Modelling of Geothermal Reservoirs

J. C. Armenteros$^{(1)}$, J. Hernández$^{(1)}$, B. Hernández$^{(1)}$, I. Kakouris$^{(2)}$, J. Rico$^{(1)}$
(1): Universidad Complutense, Madrid.
(2): University of Oxford,Oxford.

in collaboration with :
Dr. Alessandro Speranza (Università degli Studi di Firenze, Italy)

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The aim of this article is to outline the equations that govern the process of energy production in a geothermal well and to point out the important advantages of this renewable and up-and-coming energy. In this paper, all data used was obtained from the geothermal reservoirs of Monteverdi, Landarello and Amiata, located in Toscany,the region in Europe with most geothermal potential.

In first place, we will assume some different simplifications to solve analytically the basic equations. The problem of solving isolated systems in static phase will be our first approach to more realistic and complex equations. A source term will be added later, making our one-phase geothermal well dynamic. We could obtain then an approximate velocity of extraction for not depleting the well. This figure will give an idea of the profitability of the extraction well before spending a large sum of money building it.

Our final approach to the real problem will be solving a system in which two phases of water coexist in the well and interact

1. Geothermal energy

Geothermal energy is a renewable energy derived from the natural heat of the earth's core, that continuously flows outward. When the rising hot water and steam is trapped in permeable and porous rocks under a layer of impermeable rock, it can form a geothermal reservoir. In typical continental crust, the temperature gradient in depth usually increase on average by ≈ 0.03 C/m.

According to the pressure and temperature, we can distinguish between three different kinds of reservoirs: vapor dominated (with a fairly low pressure), liquid dominated (with a very high pressure) and in a mixture of liquid and gas phases.

In addition, the exploitation of Geothermal energy is highly beneficial for the following reasons:

- Provides clean and safe energy using little land
- Is renewable and sustainable
- Generates continuos, reliable "baseload" power
- Conserves fossil fuels and contributes to diversity in energy sources

The aim of all this stuff is then improve the knowledge of the existing reservoirs and their recharging time for not to reach the edge of exploit. This topic is even more interesting in high geothermal gradient places such as Toscany, one of the most powerful in Europe.

2. Main equations

A case in point is flow to a well, which in sufficient proximity to the pipe may approximate radial symmetry, meaning that at any given time, parameters such as pressure and temperature depend only on distance R from the well, and possibly also on depth x however it may even be possible to neglect x-dependence and approximate the flow system as 1D radial only.

The symbols that are going to be used in the article are:

ρ : *density* M : *molecular mass* κ : *permeability*
v : *velocity* P : *pressure* ϕ : *porosity*
Ψ_{in} : *source term* T : *temperature* g : *gravity*

The main equations necessary for modelling the g.r. are:

1. Mass conservation law:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = \Psi_{in} \tag{1}$$

   For the mass contained in a volume, the net balance owing to the flux inflow (Ψ_{in}) through the surrounding surface could be put into a differential form using the Gauss' theorem.

2. The Darcy's law

$$v = -\frac{\kappa}{\phi\mu}\left(\frac{\partial P}{\partial x} + \rho g\right) \tag{2}$$

   relates the fluids motion in a porous medium. The minus sign precedent in the second term is due to the flow is from higher pressure to lower ones, analogous to the Fourier heat transfer, but including the gravity force.

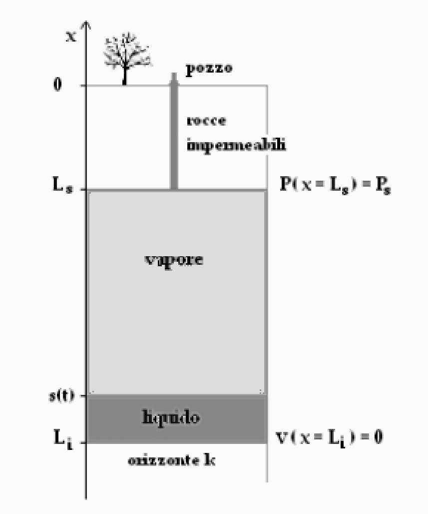
   Here we have κ is the absolute permeability, that is, a measure of the ease with which fluid can migrate through a porous medium; μ is the viscosity, which is a measure of the internal friction of the flowing fluid; and the porosity ϕ as the pore velocity with which the fluid parcels are actually flowing.

