MODELLING A CADAVER DECOMPOSITION ISLAND TO ESTIMATE TIME OF DEATH

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Abstract. Propagation of fluids from a decomposing cadaver produces a characteristic stain on the landscape known as a cadaver decomposition island. This model predicts how this fluid propagates through the soil, and assesses the feasibility of its use as a predictor for the time of death of the cadaver from measurement of the island.

1. Introduction

In modern forensic science, soil is no longer regarded simply as a medium that is transferred to and from a crime scene, but has become a potential key in the investigation of decomposed remains.

If a cadaver is not immediately consumed, then it is subject to decomposition by insects and microbes. During the process, the body releases chemical components into the soil which arise by autolysis (self-digestion of the cells) and by putrefaction (the anaerobic decomposition of animal proteins). The decompositional products in this fluid can remain trapped within the soil matrix for extended periods of time. The entry of these materials into the soil provides a very local source of nutrients resulting in what is known as a cadaver decomposition island (CDI). This island is associated with increased soil microbial biomass and microbial activity. In particular, the degradation of proteins, lipids and carbohydrates will yield carbon-based, nitrogen-based, and phosphorous-based products which may be retained in the surrounding soil. Visually, the release of cadaveric fluids results in the formation of a CDI that is visible as dead plant material. Approximately 80 days after death the CDI is surrounded by an area of increased plant growth which might be used as a marker for the onset of the 'Dry' stage of decomposition. [2].

The total amount of fluid transmitted to the soil and the rate at which it is applied throughout the decomposition cycle is determined by the size of the carcass, and the environment in which decomposition is taking place.

1.1. **Soil Fundamentals.** Soil is a porous medium (Section 2.1) consisting of a solid matrix and a void space which is also referred to as the pore space. This void space is filled with one or more miscible fluids which will be referred to as water and air. A soil is said to be *unsaturated* if voids are present. Saturated zones can occur when all of the pores within them are filled and in this case, the interfaces between the saturated and unsaturated regions become free boundaries. In this case the water motion is referred to as *saturated-unsaturated flow.*

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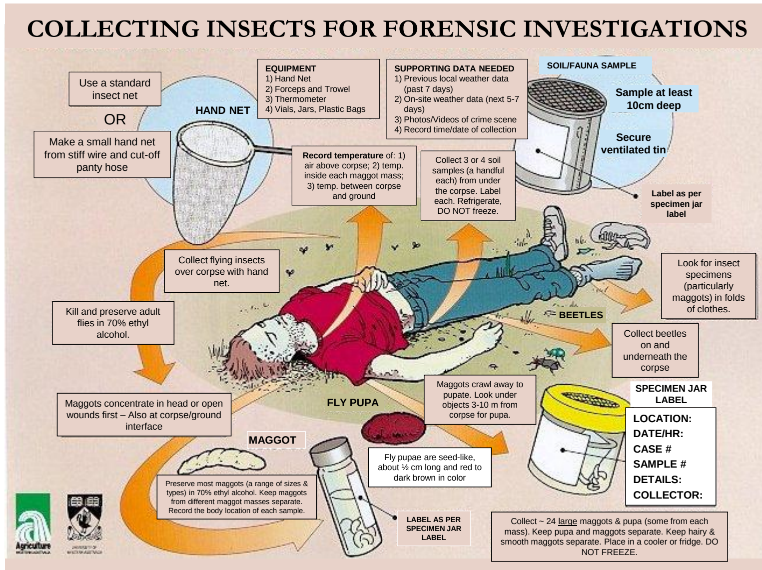

