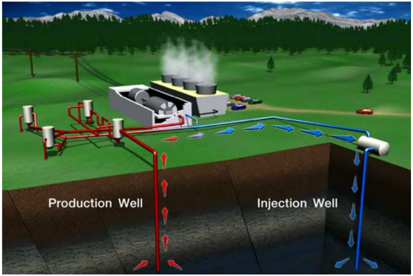
- Geothermal energy is due to the heat deep under the ground.
- Reservoirs are just a deposit at high temperature and pressure of:
  - Vapour.
  - Water.
  - Both.
- They are useful sources of energy because of their renewability.



How can energy be obtained?

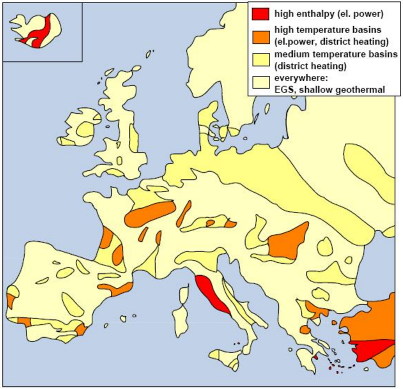
- Depending on the depth and temperature:

1. **Low** ΔT or **low** $x \Rightarrow$ **Low** henthalpy $\longrightarrow$ heating purposes
2. **High** ΔT or **high** $x \Rightarrow$ **High** henthalpy$\longrightarrow$ production wells power



Why are we interested in modelling geothermal reservoirs behaviour?

- Efficient energy extraction.
- Estimate recharging time of reservoir.
- Alternative location of existing reservoirs.



Main equations

Isobaric surfaces around the well, so we can simplify to a 1D model:

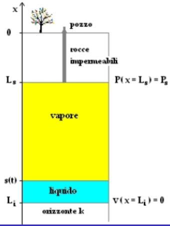
MASS CONSERVATION
$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = \Psi_{in} \quad (1)$$

DARCY'S LAW
$$v = -\frac{\kappa}{\phi\mu}\left(\frac{\partial P}{\partial x} + \rho g\right) \quad (2)$$

IDEAL GASES' LAW
$$\rho_g = \frac{P_g}{rT}\ ,\ r = \frac{\mathcal{R}}{M} \quad (3)$$

where:

- ρ: density
- v: velocity
- Ψ_{in}: source term
- M: molecular mass
- $\mathcal{R}$: Rydberg constant
- g: gravity
- P: pressure
- T: temperature
- κ: permeability
- ϕ: porosity



Isolated gas model

LINEAR DEPENDENCE $T = T(x) = T_i - \frac{T_i - T_s}{L_i - L_s}(x - L_i)$

So, as we consider steady state ($v_g = 0$):

$$\frac{\partial \rho_g}{\partial t} + \frac{\partial}{\partial x}(\rho_g v_g) = 0$$

$$v_g = -\frac{\kappa}{\phi\mu}(\frac{\partial P_g}{\partial x} + \rho_g g) = 0 \Rightarrow \frac{\partial P_g}{\partial x} = -\rho_g g$$

$$P_g = \rho_g r T$$

we obtain:

$$\frac{\partial P_g}{\partial x} = -\frac{P_g}{rT} g \tag{4}$$

- Let consider that $L_s = -1300$ m, $L_i = -3000$ m, $T_s = 610$ K, $T_i = 520$ K; so:

$$P(x) = P_s \left(\frac{T(x)}{T_s} \right)^{-\alpha}, \ \alpha = \frac{g}{r} \frac{L_s - L_i}{T_i - T_s} \approx 0.52$$

  that is, usually under the saturated experimental pressure:

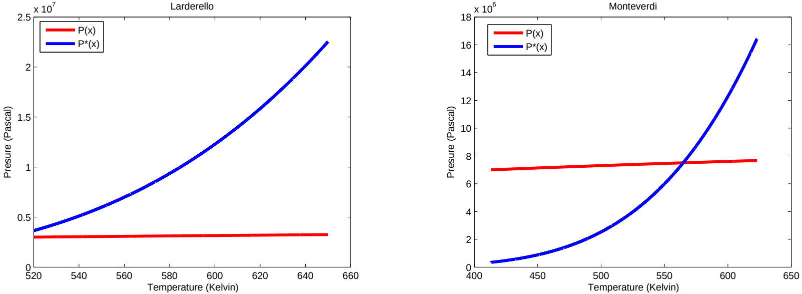
$$P^*(T) = 961.7 \mathrm{e}^{17.35 \frac{T - 273.15}{T}}$$