3. The Clausius-Clapeyron's equation of ideal gases complete the basic system of equations that we are going to take into account. Maybe more commonly write as $pV = nRT$, where n is the number of moles of the gas; after some manipulations gets into the equation:

$$\rho_g = \frac{P_g}{rT} \tag{3}$$

   , that is more suitable for our purposes however this equation is just an assumption of our gas behavior.

Our well starts at $x = 0$ point an decrease in height until $x = L_s$, where we find the beginning of the reservoir at $x = L_s$, whose first phase is gas because the pressure P_s is lower than below due to the weight over. More deeply, we find the phase transition location at $x = S(t)$ and the edge at $x = L_i$, where we will assume the liquid is static if there is no convective considerations (that could enlarge the heat flow to the top).



3. Isolated gas model

Firstly, we will treat an isolated vapor reservoir in stationary state, so we will assume that the gas velocity is null in the Tracy's law as well that there is no sources in the equation (1) of the mass conservation. Another supposition we are going to do is the linear relationship of the temperature in depth according to the expression:

$$T = T(x) = T_i - \frac{T_i - T_s}{L_i - L_s}(x - L_i)$$

So as we consider the steady state, $v_g = 0$

$$\frac{\partial \rho_g}{\partial t} + \frac{\partial}{\partial x}(\rho_g v_g) = 0$$

$$v_g = -\frac{\kappa}{\phi\mu}(\frac{\partial P_g}{\partial x} + \rho_g g) = 0 \Rightarrow \frac{\partial P_g}{\partial x} = -\rho_g g$$

Using the equation (3) $P_g = \rho_g r T$, we obtain

$$\frac{\partial P_g}{\partial x} = -\frac{P_g}{rT} g$$

This differential equation can be easily solved with separated variables.

Let consider that $L_s = -1300$ m, $L_i = -3000$ m, $T_s = 610$ K, $T_i = 520$ K; so:

$$P(x) = P_s \left(\frac{T(x)}{T_s}\right)^{-\alpha} \quad , \alpha = \frac{g}{r}\frac{L_s - L_i}{T_i - T_s} \approx 0.52$$

that is, usually under the saturated experimental pressure:

$$P^*(T) = 961.7 \mathrm{e}^{17.35 \frac{T-273.15}{T}}$$

According to the typical values we could consider, the non-linear relationship we reach is usually under the vapor-saturated pressure. There could be a crossing between the plots and in that case, a phase transition occurs if we are below the critical point.

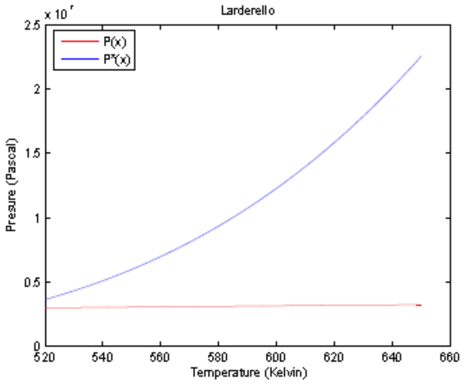
4. Isolated liquid model

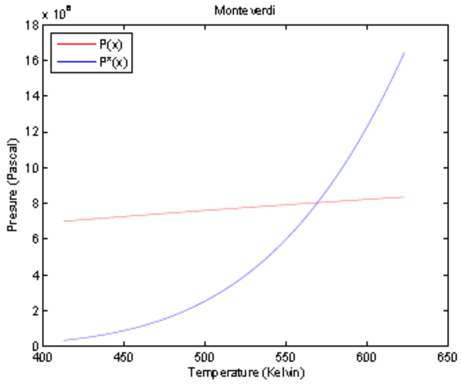
Under static conditions (no flow), we can assume that the liquid is non-compressible, so the density ρ_l doesn't vary in depth.

Solving the problem in the steady state, that is $v_l = 0$, we get to

$$\frac{\partial P_l}{\partial x} = -\rho_l g \Rightarrow P_l(x) = P_s - \rho g(x - L_s)$$

As the fluid pressure is given by the weight of the fluid over according to the linear equation above, we will have a plot upper the saturation pressure so that the reservoir is liquid dominated only (there is no crossing in the temperature interval considered).