Figure 1. Schematic illustration of a forensic study

Water is essentially incompressible so it's density, ρ_w, can be taken as constant. There are three fundamental properties of soil depending upon the proportion of water and voids within a given sample of soil. These include the *porosity*, ϕ, the volumetric water content or *soil moisture*, θ and the *water saturation*, S_w, defined as

$$\phi = \frac{V_v}{V_r}, \qquad \theta = \frac{V_w}{V_r}, \qquad S_w = \frac{V_w}{V_v}$$

where V_r, V_v, V_w are the reference volume, volume of voids and volume of water respectively. From these definitions it is apparent that $\theta = S_w\phi$ and furthermore, that $0 \leq \phi \leq 1$ where $\phi = 0$ corresponds to an impermeable medium and $\phi = 1$ is essentially a fluid medium. With respect to the soil moisture, capillary forces hold the water in the pores against gravity essentially through water molecules being attracted to each other (cohesion) and attraction of water molecules to the walls of the pores (adhesion). These forces ensure that some residual moisture θ_0 remains in the water after gravitational forces have ceased. At the other extreme, if all the pores are filled then the corresponding moisture is called the saturation moisture θ_s. This gives the following natural hierarchy

$$0 < \theta_0 \leq \theta \leq \theta_s = \phi.$$

1.2. **Assumptions.**

- Soil is assumed to be an isotropic medium, notwithstanding that it is, in reality, highly heterogeneous.
- Pressure is assumed to be due solely to the overburden of homogeneous soil, and to capillary action, which is determined by the soil structure.
- Decomposition fluid is assumed to be incompressible, to have slow flow, and to have a low Reynolds number and therefore Darcy's Law to be used.
- Fluid mass is assumed to be constant.

2. Mathematical description of the problem

2.1. **Porous Medium.** Soil, fissured rock, cemented sandstone, limestone, sand, foam rubber, bread, concrete, bricks, paper towels, lungs and kidneys are just a few examples of the large variety of porous materials experienced in practice. All of these materials have common properties that lead us to classify them into a single class: porous media.

By porous medium we mean a material medium made of heterogeneous or multiphase matter. At least one of the considered phases is solid. The solid phase is usually called the solid matrix. The space within the porous medium domain that is not part of the solid matrix is named void space or pore space. The flow of one or more fluids occurs in the interconnected pores/voids through the material. In single phase flow the pores/voids are filled by a single fluid. In a complex situation (two phase flow) the pores/voids are occupied by gas and liquid phases.

2.1.1. *Porosity* ϕ. The presence of void space distributed within the solid matrix is characterized by the porosity of the porous medium. As explained in Section 1.1 the porosity ϕ is defined as the total void volume divided by the total volume occupied by the solid matrix and void volumes. Mathematically

$$\phi = \frac{V_v}{V_m},$$

where V_v and V_m are the volume of void space and the total volume of the material respectively. Pores may be connected to other pores in which case they are said to be interconnected. On the other hand some pores may appear in isolation, so that they are not connected to other pores. It is clear that flow will occur through the interconnected pores. We therefore define the effective porosity as

$$\phi_{\text{eff}} = \frac{V_{v,c}}{V_m},$$

where $V_{v,c} \leq V_v$ is the volume of the connected pores only.

2.2. **Pressure in the porous medium.** The general problem of fluid draining vertically into a porous medium is formulated in Bear [1] and is being analysed in this project for the case of a fixed volume release of Cadaver fluid into a porous soil structure around it. Consider the soil structure as shown in Figure 2. The $z-$coordinate is into the soil as shown. The soil medium is of porosity ϕ and permeability k and Cadaver fluid drains through this medium.