- Posible crossings $\Rightarrow$ phase transition $\rightarrow$ Monteverdi

Isolated liquid model

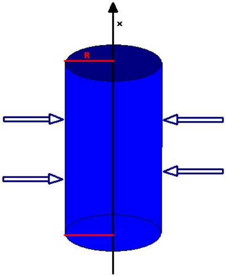
- Almost constant density ($\rho_l \approx$const.):
- Increase in presure is lineal:

$$\frac{\partial P_l}{\partial x} = -\rho_l g \Rightarrow P_l(x) = P_s - \rho g (x - L_s) \qquad (5)$$



Source term

- Flux is due to all water crossing the boundary R in radial direction (not in x-direction).
- $\bar{P}$ is the average pressure of the sourroundings. Let consider that $\bar{P} \approx P^* \rightarrow$ surrounded by water
- Proportional to the radial pressure gradient: $\Psi_{in} \propto \frac{\bar{P} - P_g}{R}$.
- Relationship constant is: $C\frac{\kappa}{\phi\mu}$, where R is a characteristic distance and C is adjusted in order to obtain a characteristic time of recharge of 1 year.



Undimensionalization and aditional conditions

- Adimensional transformations to correct weights properly with scaling factors:

$$\tilde{x} = \frac{x - L_s}{L_s - L_i}, \quad \tilde{t} = \frac{t}{t_c}, \quad \tilde{P} = \frac{P}{P_0}, \quad \tilde{T} = \frac{T}{T_0}$$

- Resulting model:

$$\frac{1}{\tilde{T}}\frac{\partial \tilde{P}}{\partial \tilde{t}} - \frac{\partial}{\partial \tilde{x}}\left(\frac{\tilde{P}}{\tilde{T}}\left[\frac{\partial \tilde{P}}{\partial \tilde{x}} + \frac{\tilde{P}}{\tilde{T}}\right]\right) = (\tilde{P}^* - \tilde{P}) \qquad (6)$$

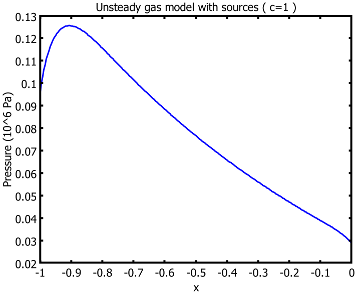
with:

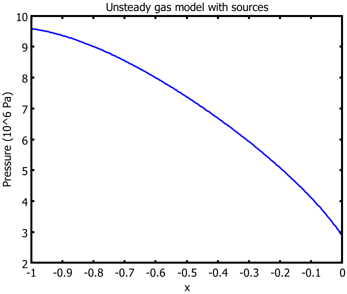
BOUNDARY CONDITIONS
$$\begin{cases} P(x = L_s) = P_s \\ P(x = L_i) = P_i \end{cases} \qquad (7)$$

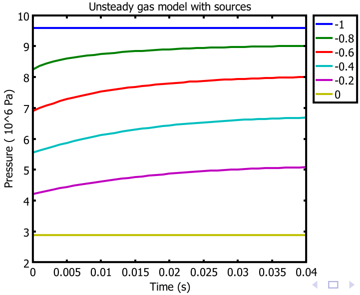
INITIAL CONDITION
$$P(t = 0) = P^0(x) \qquad (8)$$

Unsteady state

- $P^0(\tilde{x}) = P_i - (P_i - P_s)(1 + \tilde{x})$ is linear between the two boundary limits.
- In the equilibrium state, there can't be a maximum in the $P \leftrightarrow x$ plot, because that means there is a point in the well where we have opposite gas velocity directions and is inconsistent with the linear assumption of the temperature.


