IV UCM Modelling Week (14-22 June 2010)

Calibration of single-factor HJM models of interest rates

Problem raised by

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Time value of money

- Money today is worth more than the promise of money in the future.
- The price on date *t* of a monetary unit deliverable on a future date *T* is

$$P_t(T) = \exp\left[-\int_t^T f_t(S-t)\mathrm{d}S\right]$$

- where *f* is called the 'instantaneous forward interest rate'.
- The quantity $r_t = f_t(0)$ is the short-term interest rate or 'short rate', and

$$\frac{\mathrm{d}B_t}{B_t} = r_t \mathrm{d}t$$

- defines the growth of a 'current account' used as numeraire.

Heath-Jarrow-Morton Framework

- We want a model that is as simple as possible but no simpler than necessary
  - as few underlying stochastic factors as possible
  - consistent with absence of arbitrage opportunities (‘no free lunch’ principle)
  - can potentially accommodate any observed term structure of interest rates
  - stationary
- The answer is the Heath-Jarrow-Morton model

$$\mathrm{d}f_t(T-t) = \sigma_t(T-t)\left[\Sigma_t(T-t)\mathrm{d}t + \mathrm{d}\tilde{W}_t\right]$$

- where

$$\Sigma_t(\tau) := \int_0^\tau \sigma_t(v)\mathrm{d}v$$

Short-rate models

- Under no-arbitrage hypotheses:
  - Bond prices satisfy

$$P_t(T) = \mathrm{E}\left[\exp\left(-\int_t^T r_s \mathrm{d}s\right)\right]$$

  - HJM implies that

$$r_t = f_0(t) + \frac{1}{2}\Sigma^2(t) + \int_0^t \sigma(t-s)\mathrm{d}\tilde{W}_s$$

- Conversely, if a stochastic differential equation for the short rate is given, this defines a HJM model.

Forward-curve fitting

- The forward curve satisfies

$$f_t(\tau) = f_0(t+\tau) + \frac{1}{2}\left[\Sigma^2(\tau+t) - \Sigma^2(\tau)\right] + \int_0^t \sigma(\tau+t-s)\,\mathrm{d}\tilde{W}_s$$

- If the volatility is in the 'exponential-polynomial family' so is the forward curve
- Some examples of this are known from the literature:

$$f_t(\tau) = \begin{cases} a_t + (b_t + c_t\tau)e^{-\alpha_t\tau} & \text{Nelson–Siegel} \\ a_t + (b_t + c_t\tau)e^{-\alpha_t\tau} + d_t\tau e^{-\beta_t\tau} & \text{Svensson} \end{cases}$$

- We propose to fit an exponential-polynomial volatility structure to bond market data, and work from there using the HJM framework.

Suggested research

- Discussion of the difference between the asset pricing problem and the risk/portfolio management problem. Price of risk.
- Discussion of the single-factor HJM model, and the relationship between the term volatility structure and the equivalent short rate model. Role of initial conditions and price of risk. Relationship with the Nelson-Siegel parametrization.
- Descriptive statistics of a sample history of an interest rate curve. Stationarity, independence, normality, principal component analysis.
- Fitting a single-factor model to the data. Choice of the number of parameters of the model. Goodness of fit and avoiding overfitting.
- Discussion of drift estimation, in particular statistical significance and stationarity.
- Back-testing of fitted model: comparison with historical data using descriptive statistics (qualitative and quantitative).
- Forecasting and scenario analysis with fitted model: simulation of future histories (Monte Carlo simulation); estimating price and risk (e.g VaR) of simple interest-rate instruments and/or portfolios.