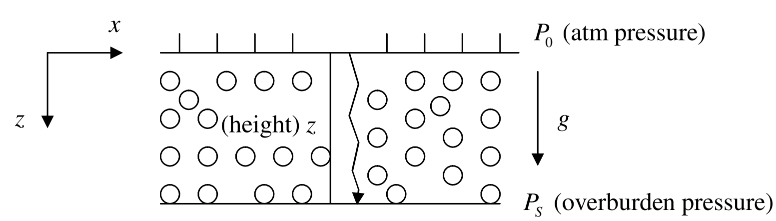

Figure 2. A sketch of a soil structure

The overburden pressure, P_s, which is the pressure felt at any height z in the porous medium by a fluid parcel, is

$$P_s = P_0 + \rho_s g z \tag{1}$$

where ρ_s is the density of soil, g the acceleration due to gravity and P_0 the atmospheric pressure. Also, the net fluid pressure driving the fluid flow in the porous medium is

$$P = P_s - P_e(\theta) \tag{2}$$

where $P_e(\theta)$ is the capillary pressure due to surface tension and adhesion that prevents the fluid flowing downwards. Substituting (2) into (1), we obtain

$$P = P_0 + \rho_s g z - P_e(\theta). \tag{3}$$

Typically the capillary pressure and the water pressure are rescaled by the factor $\rho_w g$ to give an effective height or *head*. The capillary head pressure is $\psi = P_c/\rho_w g$ and the water head pressure is $h = P_w/\rho_w g$. These concepts are combined under the assumption that the moisture of the soil depends on the pressure so that $\theta = \theta(P_w)$. If we assume that the soil is not deformed as the moisture moves through it then the porosity is not a function of the pressure. However this implies that $S_w = S_w(P_w)$ which is known as the *retention* curve of a given soil.

A heuristic form for the capillary pressure, $P_e(\theta)$, can be written as

$$P_e(\theta) = P_0 \left(\frac{\theta_0 - \theta}{\theta_0} \right), \tag{4}$$

where P_0 is atmospheric pressure and θ_0 is the residual soil moisture after gravity forces have ceased.

In equation (4), we see that an increase in the moisture content of the soil, θ, decreases the capillary pressure, P_e. This is as expected since the effect of gravity on the fluid becomes significant and fluid drains easily because of the increased weight of the fluid above fluid parcels trapped in pore spaces.

2.3. **Darcy's Law.** Fluid flow in porous medium is described by Darcy's law, which was formulated by Henry Darcy in 1856 while investigating water flow through beds of sand connected with the fountains of the city of Dijon, France.

We will derive Darcy's law by taking a porous medium of cross sectional area A and length L. Fluid will be made to flow through it at a rate $\vec{Q}$. When a steady state is achieved, the pressure gradient ∇P is related to $\vec{Q}$ by the formula

$$\vec{Q} = -\frac{A}{\mu} \vec{K} \cdot \nabla P, \tag{5}$$

where μ is the dynamic viscosity of the fluid and $\vec{K}$ is a second order permeability tensor which is independent of the fluid nature but depends on the geometry of the medium. If we define $\vec{v} = \vec{Q}/A$ as the Darcy velocity then we find that

$$\vec{v} = \frac{-\vec{K} \cdot \nabla P}{\mu}. \tag{6}$$

We deduced Darcy's law (6) by assuming steady flow of a Newtonian fluid that is only driven by a pressure gradient. In the case when the fluid is driven by other forces than the pressure gradient, we can include them in our analysis by replacing ∇P with the sum of all driven forces F per unit volume. The most common case encountered is a fluid driving by gravity $\vec{g}$ and pressure gradient for which Darcy's law may be written as

$$\vec{v} = -\frac{1}{\mu} \vec{K} \cdot \left[\nabla P - \rho \, \vec{g} \right]. \tag{7}$$

If the porous medium is isotropic, then permeability reduces to a scalar K and (6) simplifies to

$$\vec{v} = \frac{-K}{\mu} \nabla P. \tag{8}$$

In the case of a one dimensional flow $\vec{v} = u$, (8) reduces to

$$u = -\frac{K}{\mu} \frac{\partial P}{\partial x},$$

where $\partial P / \partial x$ is the pressure gradient in the flow direction.