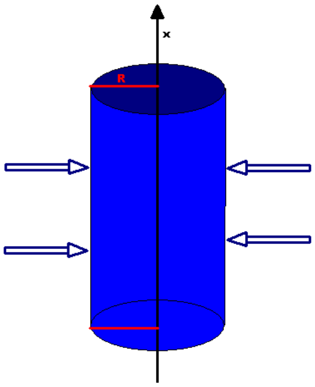


- Larderello : We consider the parameters given in the Larderello geothermal reservoir where $Li = -3500,\ Ls = -1300,\ Ti = 650,\ Ts = 520,\ Ps = 3e6,\ r = 4.6e2$, under the assumption that we only have gas phase. As we can see the line of $P(x)$ is lying below $P^*(x)$ line and this denotes that the reservoir is in a gas state and this is consistent with our initial assumptions.

- Monteverdi : We consider the parameters given in the Monteverdi geothermal reservoir where$Li = -5000,\ Ls = -750,\ Ti = 623,\ Ts = 413,\ Ps = 7e6,\ r = 4.6e2$, under the assumption that we only have gas phase. As we can see the line of $P(x)$ is crossing the line of $P^*(x)$ which is an indication that in this reservoir we do not only have gas phase and the two phases, liquid and gas, coexist.

5. Gas model with source term

The source term in the mass conservation given by the equation (1), is due to the radial water movement from a characteristic distance R but not in $x-$direction cause there is one opposite sense direction of the caps unitaries that cancel the flux there. So the flux comes from every direction equal and we can collapse all water source into the x axis.



We will define $\bar{P}$ as the average pressure of the surroundings at this distance R, so that we can consider that it is the saturated pressure and water further from R is in liquid phase. That is, at distance R, the water is in phase transition.

In the same way that we obtain the Darcy's law, we can similarly make that the source term is proportional to the pressure gradient given in this isobaric curve of radius R:

$$\psi_{in}\alpha(\overline{P} - P_g)$$

Relationship constant is: $C\frac{k}{\phi R}$

One over this relationship constant is the recharging time, the time needed for the reservoir to get to the equilibrium state. An adequate constant

C should be added also in order to set up the characteristic recharge time to 1 year. This effectiveness constant has its own physical sense that could be shown when we consider in ours model that there is no flux so that $\frac{\partial P_g}{\partial t} = \alpha(\bar{P}_g - P_g) \Rightarrow \bar{P}_g(t) = P_g(t) + (\bar{P}_g(0) - P_g(0))\mathrm{e}^{-alphat} \Rightarrow \bar{P}_g \to P_g$ in the equilibrium ($t_c = \frac{1}{\alpha}$).

The first step after construct the main equations we are going to use, is to find the suitable scaling factors in order to correct the weights of each term in the equation obtained. These are the unidimensional magnitudes we are going to manipulate to try to obtain a simpler model, thanks to equals every coefficient to one; and evaluate the characteristic values of the depth, time, pressure and temperature.

$$\tilde{x} = \frac{x - L_s}{L_s - L_i}, \quad \tilde{t} = \frac{t}{t_c}, \quad \tilde{P} = \frac{P}{P_0}, \quad \tilde{T} = \frac{T}{T_0}$$

Resulting model:

$$\frac{1}{\tilde{T}}\frac{\partial \tilde{P}}{\partial \tilde{t}} - \frac{\partial}{\partial \tilde{x}}\left(\frac{\tilde{P}}{\tilde{T}}\left[\frac{\partial \tilde{P}}{\partial \tilde{x}} + \frac{\tilde{P}}{\tilde{T}}\right]\right) = (\tilde{P}^* - \tilde{P})$$

with:

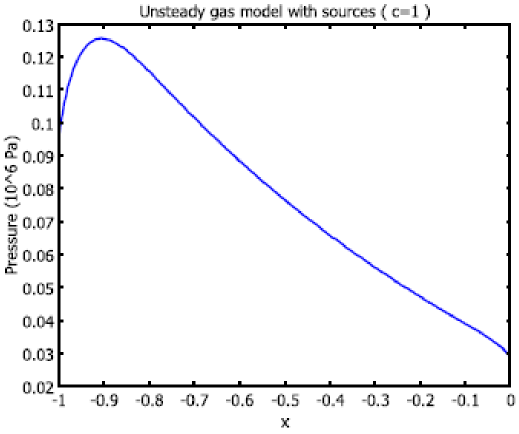
$$\textbf{BOUNDARY CONDITIONS} \qquad \begin{cases} P(x = L_s) = P_s \\ P(x = L_i) = P_i \end{cases}$$

