

# A DAY IN FUNCTION SPACES AND INTERPOLATION THEORY

*Wednesday, September 17, 2014*

Organized by F. Cobos and L.M. Fernández-Cabrera

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Departamento de Analisis Matemático  
Universidad Complutense de Madrid

**Seminario 222**

10:00 - 10:50

# Global solutions of Navier-Stokes equations for large initial data belonging to distinguished function spaces

Hans Triebel

University of Jena (Germany)

The classical Navier-Stokes equations in  $\mathbb{R}^n$ ,  $n \geq 2$ , can be reformulated as

$$\begin{aligned}\partial_t u - \Delta u + \mathbb{P} \operatorname{div} (u \otimes u) &= 0 && \text{in } \mathbb{R}^n \times (0, \infty), \\ u(\cdot, 0) &= u_0 && \text{in } \mathbb{R}^n,\end{aligned}$$

where  $u(x, t) = (u^1(x, t), \dots, u^n(x, t))$  is the unknown velocity,  $\partial_t = \partial/\partial t$ ,  $\partial_j = \partial/\partial x_j$  with  $j = 1, \dots, n$  and

$$\operatorname{div} (u \otimes u)^k = \sum_{j=1}^n \partial_j (u^j u^k). \quad k = 1, \dots, n.$$

$\mathbb{P}$  is the Leray projector,

$$(\mathbb{P}f)^k = f^k + R_k \sum_{j=1}^n R_j f^j, \quad k = 1, \dots, n,$$

based on the (scalar) Riesz transforms  $R_k$ ,

$$R_k g(x) = \lim_{\varepsilon \downarrow 0} \int_{|y| \geq \varepsilon} \frac{y_k}{|y|^{n+1}} g(x-y) dy, \quad x \in \mathbb{R}^n.$$

We ask for solutions  $u(x, t)$  global in time in (vector-valued) spaces  $u(\cdot, t) \in A_{p,q}^s(\mathbb{R}^n)_n$  of Besov-Sobolev type for suitably given initial data  $u_0 \in A_{p,q}^\sigma(\mathbb{R}^n)_n$ ,  $\sigma < s$ . We are mainly interested in the interplay between multiplication algebras for  $A_{p,q}^s(\mathbb{R}^n)$ , Haar expansions for  $A_{p,q}^\sigma(\mathbb{R}^n)$  and representations of some homogeneous spaces  $\dot{A}_{p,q}^\sigma(\mathbb{R}^n)$  in terms of heat kernels.

11:00 - 11:50

## Characterizations of Periodic Besov Spaces of Measurable Functions

Hans-Jürgen Schmeisser

University of Jena (Germany)

The classical approach to periodic Besov spaces  $B_{p,q}^s(\mathbb{T}^d)$  based on differences (moduli of continuity) has been extended to the case  $0 < p < 1$ ,  $0 < q \leq \infty$ , and  $s > 0$ . If  $s > d(\frac{1}{p} - 1)$  then these function spaces can be considered as subspaces in  $D'(\mathbb{T}^d)$ , the space of periodic distributions, and various equivalent characterizations are well-known.

In this talk we focus on the case  $0 < s \leq d(\frac{1}{p} - 1)$ . It is our aim to present equivalent characterizations based on

- polynomial  $\mathcal{K}$ -functionals
- general moduli of smoothness

and

- constructive approximation methods.

This is joint work with K. Runovski (Sevastopol).

12:00 - 12:50

## Embeddings of weighted spaces of Besov and Morrey type

Dorothee D. Haroske

University of Jena (Germany)

We study embeddings of weighted spaces of Morrey type,  $\mathcal{M}_{u,p}$ ,  $0 < p \leq u < \infty$ , and spaces of Besov-Morrey type,  $\mathcal{N}_{u,p,q}^s$  and  $B_{p,q}^{s,\tau}$ ,  $s \in \mathbb{R}$ ,  $\tau \geq 0$ ,  $0 < q \leq \infty$ , all defined on  $\mathbb{R}^n$ . Mainly we consider weights from the Muckenhoupt class  $\mathcal{A}_\infty$ , with  $w_\alpha(x) = |x|^\alpha$ ,  $\alpha > -n$ , as typical example, and obtain necessary and sufficient conditions for such an embedding. This is joint work with Leszek Skrzypczak (Poznań), Wen Yuan (Beijing) and Dachun Yang (Beijing).

15:30 - 16:20

## Compact embeddings of weighted Sobolev spaces

Therese Mieth

University of Jena (Germany)

Let  $B$  be the unit ball in  $\mathbb{R}^n$  and  $\psi$  be a continuous, slowly varying function on  $(0, 1]$  with  $\psi(1) = 1$  and bounded from below.

We define the weighted Sobolev space  $E_{p,\psi}^m(B)$ ,  $1 \leq p < \infty$ ,  $m \in \mathbb{N}$ , as the completion of  $C_0^m(B) = \{f \in C^m(B) : \text{supp } f \text{ compact}\}$  in the norm

$$\|f\|_{E_{p,\psi}^m(B)} := \left( \int_B |x|^{mp} \psi^p(|x|) \sum_{|\alpha|=m} |D^\alpha f(x)|^p dx \right)^{1/p}.$$

Then the continuous embedding

$$\text{id} : E_{p,\psi}^m(B) \hookrightarrow L_p(B)$$

is compact if, and only if,  $\lim_{t \rightarrow 0} \psi(t) = \infty$ . We investigate the influence of the growth rate of  $\psi$  on the compactness of the embedding, measured in terms of entropy and approximation numbers.

16:30 - 17:20

## Compact operators interpolated under logarithmic methods

Fernando Cobos

Universidad Complutense de Madrid

In 1960 Krasnosel'skiĭ proved a reinforced version of the Riesz-Thorin theorem, to the effect that, in the usual hypotheses on the exponents and the additional assumption  $q_0 < \infty$ , if  $T$  is a linear operator such that  $T : L_{p_0} \longrightarrow L_{q_0}$  compactly and  $T : L_{p_1} \longrightarrow L_{q_1}$  boundedly, then  $T : L_p \longrightarrow L_q$  is also compact.

Very recently Edmunds and Opic have established a limiting variant of Krasnosel'skiĭ theorem for finite measure spaces showing that compactness of  $T$  is preserved when  $T$  acts between Lorentz-Zygmund spaces which are very close to  $L_{p_0}$  and  $L_{q_0}$ .

In this talk we show abstract versions for Banach couples of the results of Edmunds and Opic. The talk is based on results of joint papers with L.M. Fernández-Cabrera and A. Martínez and with A. Segurado.

17:30 - 18:20

## Problem session