2.4. **Mass Balance Equation.** Let us consider a control volume V located in a fluid flow field as shown in Figure 3, with boundary Ω. The law of conservation of mass for a homogeneous fluid with respect to the control volume is stated as

$$\text{(Rate of mass accumulation)} = \text{(Rate of mass in)} - \text{(Rate of mass out)} \tag{9}$$

The terms in the above equation can be expressed as follows. The rate of accumulation of mass in any volume dV is $\frac{\partial \rho_f}{\partial t} dV$, where ρ_f is the density of the fluid. The total rate of mass accumulation in the control volume V can be obtained by integrating $\frac{\partial \rho_f}{\partial t}$ over V,

$$\frac{\partial}{\partial t} \int_V \rho_f \, dV. \tag{10}$$

The rate at which mass flows across an infinitesimal surface $d\Omega$ in the control volume surface is equal to $\rho_f \vec{v} d\Omega \cos\vartheta$, where ϑ is the angle between the velocity vector $\vec{v}$ and the outward unit normal vector $\vec{n}$ to $d\Omega$. Mathematically mass efflux is

$$\begin{aligned} \rho_f \, \vec{v} \, d\Omega \, \cos\vartheta &= \rho_f \, d\Omega \, |\vec{v}| \, |\vec{n}| \, \cos\vartheta \\ &= \rho_f \, \vec{v} \cdot \vec{n} \, d\Omega. \end{aligned}$$

The rate of mass flowing in through $d\Omega$ is $-\rho_f \, \vec{v} \cdot \vec{n} \, d\Omega$, where the negative sign is because of the outward normal vector $\vec{n}$. The total net rate of mass influx into the control volume V can be obtained by integrating $-\rho_f \, \vec{v} \cdot \vec{n} \, d\Omega$ over the control volume surface Ω as

$$-\int_\Omega \rho_f \, \vec{v} \cdot \vec{n} \, d\Omega. \tag{11}$$

According to Gauss's divergence theorem, the surface integral (11) will be transformed into a volume integral as

$$-\int_\Omega \rho_f \, \vec{v} \cdot \vec{n} \, d\Omega = -\int_V \nabla \cdot (\rho_f \vec{v}) \, dV. \tag{12}$$

Substituting (10) and (12) into (9) gives

$$\int_V \left\{ \frac{\partial \rho_f}{\partial t} + \nabla \cdot (\rho_f \vec{v}) \right\} dV = 0,$$

$$\Rightarrow \qquad \frac{\partial \rho_f}{\partial t} + \nabla \cdot (\rho_f \vec{v}) = 0. \tag{13}$$

This equation is called the continuity equation.

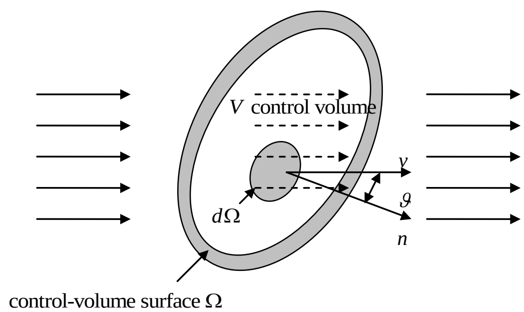

FIGURE 3. Control volume through which fluid flows

Now we will use equation (13) to derive the mass balance equation or continuity equation for porous media. For this let us multiply equation (13) by the porosity ϕ, which is assumed to be constant in space, so we have

$$\phi \frac{\partial \rho_f}{\partial t} + \nabla \cdot (\rho_f \phi \vec{v}) = 0. \tag{14}$$