Unsteady gas model with sources (c=1)
Pressure (10^6 Pa)
x
0.13
0.12
0.11
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
-1
-0.9
-0.8
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0


Unsteady gas model with sources
Pressure (10^6 Pa)
x
10
9
8
7
6
5
4
3
2
-1
-0.9
-0.8
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0


Unsteady gas model with sources
Pressure (10^6 Pa)
Time (s)
10
9
8
7
6
5
4
3
2
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
-1
-0.8
-0.6
-0.4
-0.2
0

Free boundary model

- We model gas setting the saturation point at $\tilde{x} = -0.8$, where the gas phase changes into liquid and viceversa.
- At this point we'll take free boundary condintion using a moving surface $S(t)$, so that there is a mass and momentum conservation across it.

$$\text{MASS' CONSERVATION} \quad \overbrace{\rho_g(v_g - \dot{S})}^{\phi_g} = \overbrace{\rho_l(v_l - \dot{S})}^{\phi_l}$$
$$\text{MOMENTUM'S CONSERVATION} \quad \phi_g v_g + P_g = \phi_l v_l + P_l$$

and we get to (using that at the bottom $v_l = 0$):

$$\boxed{-\phi_g = -\rho_l \dot{S}} \qquad \boxed{P_l - P_s = \phi_g v_g} \tag{9}$$

met as the **Rankine-Hugoniot equations**

PHASE TRANSITION SURFACE

$$\dot{S} = \frac{\kappa}{\phi \mu_g} \left[\frac{\rho_g}{\rho_l} \left(\frac{\partial P_g}{\partial x} + \rho_g g \right) - \frac{\mu_g}{\mu_l} \left(\frac{\partial P_l}{\partial x} + \rho_l g \right) \right]$$

GAS

$$\frac{\partial P_g}{\partial t} - \frac{\kappa T}{\phi \mu_g} \frac{\partial}{\partial x} \left[\frac{P_g}{T} \left(\frac{\partial P_g}{\partial x} + \frac{P_g g}{rT} \right) \right] = C \frac{\kappa r T}{\phi \mu_g R} (\bar{P}_g - P_g)$$

LIQUID

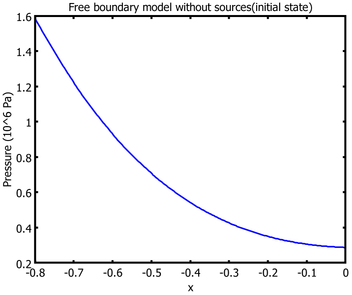
$$-\rho_l \frac{\partial^2 P_l}{\partial x^2} = \frac{C}{R} (\bar{P}_l - P_l)$$

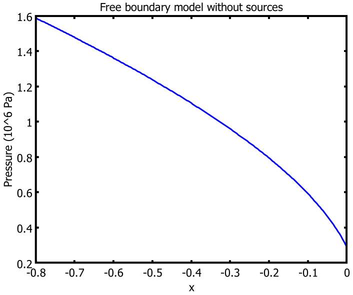
BOUNDARY CONDITIONS

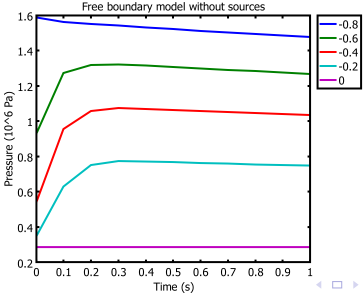
$$P_g(x = L_s) = P_s$$
$$P_l(x = S(t)) = \bar{P}^* + \left(\rho_g(v_g - v_l)^2\right)\Big|_{x=S(t)}$$
$$v_l(x = L_s) = 0$$
$$P_g(x = S(t)) = \bar{P}^*$$

INITIAL CONDITION

$$P_g(t = 0) = P_g^0(x)$$
$$P_l(t = 0) = P_l^0(x)$$
$$S(t = 0) = S^0$$


Free boundary model without sources(initial state)
Pressure (10^6 Pa)
x


Free boundary model without sources
Pressure (10^6 Pa)
x


Free boundary model without sources
Pressure (10^6 Pa)
Time (s)
-0.8
-0.6
-0.4
-0.2
0

Conclusions

- Adding the source term to the free boundary equations, it becomes more complicated to solve.
- The solution of the full problem is part of the ongoing research regarding the Geothermal reservoirs.
- Simple 1D can help to understand how things go.

Thanks for paying attention!

END