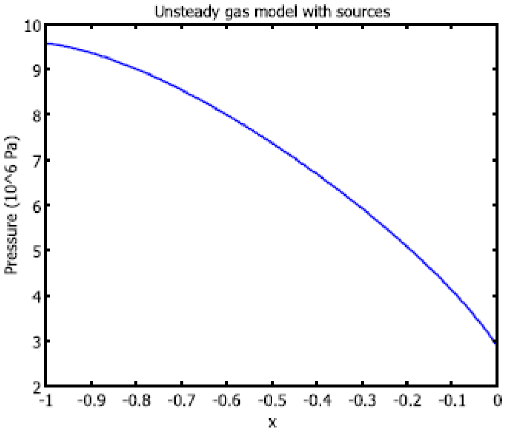
$$\textbf{INITIAL CONDITION} \qquad P(t = 0) = P^0(x)$$

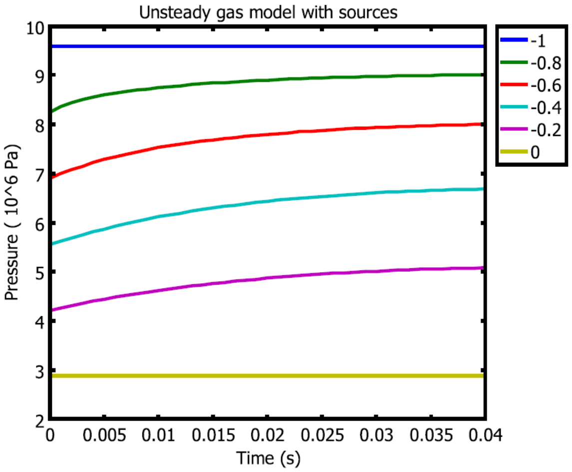
The initial condition will certainly be a linear relationship between the two edges of the reservoir phases or boundary limits cause we need the pressure in all points defined

$$P^0(\tilde{x}) = P_i - (P_i - P_s)(1 + \tilde{x})$$

In the equilibrium state, there can't be a maximum in the $P - x$ plot, because that means there is a point in the well where we have opposite gas velocity directions and is inconsistent with the linear assumption of the temperature. So we need to adjust the value of c to not have a maximum in the plot of equilibrium state(c=0.01).







In the previous plot are drawn the evolution of some points along the time. We can see that in the very first phase of the simulation the system evolves quickly, and it gets to the equilibrium state slower as time passes.

6. Free boundary model

We'll take free boundary condition using a moving surface $S(t)$ initially set at $\tilde{x} = 0.8$(is the saturation point,where the gas phase changes into liquid and viceversa.).

$S(t)$ evolves according to: $\xi = x - S$, with ξ the vertical distance to the near particles moving across the phase change surface. So, as the flux is defined as $\phi \equiv \rho\dot{\xi}$, we can evaluate the mass conservation as well as the momentum conservation with pressure as the only stress.

$$\begin{array}{rl} \textbf{MASS}'\textbf{CONSERVATION} & \overbrace{\rho_g(v_g - \dot{S})}^{\phi_g} = \overbrace{\rho_l(v_l - \dot{S})}^{\phi_l} \\ \textbf{MOMENTUM}'\textbf{SCONSERVATION} & \phi_g v_g + P_g = \phi_l v_l + P_l \end{array}$$

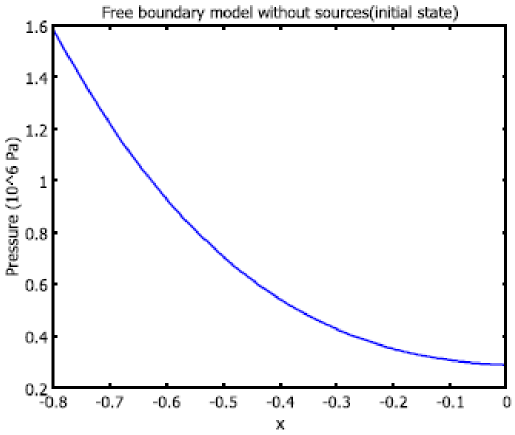
If we consider that the liquid velocity is null at the bottom ($v_l = 0$), we reach the Rankine-Hugoniot equations:

