SMOOTH LINEAR CONGRUENCES OF LINES

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A *congruence of lines* in $\mathbb{P}^N$ is an $(N-1)$-dimensional subvariety of the Grassmann variety. The *order* $o(\Sigma)$ of a congruence $\Sigma \subset G(1,N)$ is the number of its lines passing through a general point of $\mathbb{P}^N$. The points of $\mathbb{P}^N$ through which there pass infinitely many lines of a congruence Σ form its *fundamental locus* $X(\Sigma)$. When $o(\Sigma) = 1$, the fundamental locus has a natural schème structure. As an example, we see that the 2-secant lines to a twisted cubic curve form a congruence of order 1 whose fundamental locus is set-theoretically but not scheme-theoretically the twisted cubic.

A congruence of order 1 is *linear* if it is cut in $G(1,N)$ by a linear subspace of the ambient Plücker space.

We study here smooth linear congruences of order 1 whose fundamental locus is smooth and connected. We call them twice smooth linear congruences. We prove that the lines of a twice smooth linear congruence Σ are the k-secants to $X(\Sigma)$ with $k = (N-1)/(N-dim(X)-1)$. We define k as the secant index of Σ.

Twice smooth linear congruences with secant index 1 or 2 are described by elementary means. We note (this is the origin of this work) that there exist precisely four families with secant index 3 (there fundamental loci are the celebrated projected Severi Varieties), that two families with secant index 4 are known (there fundamental loci are Palatini varieties of dimension 3 and 6). We do not know any twice smooth linear congruence with secant index ≥ 5.

As a conclusion we note with amusement that any new twice smooth linear congruence would be a counter-example to the celebrated conjecture of Hartshorne concerning low codimension varieties.