Calibration of single-factor HJM models of interest rates

Problem raised by Banco Santander

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Exposition of the problem:

The Heath-Jarrow-Morton model provides a framework for discussing arbitrage-free evolution of interest rate curves. Here we propose to explore a few issues around the calibration of interest rate models for risk management purposes.

The HJM framework is rich enough to encompass any interest-rate dynamics. However, for the purposes of fitting a model to historical data a sufficiently parsimonious model should be chosen. It can be argued that a single-factor HJM model (one with a single Brownian generator) is already rich enough for most practical purposes. Such a model is equivalent to a certain "short-rate" model, which has a more intuitive interpretation than the full curve dynamics it encodes. It can also be shown that a sufficiently rich variety of interest rate curves are possible under such a model - in particular, curves of the form proposed by Nelson and Siegel, while preserving the ability to interpret the short rate model intuitively.

Though the choice of single-factor models limits the possibility of overfitting, one can make Nelson-Siegel type curves arbitrarily complex and so one has to be careful not to introduce more parameters into the model than can be fitted reliably from actual data. Finally there is the issue of how much of the variance of historical data is explained by the model.

Scheme of the work to be done:

(Any or all of the following topics can be developed and taken as motivation for further work - throughout, issues of implementation and efficiency can be discussed)

1) Discussion of the difference between the asset pricing problem and the risk/portfolio management problem. Price of risk.

2) Discussion of the single-factor HJM model, and the relationship between the term volatility structure and the equivalent short rate model. Role of initial conditions and price of risk. Relationship with the Nelson-Siegel parametrization.

3) Descriptive statistics of a sample history of an interest rate curve. Stationarity, independence, normality, principal component analysis.

4) Fitting a single-factor model to the data. Choice of the number of parameters of the model. Goodness of fit and avoiding overfitting. Discussion of drift estimation, in particular statistical significance and stationarity.

5) Back-testing of fitted model: comparison with historical data using descriptive statistics (qualitative and quantitative).

6) Forecasting and scenario analysis with fitted model: simulation of future histories (Monte Carlo simulation); estimating price and risk (e.g VaR) of simple interest-rate instruments and/or portfolios.