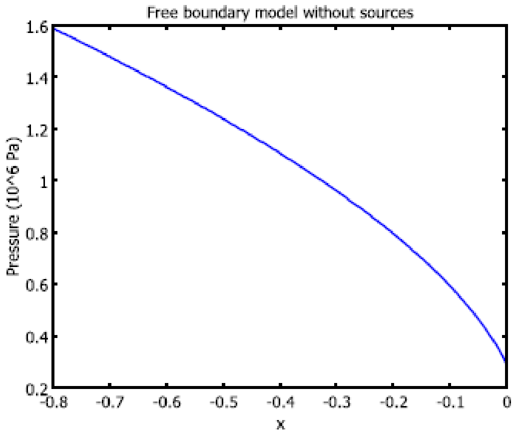
$$\boxed{-\phi_g = -\rho_l \dot{S}} \qquad \qquad \boxed{P_l - P_s = \phi_g v_g} \tag{4}$$

On the other hand, we have to point that it is not possible to have that $P_g|_{x=S(t)} = P_l|_{x=S(t)} = P^*(x = S(t))$ cause it implies no movement or no density variation, that is thermodynamically false because there should be a pressure gradient. So, we obtain the equivalent boundary condition $P_l|_{x=S(t)} = P_g|_{x=S(t)} + (\rho_g v_g^2)\big|_{x=S(t)}$ and when evaporating, the phase transition surface moves down. Additionally, we have that $P_g|_{x=S(t)} = P^*(x = S(t)) = \bar{P}^*$ cause the density and velocity of the gas are pretty small on the surface $S(t)$, what complete the system of equations.

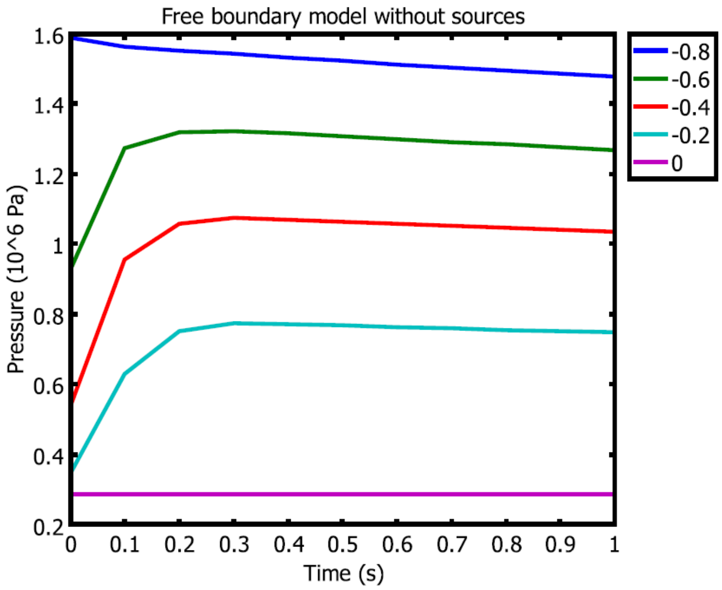
This system is very difficult to solve with sources and without any assumption over the velocity of the liquid at the bottom. The only results obtained are referred to this simplifications, and is characterized by a rapid transition to a stationary state that evolves after by increasing the gas phase(the liquid water evaporates).

The plot on the left is the initial state of the system, and on the right hand side is the stationary state. It can't be seen here, but the blue line is elongating to the left, which represents an increasing of the gas phase in the well as liquid water evaporates.





In the following plot, we represent for a set of points in the line, its evolution along the time. We point out the same as before, the system evolves rapidly to the stationary state, and then it's when it begin to increase the gas phase.



The complex model is a problem of ongoing research, but this results presented in this paper with simpler models could help to understand how a geothermal well works and to determine an approximate a velocity of extraction for not depleting the well. Also, with the equations presented above, one could have a guess of what is going to reach before beginning the very costly construction of a well, so that only when properties of the reservoir were appropriate, the well is built.