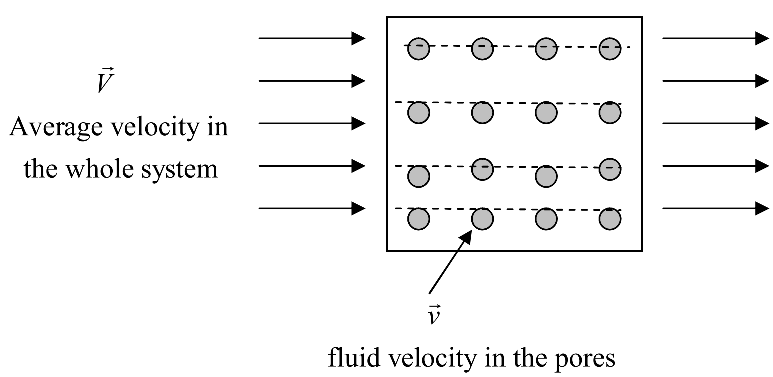

FIGURE 4. Fluid flow in porous media

According to the Dupuit-Forchheimer relationship $\vec{v} = \phi\vec{V}$, where $\vec{v}$ is the Darcy velocity (only in pores) and $\vec{V}$ is the average velocity of the fluid in the whole system (solid matrix and voids), then equation (14) will take the form

$$\phi \frac{\partial \rho_f}{\partial t} + \nabla \cdot (\rho_f \vec{v}) = 0. \tag{15}$$

Note that ϕ is independent of time. The above equation is known as the continuity equation for a porous medium.

2.5. **Permeability as a function of water content.** The empirical formula for the soil permeability, $K(\theta)$, is given as

$$K(\theta) = K_0 \left(\frac{\theta}{\theta_0}\right)^m \tag{16}$$

where $m > 0$ is a parameter that depends on the nature of soil and θ_0 is the water quantity that remains after gravity forces have ceased. Fig 2.5 shows the linear relationship that exist by plotting the log of permeability against log of porosity.

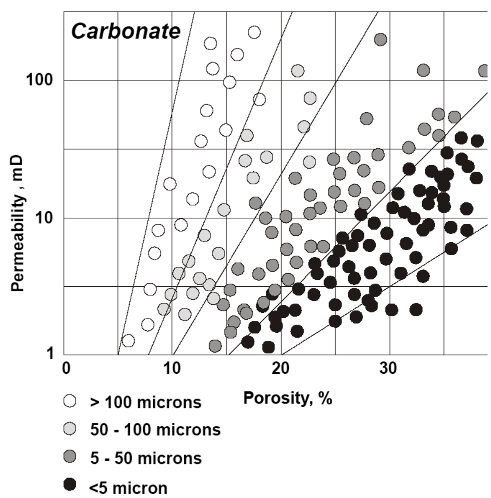

FIGURE 5. Porosity vs Permeability for different types of soil

2.6. **The complete model.** The resulting model we want to solve is

$$\frac{\partial \theta}{\partial t} + \nabla \cdot \vec{v} = 0, \tag{17}$$

$$\vec{v} = -\frac{1}{\mu} K(\theta)(\nabla P - \rho_l g \hat{e}_z), \tag{18}$$

$$P = P_0 + \rho_s g z - P_e(\theta). \tag{19}$$

Substituting equation (19) into (18) and the resulting into (17) we obtain

$$\frac{\partial \theta}{\partial t} - \frac{g}{\mu}(\rho_s - \rho_l) K'(\theta) \frac{\partial \theta}{\partial z} + \frac{1}{\mu} \nabla \cdot (K(\theta) P_e'(\theta) \nabla \theta) = 0. \tag{20}$$

Also, we substitute the expression for $P_e(\theta)$ and $K(\theta)$ from equations (4) and (16) into (20) to obtain

$$\frac{\partial \theta}{\partial t} - \frac{g}{\mu}(\rho_s - \rho_l) \frac{m}{\theta} K_0 \left(\frac{\theta}{\theta_0}\right)^m \frac{\partial \theta}{\partial z} - \frac{K_0}{\mu} \nabla \cdot \left(\left(\frac{\theta}{\theta_0}\right)^m \left(\frac{P_0}{\theta_0}\right) \nabla \theta \right) = 0, \tag{21}$$

where the parameter values are

$K_0 = 10^{-12}\, m^2,\ \mu = 10^{-3}\, Pa\, s,\ \rho_s = 1800\, kg\, m^{-3},\ \rho_l = 1000\, kg\, m^{-3},\ P_0 = 10^5\, Pa,\ \theta_0 = 0.4$

