Departamento de Matemática Aplicada

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Complete quenching for a quasilinear and singular parabolic equation

Abstract:

We present here some results extracted from [1], where the following quasilinear and parabolic problem is investigated:

$$(\mathrm{P})\left\{\begin{array}{rcll} \partial_t u - \Delta_p u + \mathbb{1}_{\{u>0\}} u^{-\beta} & = & f(x,u) & \text{in } (0,T)\times\Omega, \\ u & = & 0 & \text{on } (0,T)\times\partial\Omega, \\ u(0,\cdot) & = & u_0 & \text{in } \Omega. \end{array}\right.$$

In this problem, $T>0$, Ω is a smooth and bounded domain of $\mathbb{R}^d$ with $d\geq 1$, $\Delta_p u \stackrel{\text{def}}{=} \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian operator and $\mathbb{1}_{\{u>0\}}$ is the indicator function of the set

$$\{(t,x)\in(0,T)\times\Omega,\ u(t,x)>0\},$$

with the natural convention that $\mathbb{1}_{\{u>0\}}u^{-\beta}=0$ as $u=0$. The exponent β is fixed in $(0,1)$ and the initial datum satisfies

$$u_0 \in \mathrm{W}_0^{1,p}(\Omega)\cap \mathrm{L}^\infty(\Omega) \quad \text{and} \quad u_0\geq 0 \quad \text{a.e. in } \Omega.$$

In the right-hand side of the first equation, $f:\Omega\times\mathbb{R}\to\mathbb{R}$ is a Carathéodory function which is locally Lipschitz continuous with respect to the second variable and satisfies an appropriate growth condition.

First, we prove the existence of a solution of a such problem. For this, we start by introducing and studying a regularized problem (P_ε), where the singular absorption term $\mathbb{1}_{\{u>0\}}u^{-\beta}$ is suitably approached. By using a semi-discretization in time, we prove the existence, the uniqueness as well as some useful energy estimates for the solution of (P_ε). Thanks to a monotonicity argument, we prove the existence of a solution of (P) passing to the limit as $\varepsilon\to 0$ in the regularized problem (P_ε).

In a second time, we focus on the asymptotic behaviour of the solutions of problem (P). More precisely, we prove that the solutions of (P) vanish in a finite time in the whole domain Ω; and this even if the initial datum is positive in Ω. For this, we prove that the mapping $t\mapsto \|u(t)\|_{L^2(\Omega)}$ (where u is a solution of (P)) satisfies a differential inequality, deriving from an energy estimate on u together with a Gagliardo-Nirenberg-type inequality.

[1] J. GIACOMONI, P. SAUVY, S. SHMAREV, Quenching phenomenom for a quasilinear and singular parabolic problem.

Organizado por el grupo de investigación MOMAT y el IMI

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