Now, we introduce dimensionless variables for space and time defined by

$$\hat{x} = \frac{x}{L} \quad \text{and} \quad \hat{t} = \frac{t}{T}, \tag{22}$$

where L and T are the characteristic length and time respectively. We choose the length scale to be $L = 1\,m$, which is reasonable considering we are looking at a cadaver, and we take T as

$$T = \frac{\mu L^2 \phi_0^{m+1}}{K_0 P_0} \approx 10^4 \text{ s}, \tag{23}$$

With such scales the dominant process is diffusion and, neglecting the hat, equation (21) reduces to

$$\frac{\partial \theta}{\partial t} - \nabla \cdot (\theta^m \nabla \theta) = 0. \tag{24}$$

3. Solving the model – Similarity solutions

Rewriting equation (24) in cylindrical coordinates, assuming axial symmetry, gives

$$\frac{\partial \theta}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(r\theta^m \frac{\partial \theta}{\partial r}) + \frac{\partial}{\partial z}(\theta^m \frac{\partial \theta}{\partial z}) \tag{25}$$

Equation (25) is in appearance very similar to the "stained rug" problem. Solving first this easier version of the problem, assume glass of wine is spilt onto a rug. Assuming that the wine is spilt all in one go and at the same spot (single point source) the equation that describes the liquid content on the rug, as a function of space (in radial coordinates, as it spreads in a circulare shape) and time, $\theta(r, t)$, is

$$\frac{\partial \theta}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(r\theta^m \frac{\partial \theta}{\partial r}) \tag{26}$$

The volume of fluid on the rug is always the same, which means

$$2\pi \int_0^\infty \theta(r, t) r\, dr = I_0 \tag{27}$$

where I_0 is a constant. As equation (26) is a diffusion equation in an unbounded region, we look for a similarity solution of the form

$$\theta(r, t) = t^{-\beta} f(\eta) \tag{28}$$

where $\eta = t^{-\alpha} r$. Substituting (28) into (26) and using (27) gives

$$\alpha = \frac{1}{2 + 2m}, \quad \beta = \frac{1}{m+1}$$

and

$$f(\eta) = \begin{cases} \left(\frac{m}{4(m+1)}\right)^{1/m} (\eta_0^2 - \eta^2)^{1/m} & |\eta| < \eta_0 \\ 0 & |\eta| \geq \eta_0 \end{cases}$$

where

$$\eta_0 = \left(I_0 \left(\frac{2(m+1)}{m}\right) \left(\frac{4(m+1)}{m}\right)^{1/m} \right)^{m/2(m+1)}.$$

Finally, we have a formula for $\theta(r, t)$

(29)

$$\theta(r, t) = \begin{cases} t^{-1/(m+1)} \left(\frac{m}{4(m+1)}\right)^{1/m} (\eta_0^2 - t^{-1/(m+1)} r^2)^{1/m} & |t^{-1/(2m+2)} r| < \eta_0 \\ 0 & |t^{-1/(2m+2)} r| \geq \eta_0 \end{cases}$$

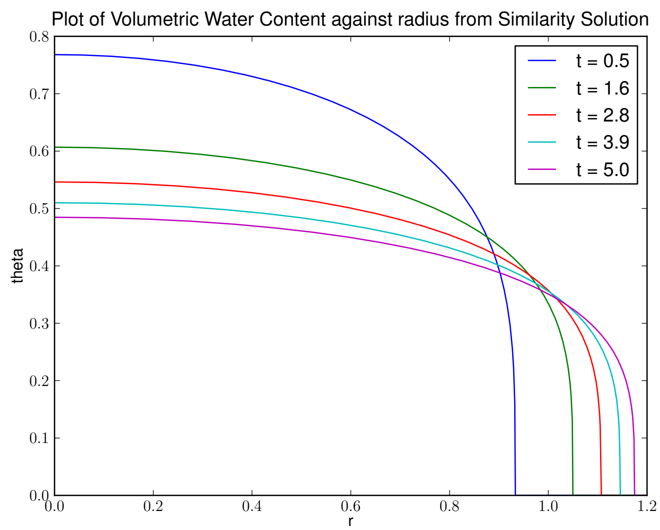

FIGURE 6. Evolution of the Volumetric Water Content with time

The position of the moving boundary (i.e. when $\eta = \eta_0$) is the key feature that gives this model predictive power. We can calculate the radius of the moving boundary as a function of time:

$$\bar{r}(t) = \eta_0 \; t^{1/(2m+2)}$$

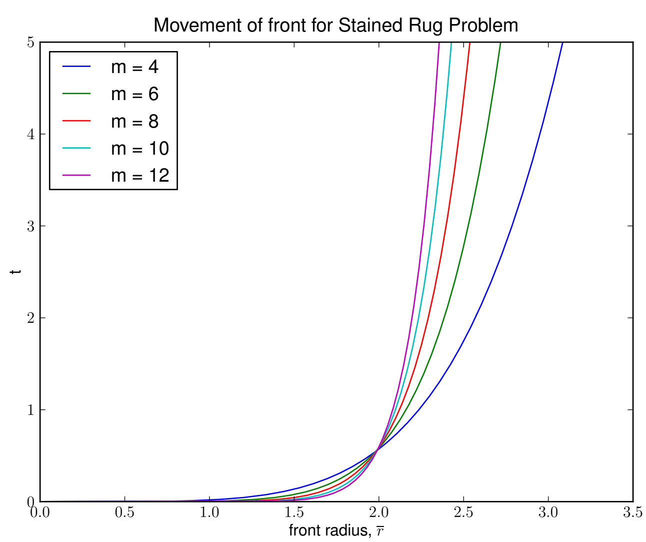

FIGURE 7. Evolution of the liquid front radius as a function of time

Clearly, the solution depends on m (i.e. on the nature of the soil). For different values of m, the slopes are different (see Figure 7) which means the predictive power is lower for longer times and for larger values of m.

Going back to the fluid propagation through soil problem, we would have wanted to find a solution for $\theta(r, z, t)$, as the fluids from a decomposing cadaver propagate through the soil also downwards and not only radially. However, we did not have enough time to find a similarity solution for equation (25).

4. Other considerations

4.1. **Initial and boundary conditions.** We have chosen to solve the pde analytically for some simple boundary and initial conditions: neither are very realistic. Typical initial conditions would be either specifying the initial moisture $\theta(\cdot, 0) = \theta_0$ or the pressure head $h(\cdot, 0) = h_0$. Appropriate boundary conditions are more complicated.

We could assume a Dirichlet boundary condition, a Neumann boundary condition or a flux, Robin-type, boundary condition. A Dirichlet condition would suggest that the the fluid pools on the surface of the soil before entering the bulk, or possibly, also, that there is some nearby open water such as a lake or stream. A Neumann condition would be in the case where we knew how much of the fluid went directly into the soil at a given time and place. Finally a flux condition would probably be most realistic. A flux condition would take the form $\vec{q} \cdot \hat{\nu} = q_n$ on some part Γ_q of the boundary of our domain. For an impermeable boundary we would have $q_n \equiv 0$. A flux condition could also be used to model the effects of rain fall.

Any of these conditions could be applied to (24) and then solved numerically.

4.2. **Modelling Body Decomposition.** To specify the flux q_n we need to investigate how the body decomposes. Full details can be found in [3]. We have assumed that we have a point source which injects all the fluid at time zero, however in the physical situation we have the fluid release is non-uniform in time and in space.

Decomposition of a cadaver begins at four hours after death and can take up to approximately 128 days for skeletonisation, although this figure depends upon temperature and ambient moisture content. Typically little fluid is released at the early stages which builds until the skin ruptures and then the amount of fluid released decreases again. Figure 8 shows an example decomposition if the cadaver is kept at a constant temperature and ambient moisture.

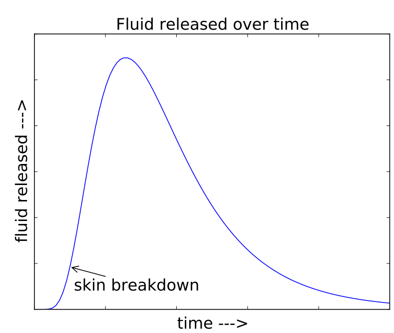

Figure 8. Amount of fluid released over time at constant temperature and ambient moisture

In the application we are looking at we would probably have diurnally periodic variation in this rate due to day-night temperature changes. Temperature and ambient moisture could vary from day to day with the local weather conditions.

Also the fluid does not leave all the body at all times so even a point source is a poor approximation. Since initially the fluid must escape the body through available orifices such as the eyes and mouth in the head and also though the anus. This effect is also extenuated by the effect of maggots which can only enter the body via the same orifices. This means initially fluid is released around the head and hips. Then as the skin breaks down the fluid released is more uniform. Parts of the body with high moisture content, for example the brain, and high enzyme content, for example the liver, breakdown more rapidly also.

4.3. **Soil Composition.** We have assumed that the soil is homogeneous in structure, but evidence from a small amount of data collected suggests that this is a poor choice. An experiment was performed in which a fixed amount of fluid was placed in a container on the top surface of the soil and allowed to flow into the bulk. The time it took for the fluid to enter the soil plus the size of the stain, depth and width, was measured. This test was performed at six times and three different sites within a field. Even when the test sites where chosen at almost the exact point, the data gives a large variation in soil properties. Across the field there was almost no correlation between the measurements.

The data suggests we should use a piecewise homogeneous model for the soil or a perhaps a probabilistic model. This would mean we would assume that different clumps of the soil would have a similar structure. Modelling the exact properties of the soil would be a major difficulty in developing a more precise model of this system.

5. Discussion

When set this problem we were asked several questions. We hope to answer these here:

- Can this stain be used to predict the time of death of the cadaver?
- How does this fluid propagate through the soil?
- Can an expression be for the expected concentration profile be found?
- Does a long time distribution profile exist?

If we are allowed to assume that the body is found on 'nice' homogeneous soil then our model could be developed into a way of predicting time of death. The important curves come from figure 7. This shows that if a forensic scientist can perform tests to calculate certain properties of the soil, K_0 and m, then they would have a curve in order to the time given the front radius. Measuring errors for this radius would mean that our model would work best for short time scales but could be used for longer time scales for working out the order of magnitude for the time of death.

If we take into account inhomogeneities in the soil, it would be difficult to extend our current model. The parameters we have chose to vary with the soil are difficult to measure experimentally so, in effect, it is very difficult to choose which curve one is using to match time of death to island radius. Also measuring the radius in natural soil would be difficult since typical stains would not be symmetric and often have a blurred edge due to other effects such as maggot infestations.

Our dimensional analysis has shown us that diffusion is the dominant effect since the convection due to gravity is insignificant. Solutions typically have a front, which we have used to create estimate curves for pmi. This analysis has only, however, been performed for the homogeneous soil and the inhomogeneities may well actually mean other effects are more important.

This does not compare too well to current forensic techniques. Errors for measuring post mortem interval using decompositional products can range from ± 2 days for soft tissue decay to ± 3 weeks using inorganics for skeletonized material. New, sophisticated hand-help devices are being planned which can be used at a crime scene to give an instant estimate of pmi also.

6. Acknowledgements

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